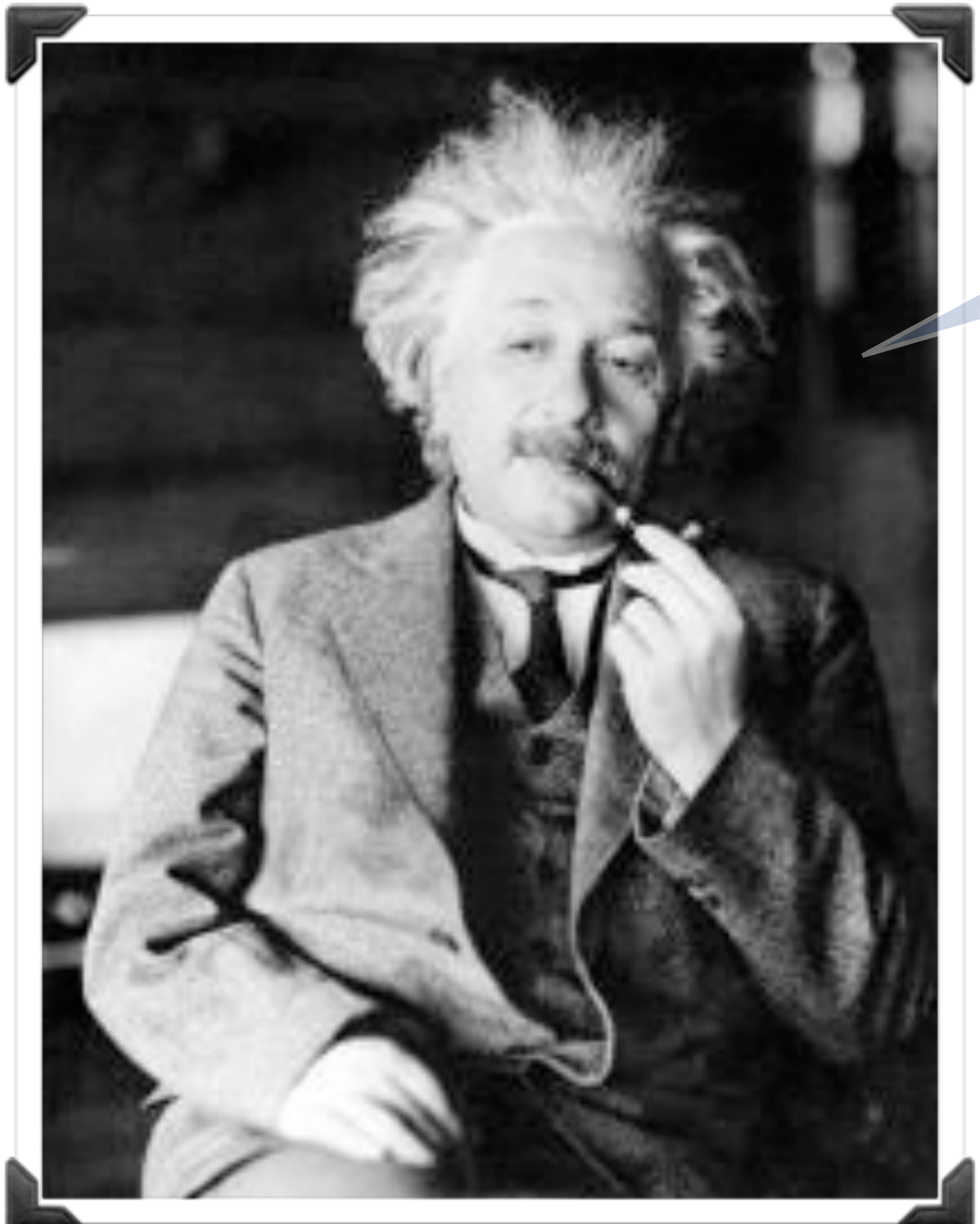


Welcome back to PHY 3305

Today's Lecture:

General Relativity

Albert Einstein
1879-1955

SPECIAL RELATIVITY

In 1905 Einstein postulated:

1. The laws of physics are the same in all inertial frames.

2. The speed of light is the same in all inertial frames

Why inertial frames only! Why don't the laws of physics hold in ALL frames.

The answer is that they can.

General Relativity is the generalization of relativity to noninertial reference frames.

GENERAL RELATIVITY

- The theory that describes gravity.
- Gravity is the geometry of 4-D spacetime.
- Basis of our understanding of the Big Bang, black holes, quasars, the cycle of stars and the evolution of the universe.
- Requires mathematics beyond the scope of this course.
- Einstein's motivation for the development of GR was not to explain any experimental enigma, but to describe all natural phenomena within the framework of special relativity.

GENERAL RELATIVITY

- Took 10 years to develop the mathematics
  - We won't do any of it here
- Published in 1916
- Special relativity told us that space and time are closely related $\Rightarrow$ spacetime
- General relativity tells us that spacetime is shaped by mass and energy, and similarly, mass and energy are "pushed" around by spacetime

REVISIT NEWTONIAN MECHANICS

Newton's Law of Universal Gravitation (gravity):

$$F_g = \frac{GM_E}{r_E^2}m \;\; = mg$$

If we are close to the earth, $g = 9.8\ m/s^2$.

Newton's 2nd Law of Motion (inertia):

$$\vec{a} = \vec{F}_{net}\frac{1}{m}$$

Are the masses in both these equations the same?

$$m_g \stackrel{?}{=} m_i$$

Yes, it has been experimentally verified to better than 1 part in 10^{12}.

THE PRINCIPLE OF EQUIVALENCE

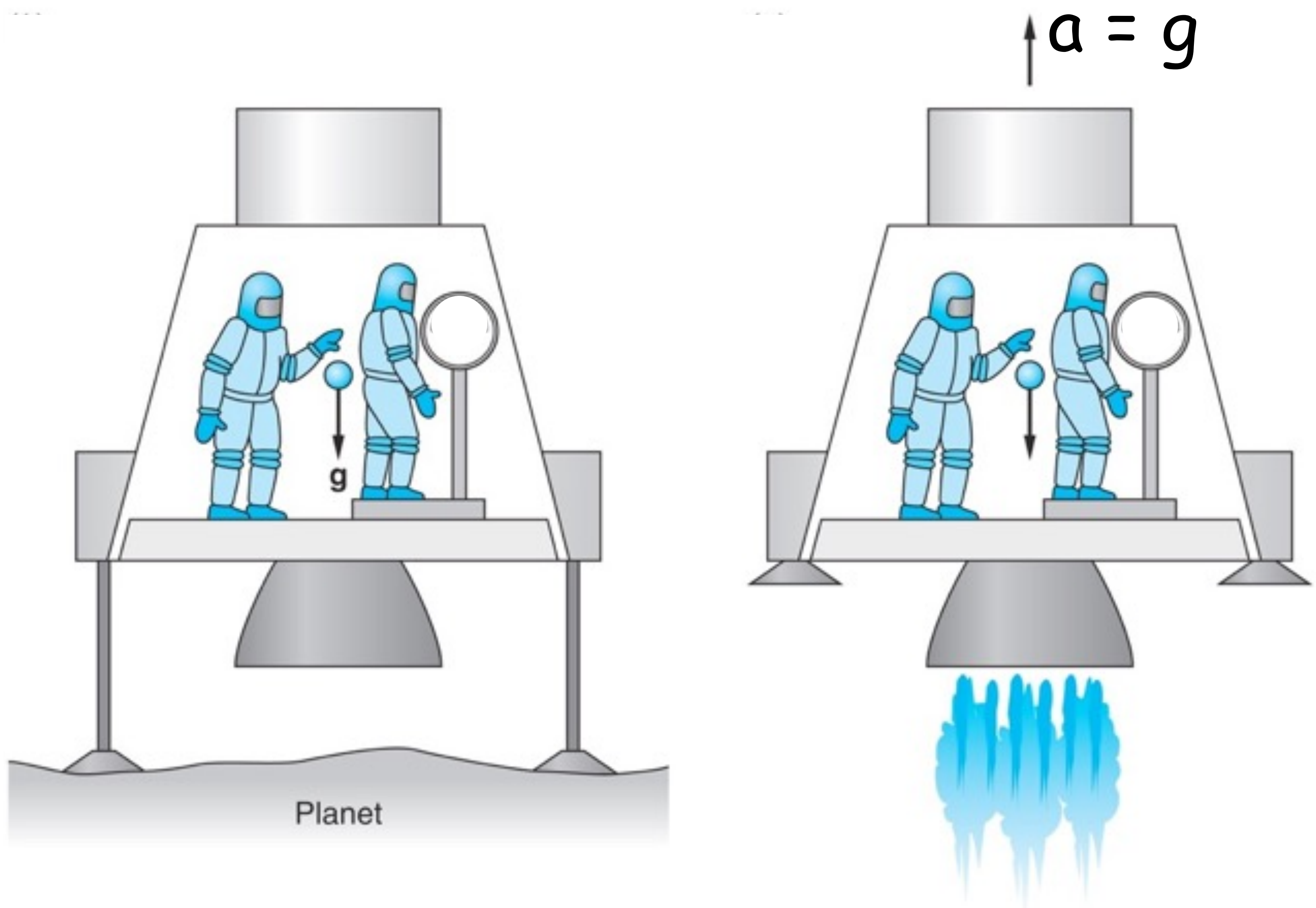
Einstein postulated:

A homogenous gravitational field is completely equivalent to a uniformly accelerated reference frame.

-or-

The form of each physical law is the same in all locally inertial frames.

PRINCIPLE OF EQUIVALENCE



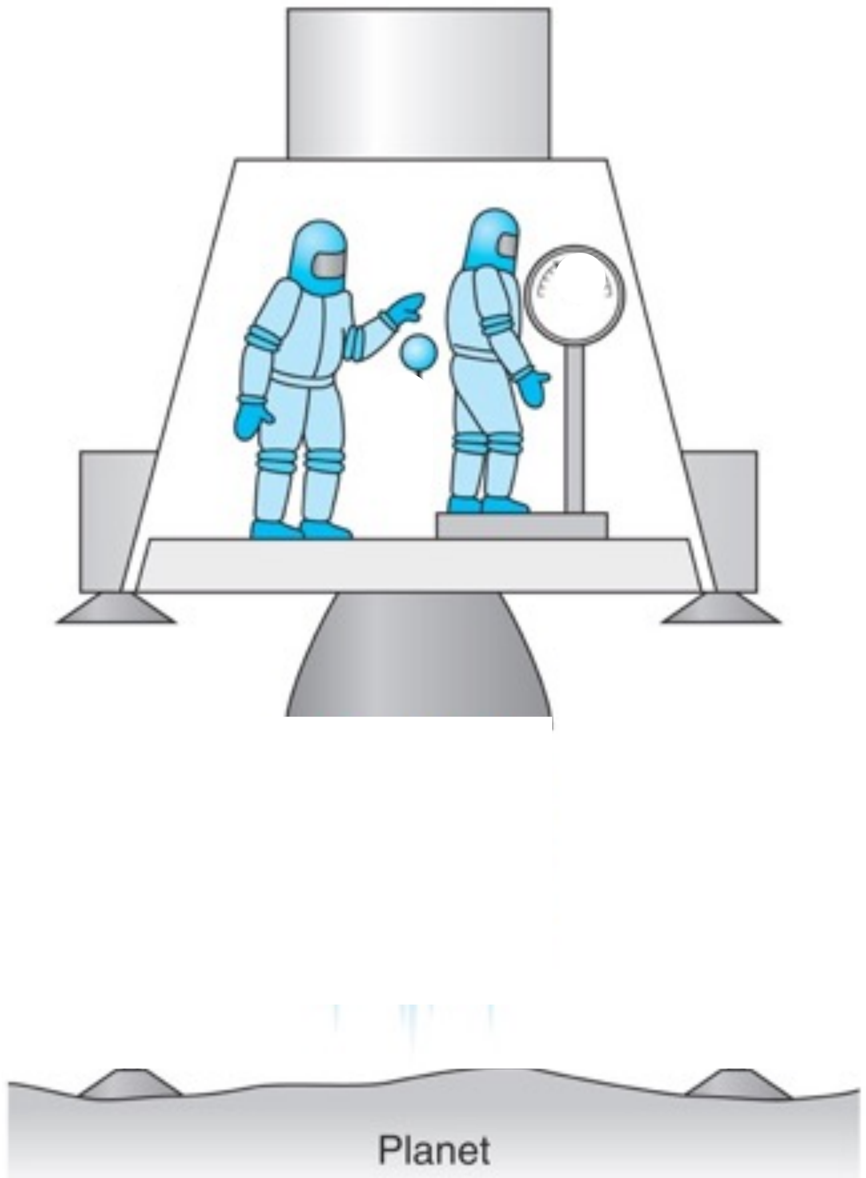
Can we tell the difference between these two frames ?

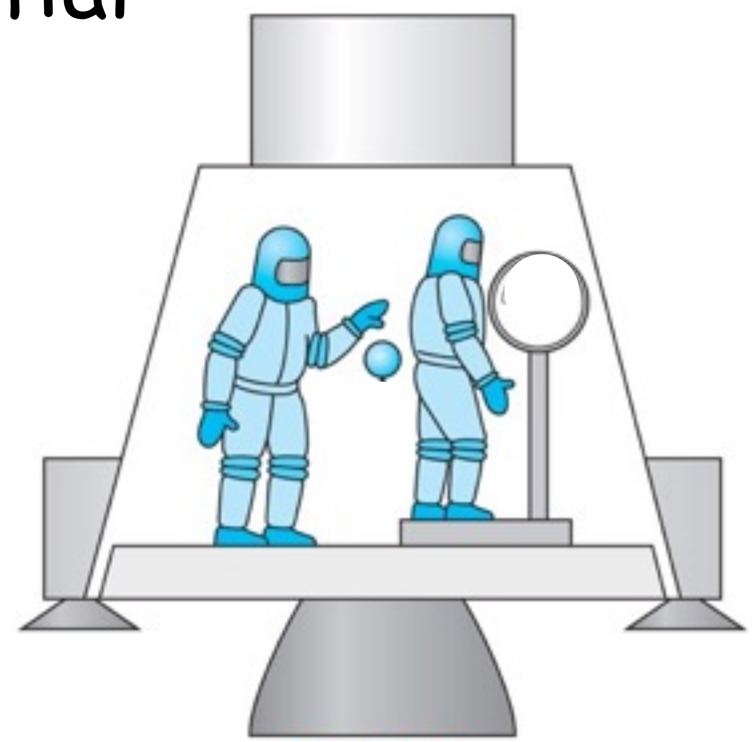
No, the normal force is the same in both situations.

PRINCIPLE OF EQUIVALENCE

free-fall

inertial

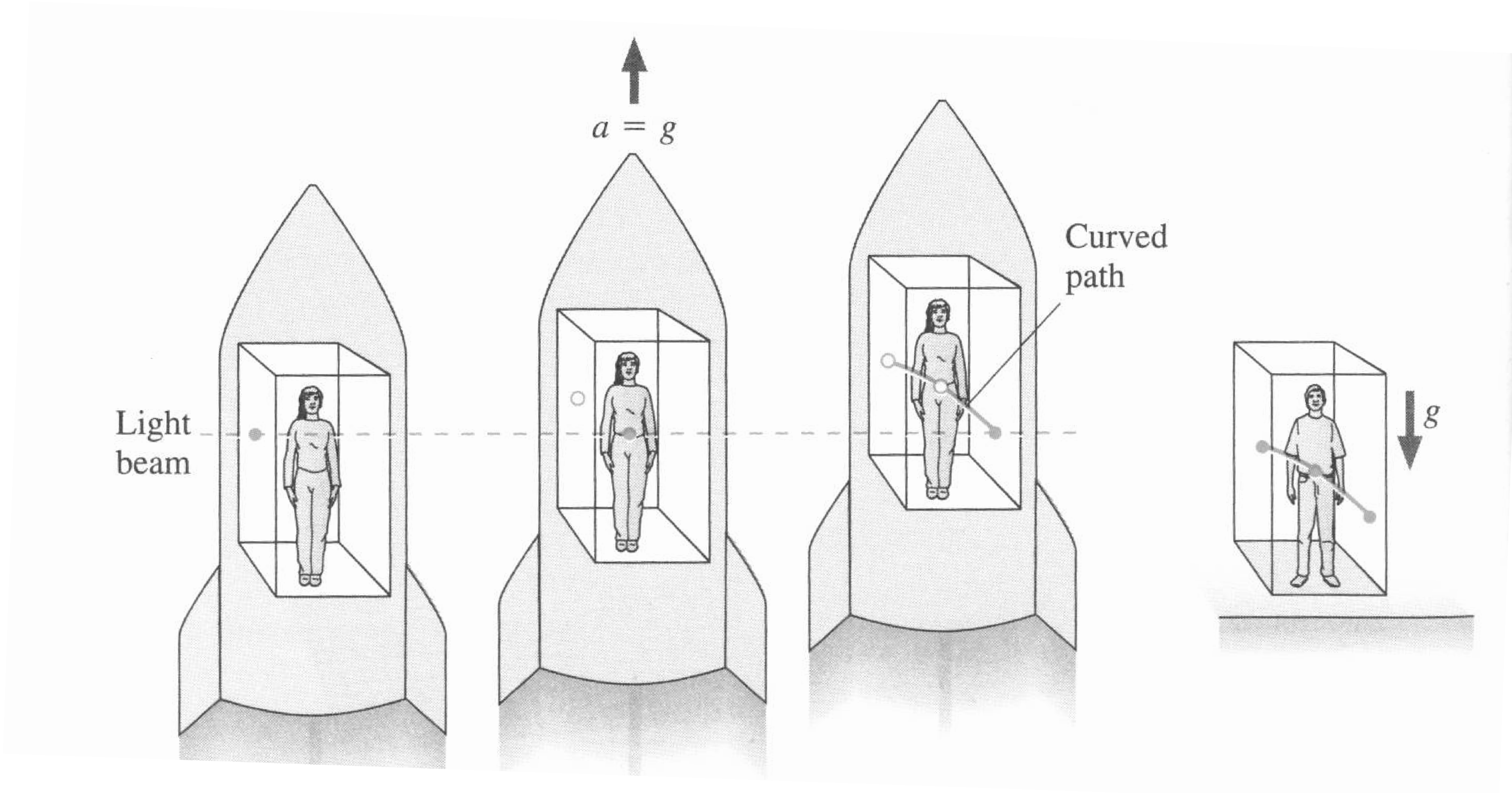




Can we tell the difference between these two frames?

No, in this case the observer would see all objects floating or moving at constant speed because no forces act.

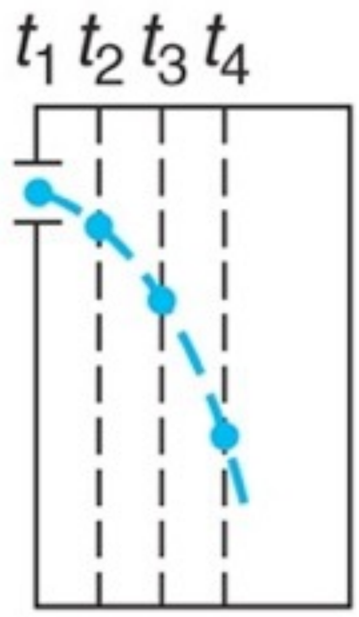
Deflection of Light in a Gravitational Field



Observing a beam of light in an accelerated reference frame.

DEFLECTION OF LIGHT IN A GRAVITATIONAL FIELD

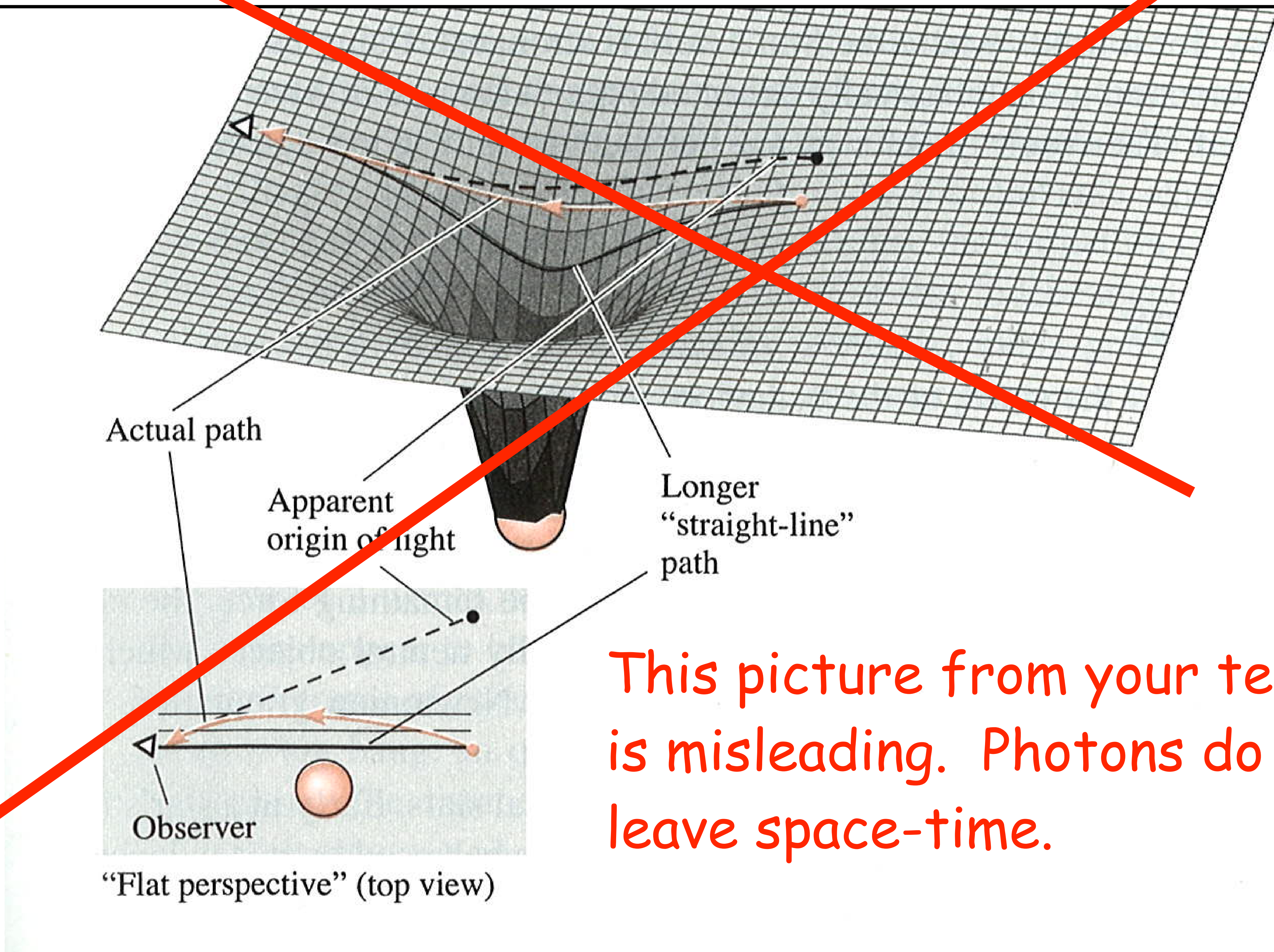
- In an accelerating frame a light beams appears to be “falling down” in a parabolic arc



- According the the principle of equivalence: there is no physical difference between an accelerating frame and a gravitational field.

- Therefore a light beam will “fall down” in a gravitational field

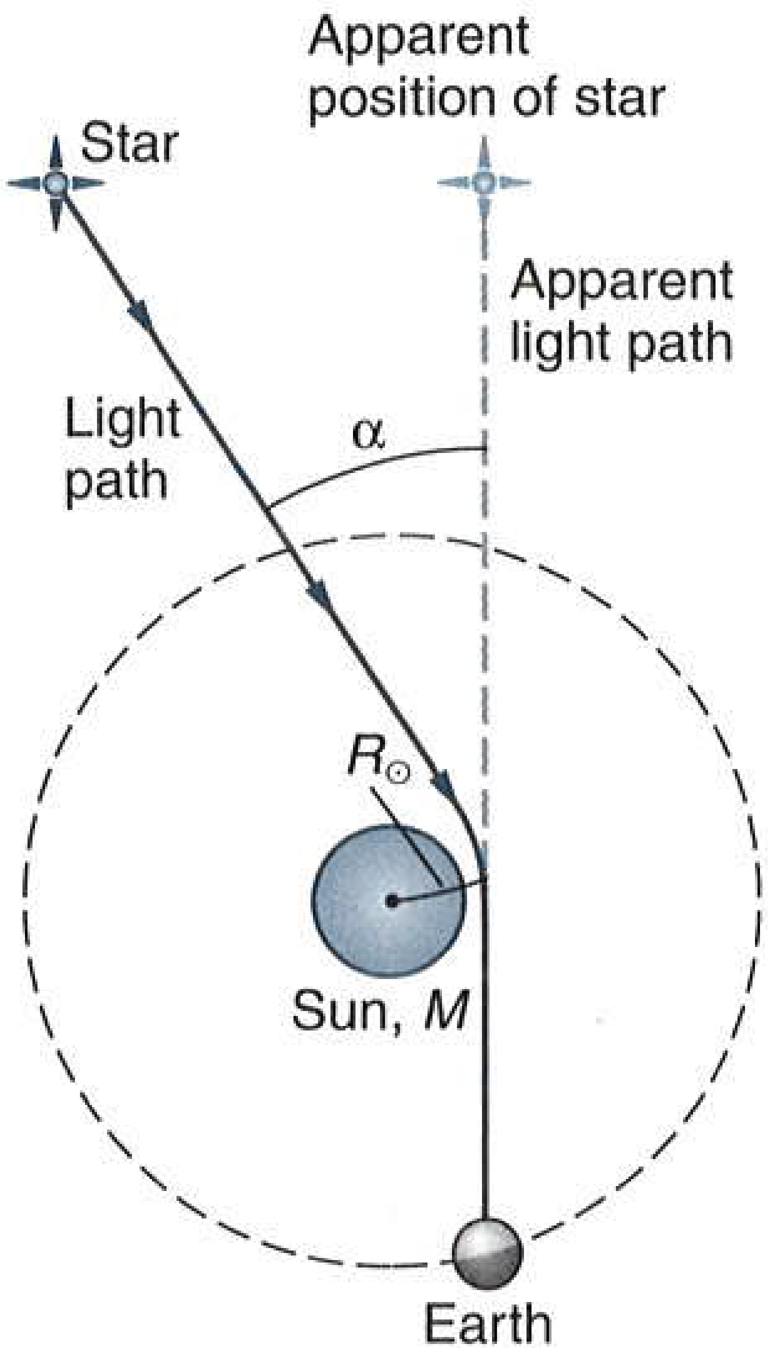
That's an 'ok' point of view, but a better answer is that light will take the shortest path (pink line).



"Flat perspective" (top view)

This picture from your textbook is misleading. Photons do not leave space-time.

A Better Picture:

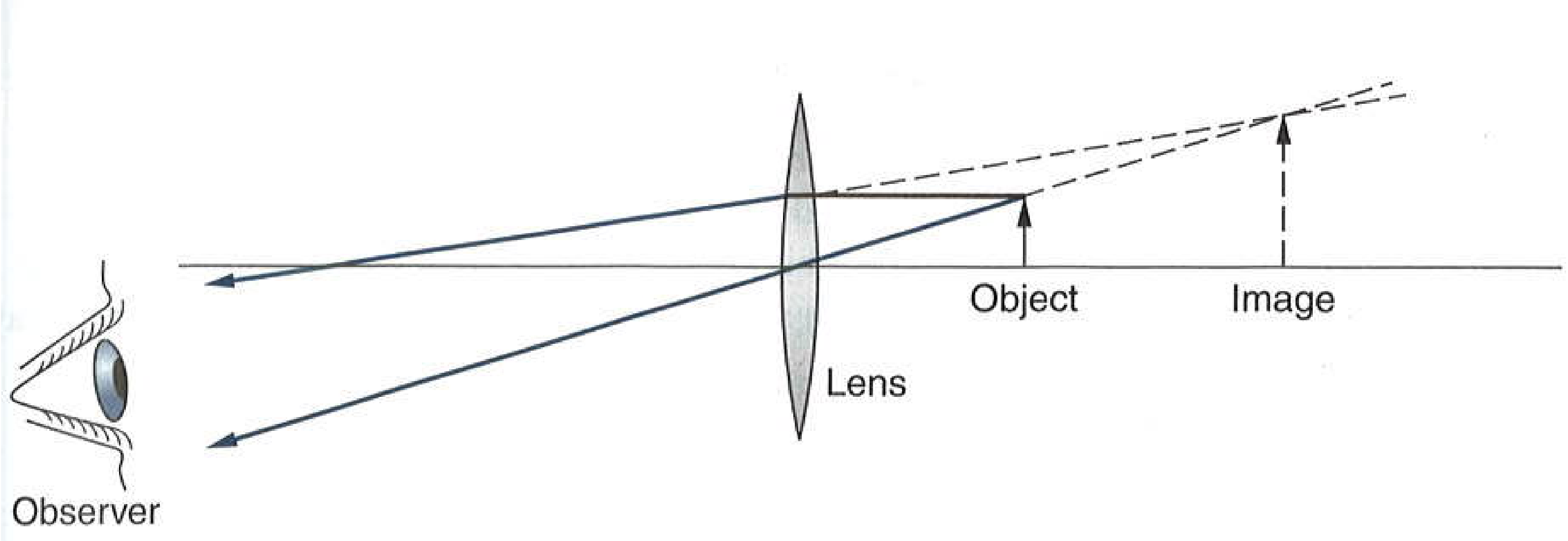


Note: This picture is an extreme exaggeration of the effect known as "Gravitational Lensing"

If you envision space-time as a fabric -- photons can not leave the fabric.

Why is this called Gravitational Lensing?

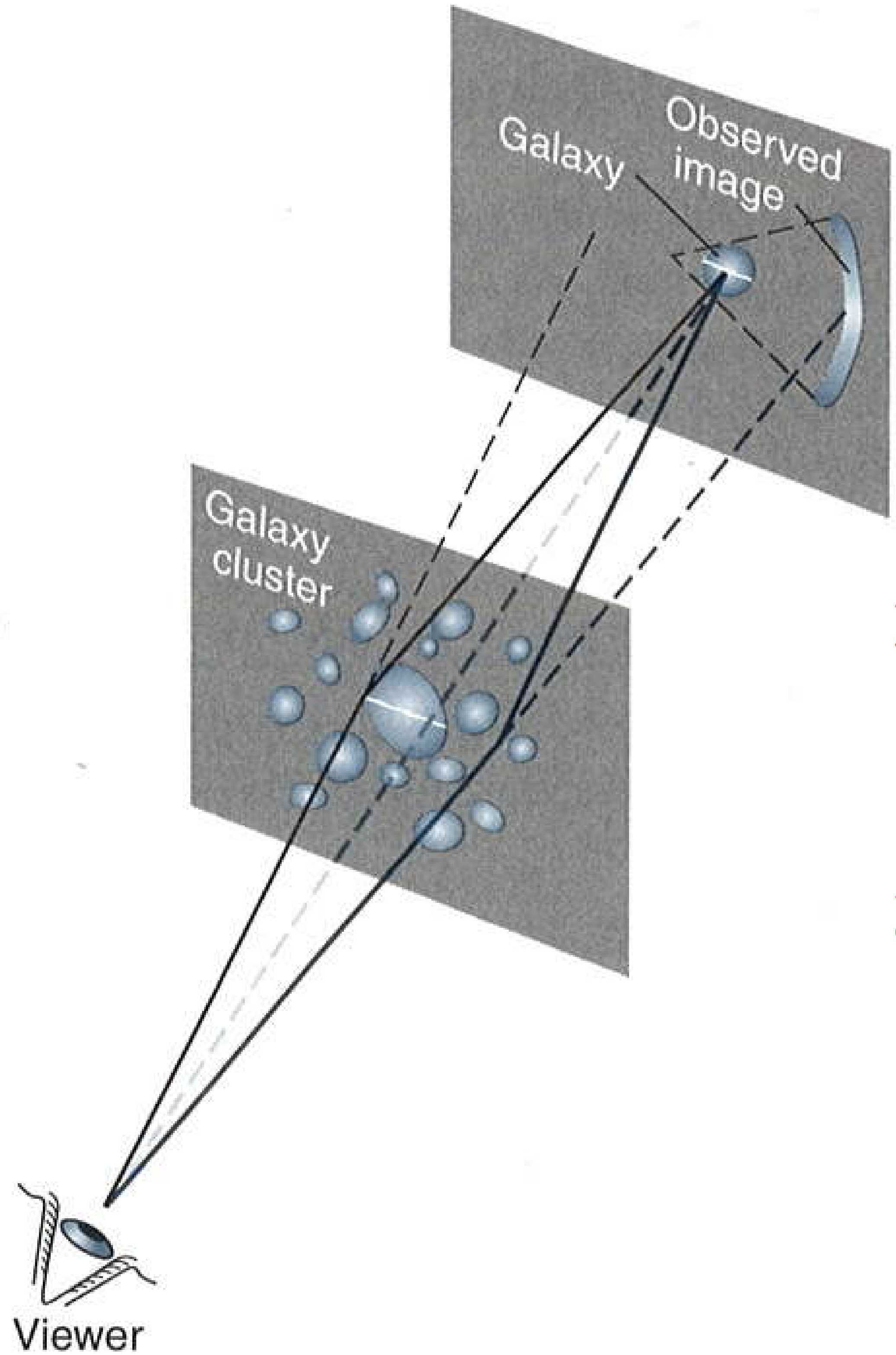
Recall how a lens works:



What do we know about the relationship between the apparent image and the actual object?

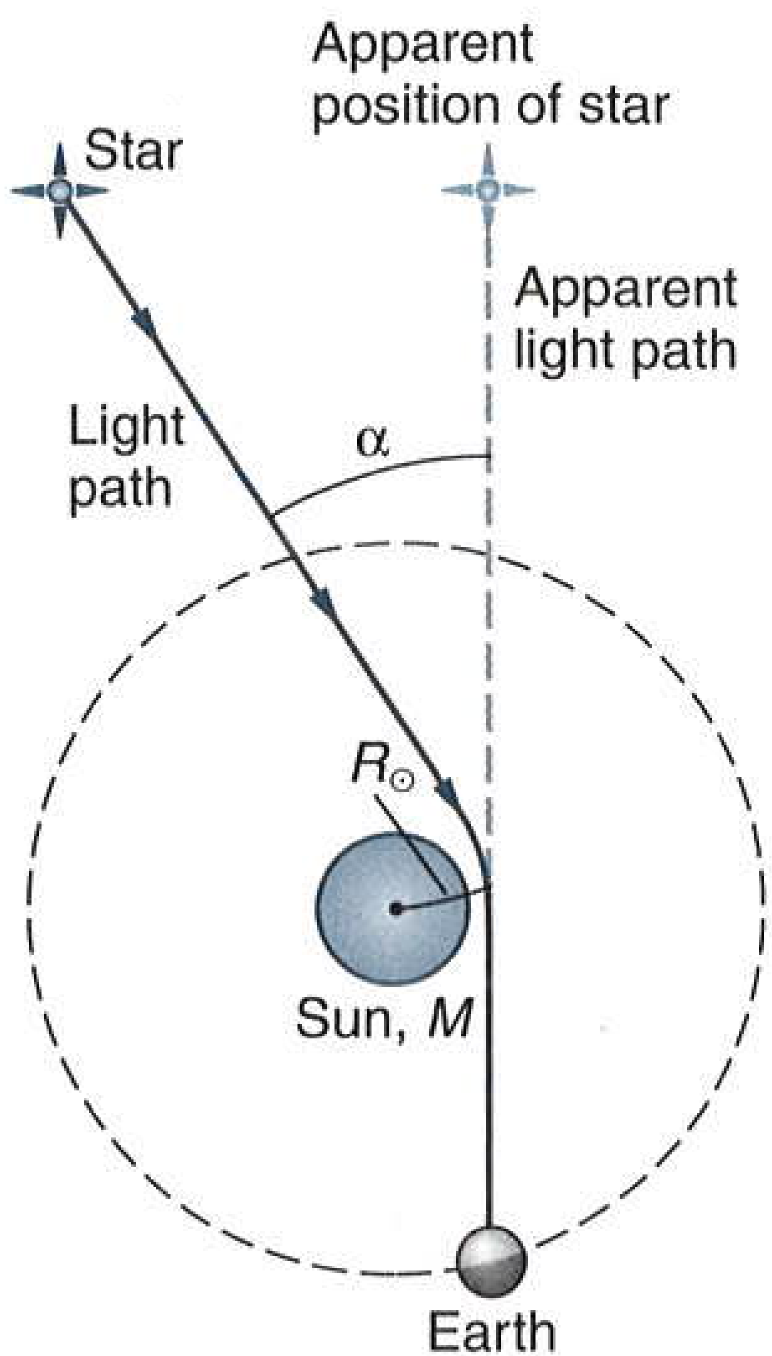
1. The apparent image is larger than the actual object.
2. The apparent object can be bent or otherwise deformed (think of a fun house).
3. The apparent image is not in the same location as the actual object.

Why is this called Gravitational Lensing?

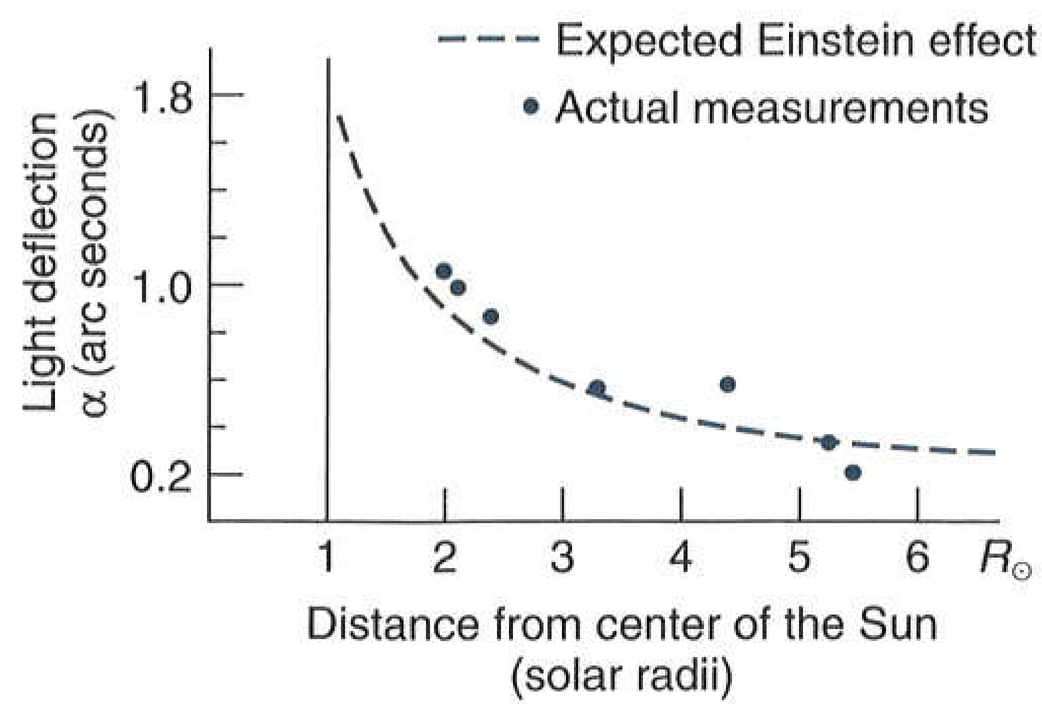


1. In gravitational lensing an object such as a large galaxy cluster acts as a lens.
2. The apparent image is not in the same location as the actual object and appears distorted compared to the actual object.

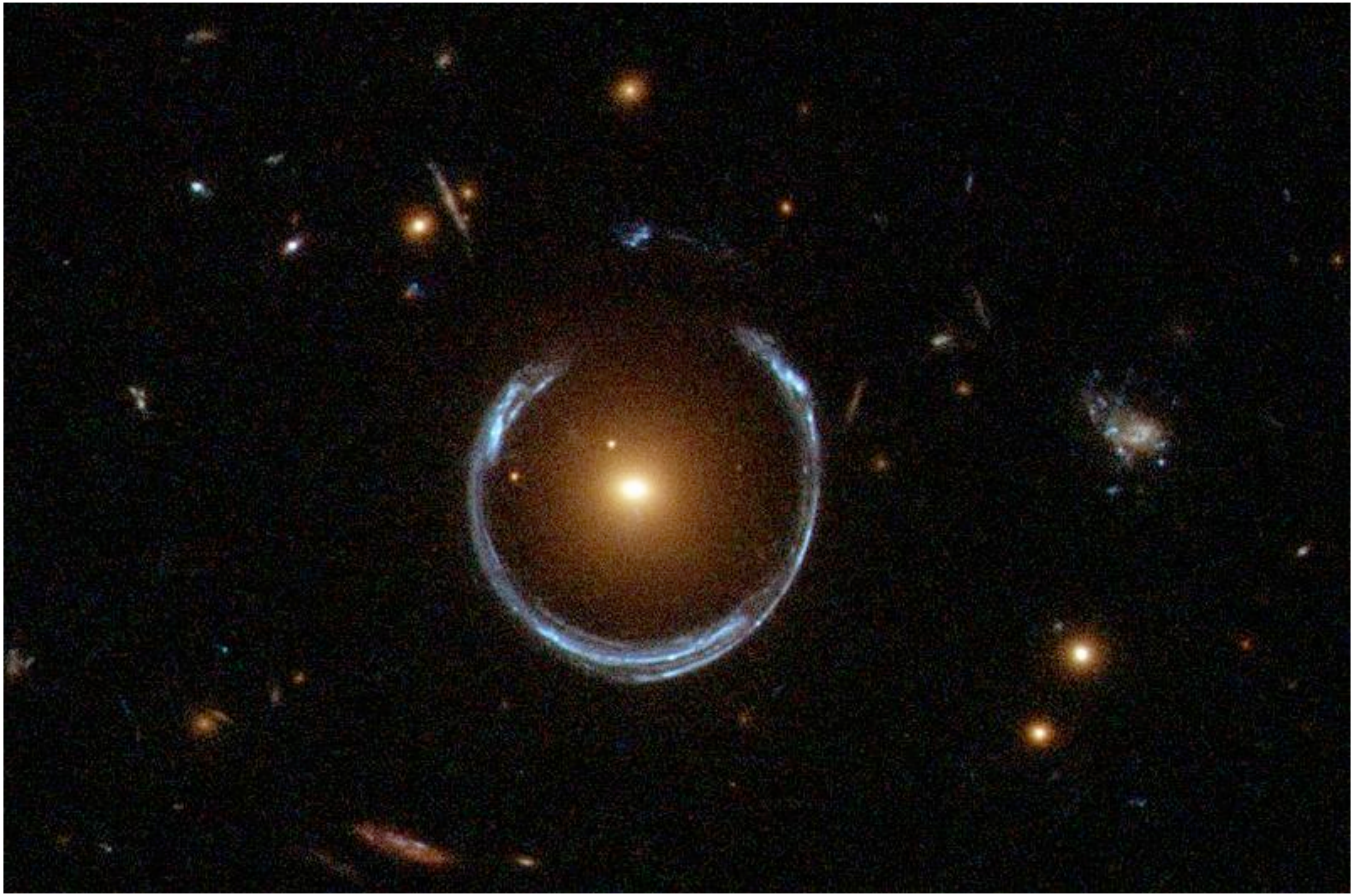
Gravitational Lensing:



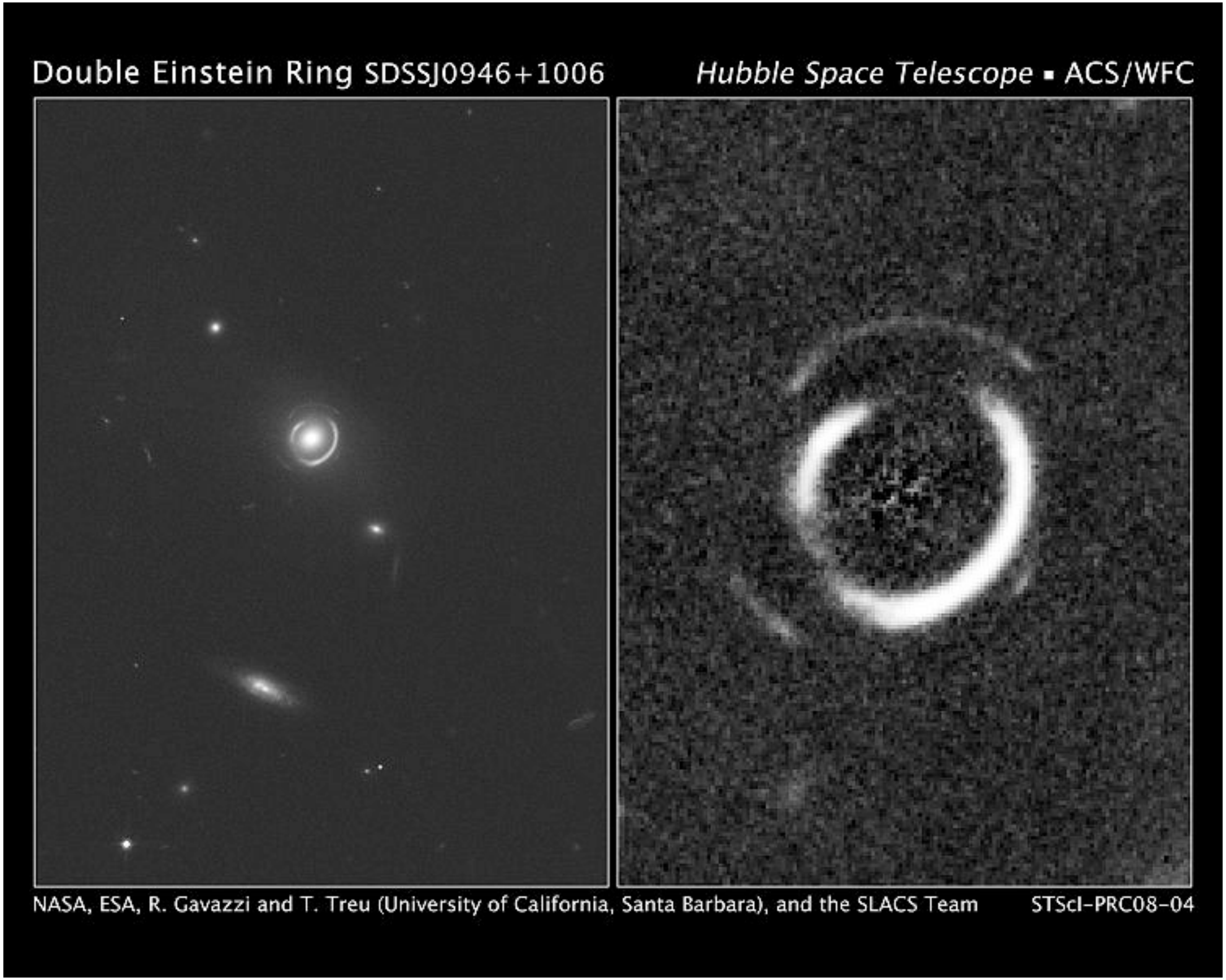
- The deflection angle (α) depends on the distance of closest approach.
- This effect was predicted by Einstein (1915) .
- The deflection of light from the sun was measured by Eddington (1919) at two points during a solar eclipse.
- The measurements agreed within 2%.
- Latest measurements are made by radio telescopes and agree with predictions within 0.1%

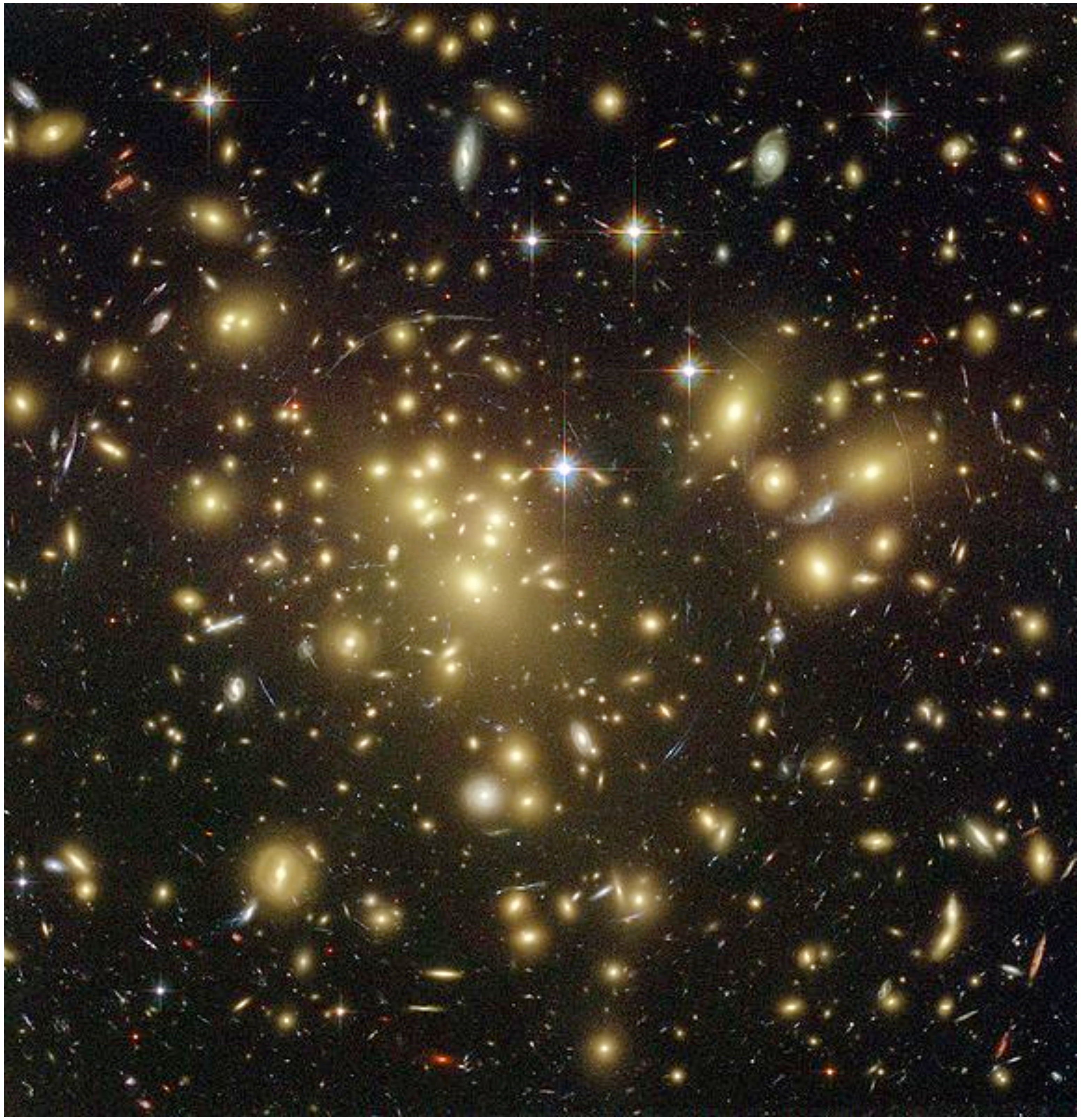


Einstein Ring (or Horseshoe)
LRG 3-757
(luminous red galaxy)

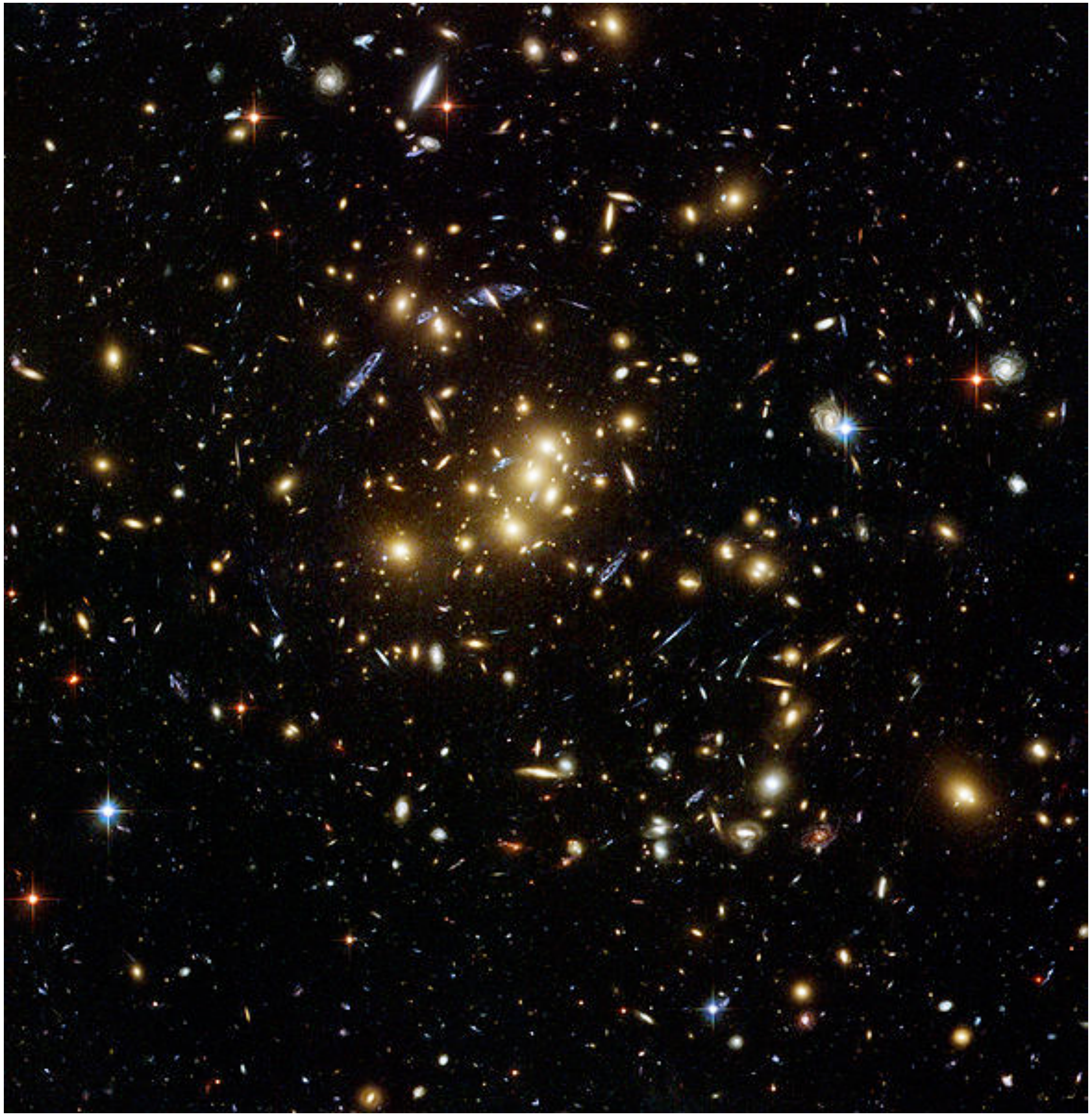


Double Einstein Ring
3 galaxies at 3, 6 and 11 billion ly





Abell 1689
Galaxy Cluster
Virgo Constellation
(2.2 billion ly)



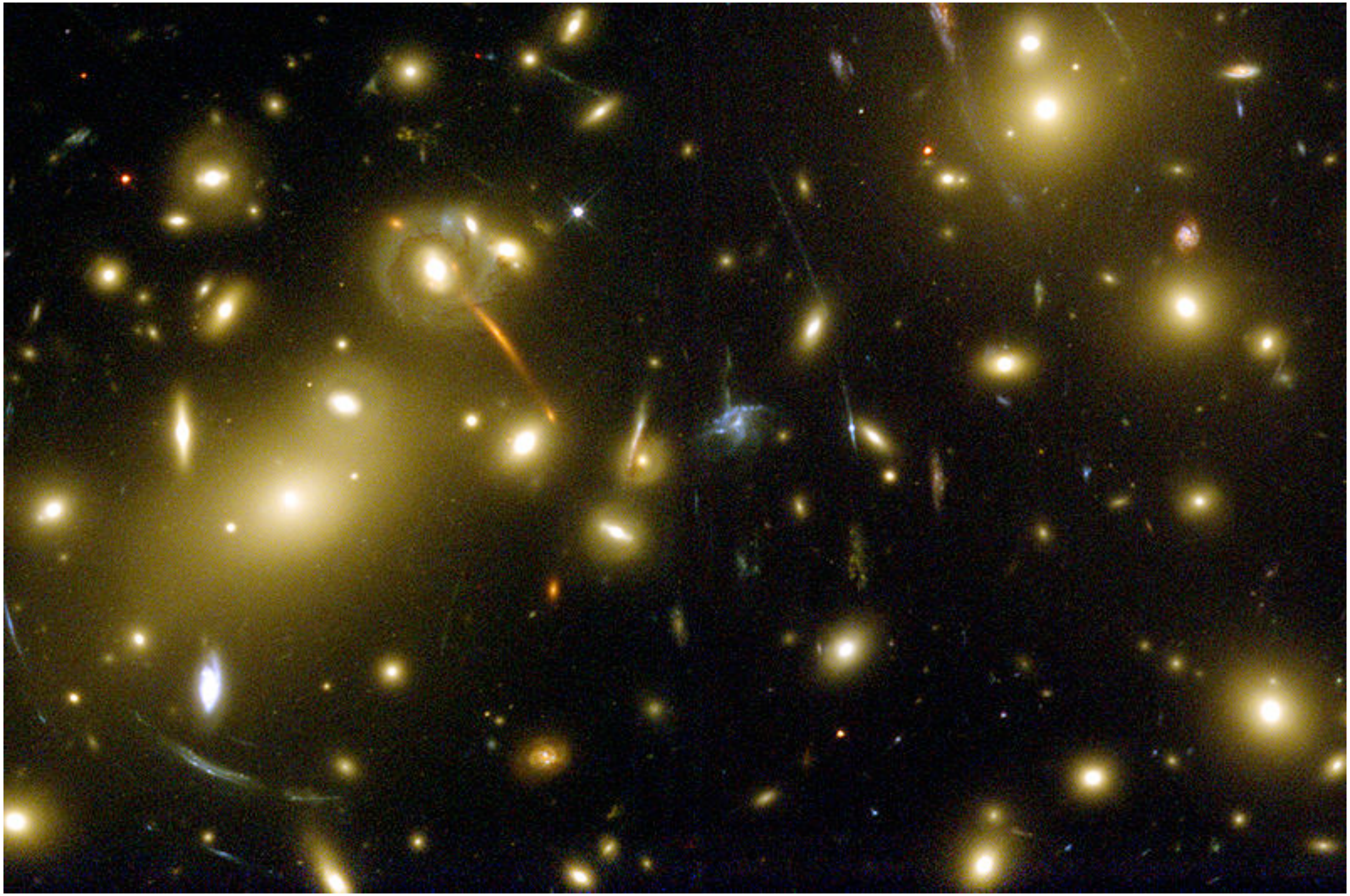
CL0024+17
Galaxy Cluster
Pisces Constellation

Abell NGC2218

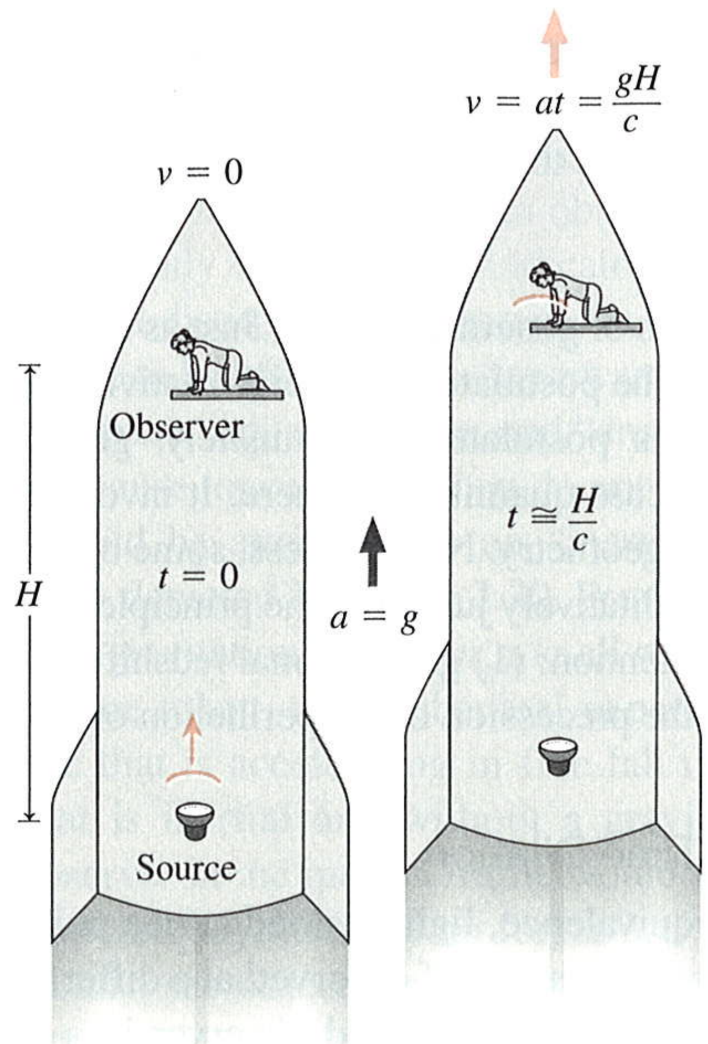
Galaxy Cluster

Draco Constellation

(2 billion light years)



GRAVITATIONAL REDSHIFT



(a) Accelerating frame

- A source in an accelerating frame emits light when its velocity is zero.
- Anna (distance H above the source) observes the light after a time H/c.
- Her upward velocity is given as v = gH/c.
- Light is observed in a frame that is moving away from the source.

Use the Doppler Effect formula to find the observed frequency.

$$f = f'\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = f'(1-\frac{v}{c})^{\frac{1}{2}}(1+\frac{v}{c})^{-\frac{1}{2}}$$

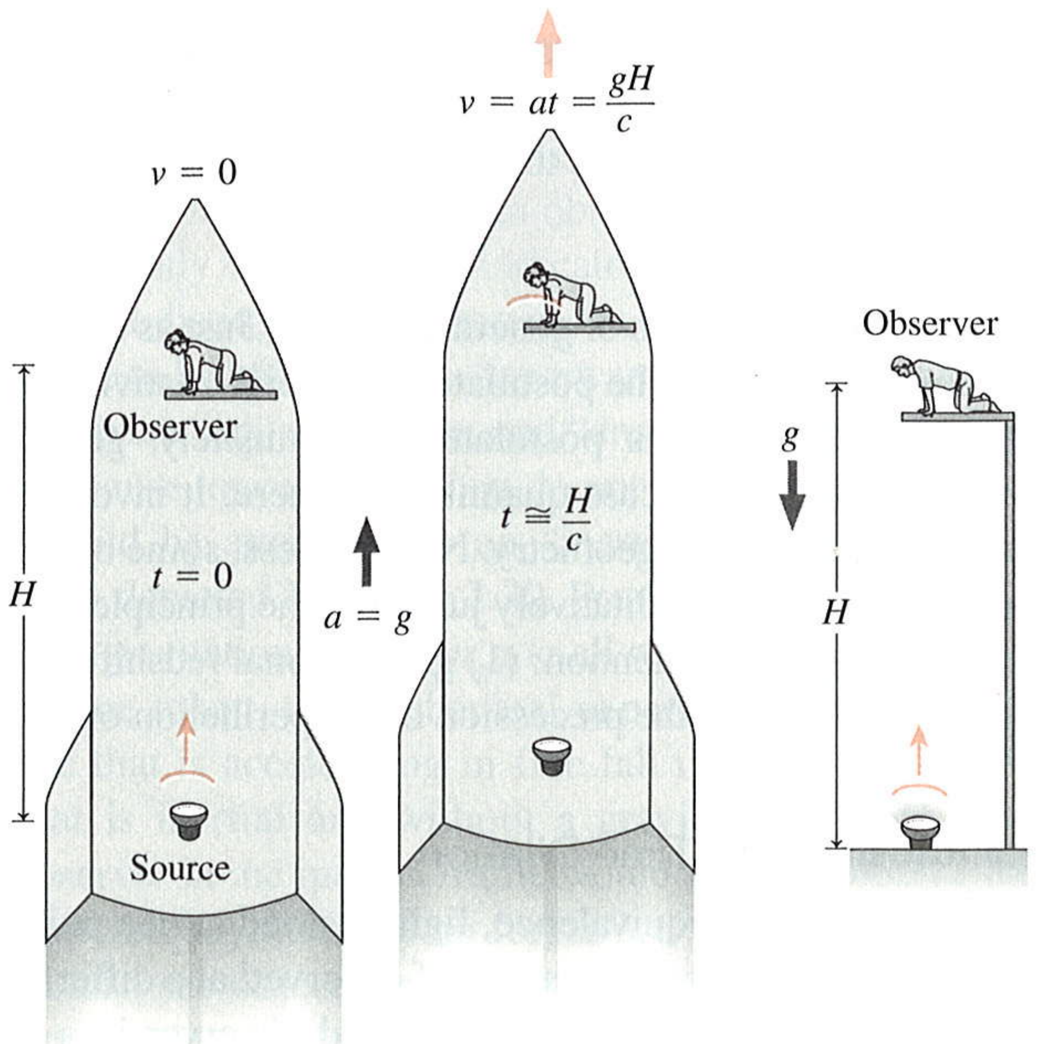
Use binomial expansion

$$f \approx f'(1-\frac{1}{2}\frac{v}{c})(1-\frac{1}{2}\frac{v}{c}) \approx f'(1-\frac{v}{c})$$

Substitute v = gH/c.

$$\boxed{f \approx f'(1-\frac{gH}{c^2})}$$

From the equivalence principle an accelerating frame is equivalent to a stationary frame in a gravitational field.



(a) Accelerating frame (b) Equivalent frame

As light moves upward in the equivalent frame, its frequency must become smaller and wavelength longer.

This is a time-dilation effect. It has nothing to do with the relative motion between the source and observe frames!

Time passing at a lower point is less than time passing at the higher point.

$$\boxed{\Delta t_{lower} = \Delta t_{higher}(1 - \frac{gH}{c^2})}$$

Experimentally verified in 1960 by shooting gamma rays from the roof of the physics building to a detector on the ground floor.

BLACK HOLES

GR predicts the existence of black holes. We can see this from examination of gravitational redshift.

$$f = f'(1 - \frac{gH}{c^2}) \longrightarrow \frac{f}{f'} = 1 - \frac{gH}{c^2}$$

For the general case of a spherical mass M.

$$\frac{f}{f'} = 1 - \frac{GM}{c^2 R}$$

Recall $g = GM/r^2$

Consider the case $R = GM/c^2$. What happens?

$f = 0$

This is a result of the equivalence principle. $f = 0$ at a critical radius. Note: This is an approximation.

The escape velocity from a gravitational field is given by

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

If

$$r = \frac{2GM}{c^2} \longrightarrow v_{esc} = \sqrt{\frac{2GM}{\frac{2GM}{c^2}}} = c$$

$$\boxed{r_s = \frac{2GM}{c^2} \qquad \text{Schwarzschild Radius}}$$

This is the radius at which the escape speed exceeds the speed of light.

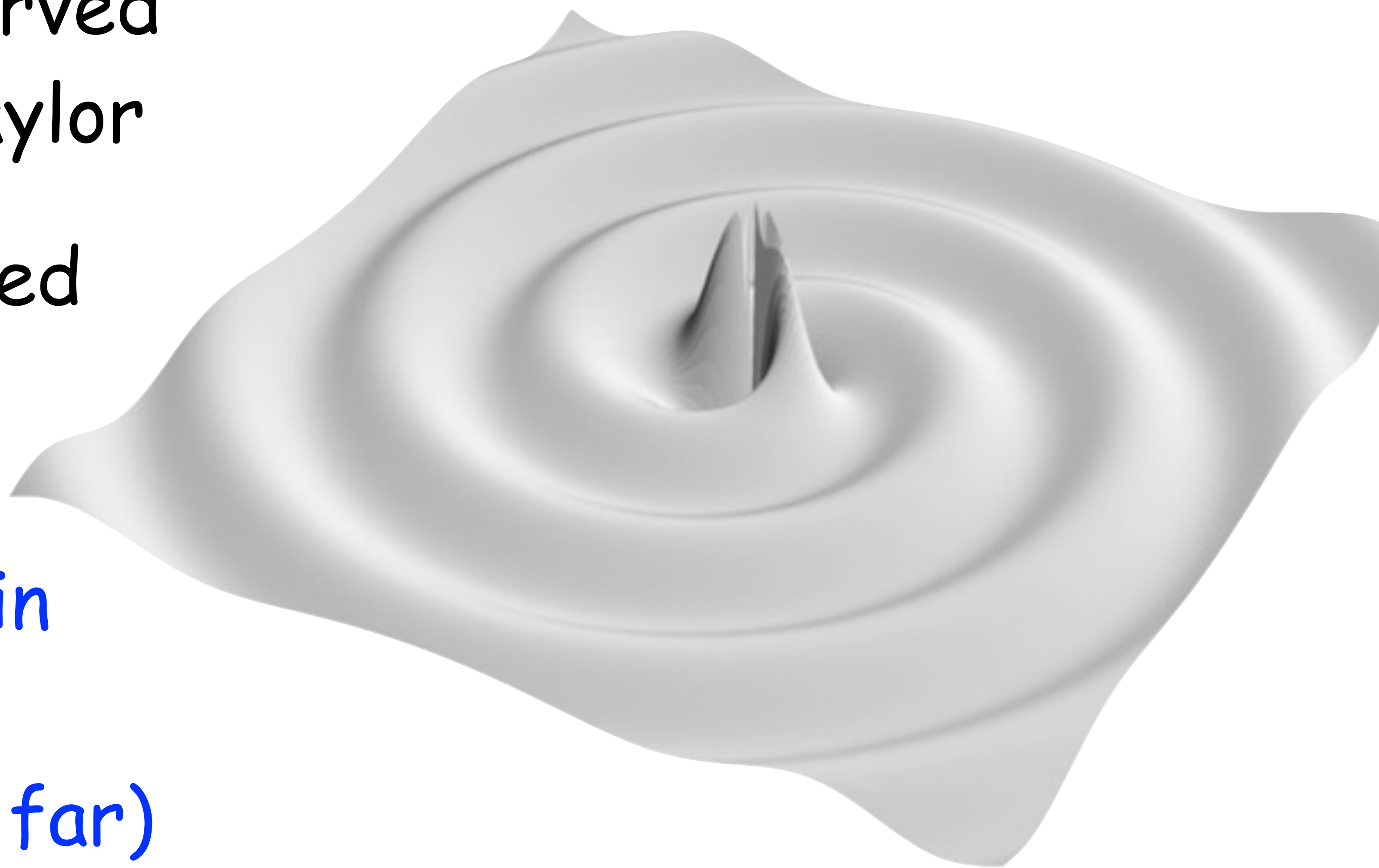
r_s is also known as the 'event horizon'

GRAVITY WAVES

- Any movement of mass in spacetime will cause ripples that propagate away from the source (just like water/sound waves). These are known as gravity waves

- In-spiral of binary star system explained by energy loss to gravity waves observed in 1974 by Hulse & Taylor

- Hulse & Taylor received the 1993 Nobel prize in physics. Only Nobel prize awarded in relation to General or Special relativity! (so far)



MICHELSON-MORLEY REVISITED

- The Michelson-Morley experiment has been recently reborn in the guise of a gravity wave search
- Gravity waves distort the length of space
- An interferometer is sensitive to tiny (<< atom) changes in the arm lengths (Pictured here LIGO).

HUBBLE CONSTANT

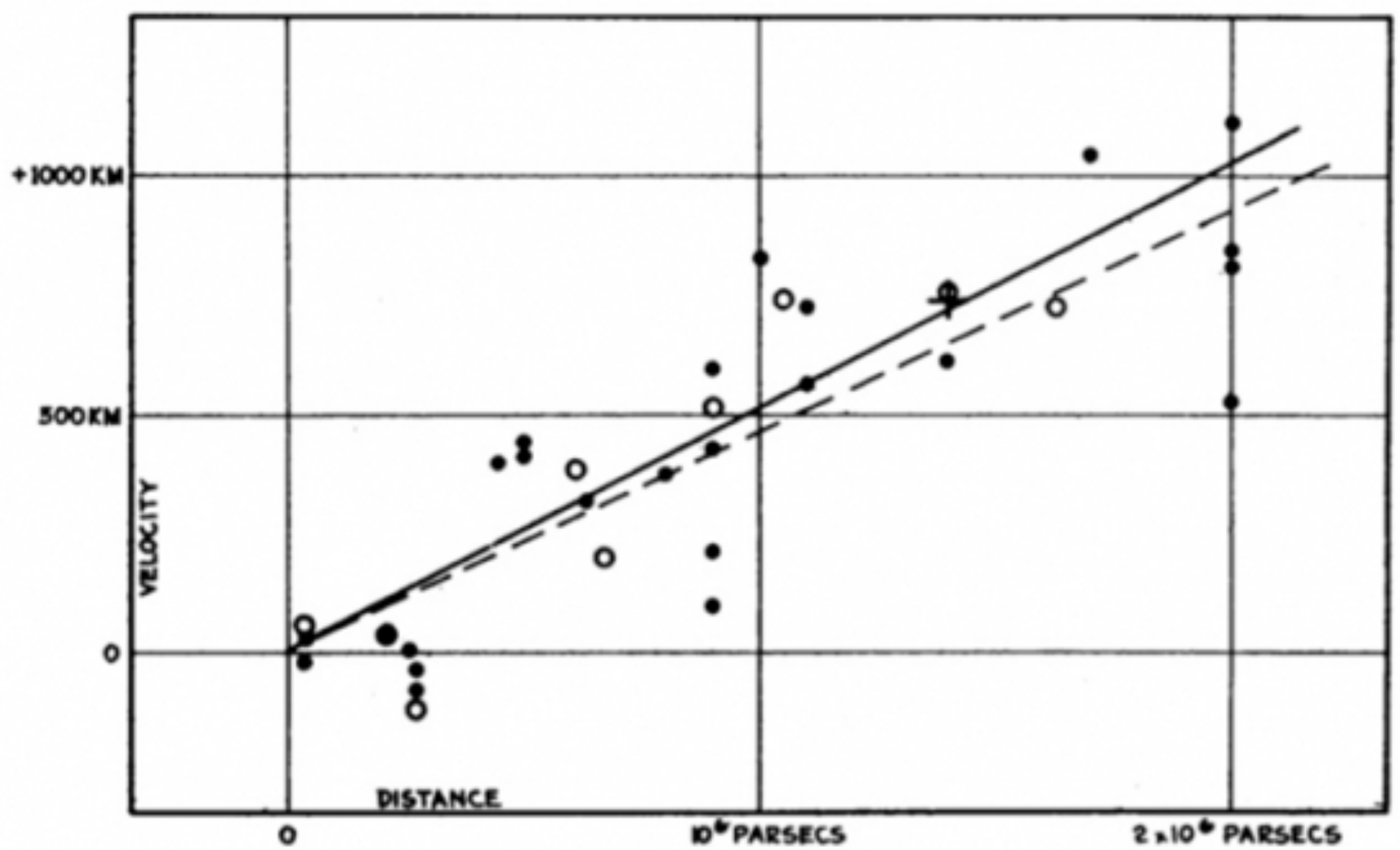


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

The farther from Earth, the greater the redshift in its spectrum.

$$v = \frac{H_0}{r}$$

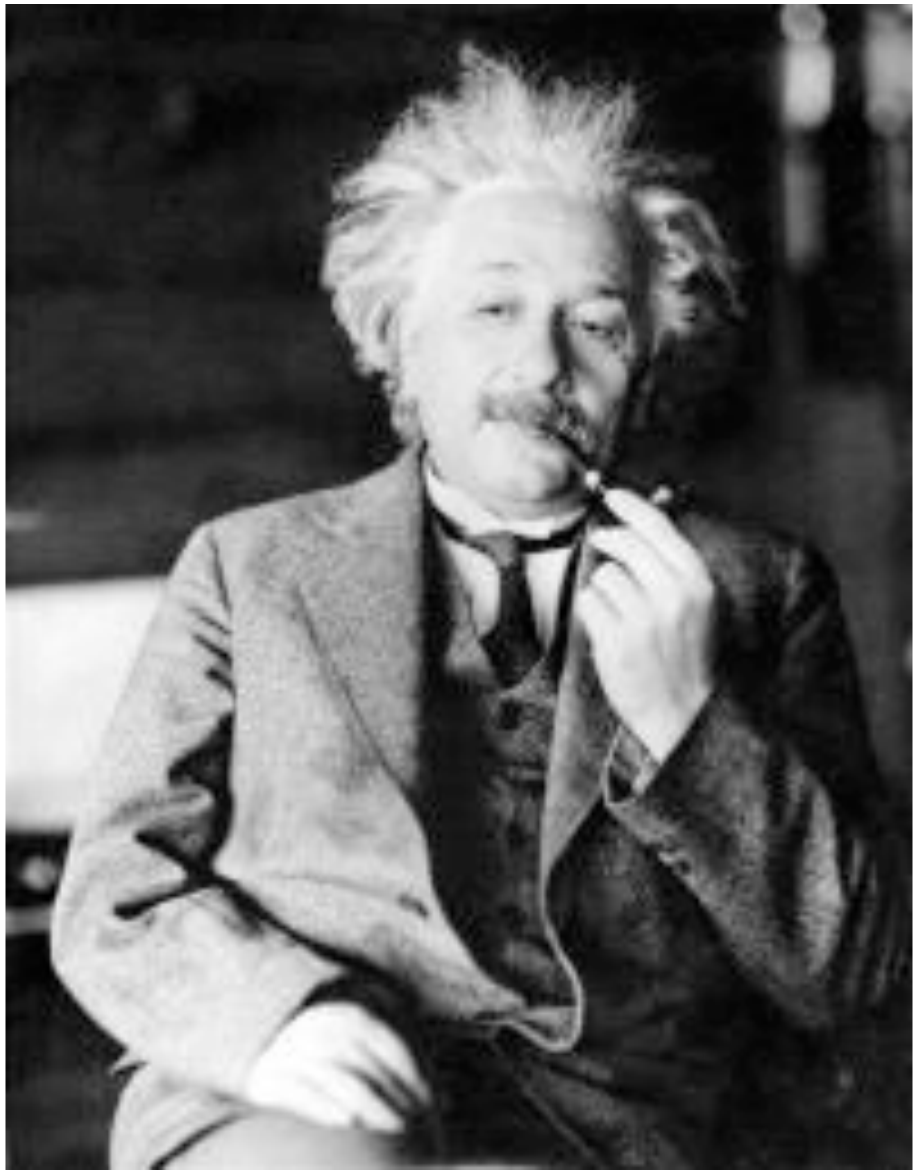
$$\frac{r}{v} = \frac{1}{H_0}$$

same for all galaxies!

$1/H_0$ is referred to as the age of the universe. Evidence suggests the age of the universe is 14 Gyr (billion years).

Einstein's Postulates:

1. The laws of physics are invariant to observers in different inertial reference frames.
2. The speed of light is the same to all observers, regardless of their motion relative to the light source.



As a consequence of Einstein's Postulates,
<u>no information travels faster than the speed of light</u>.

This claim is supported by the Lorentz Equations!

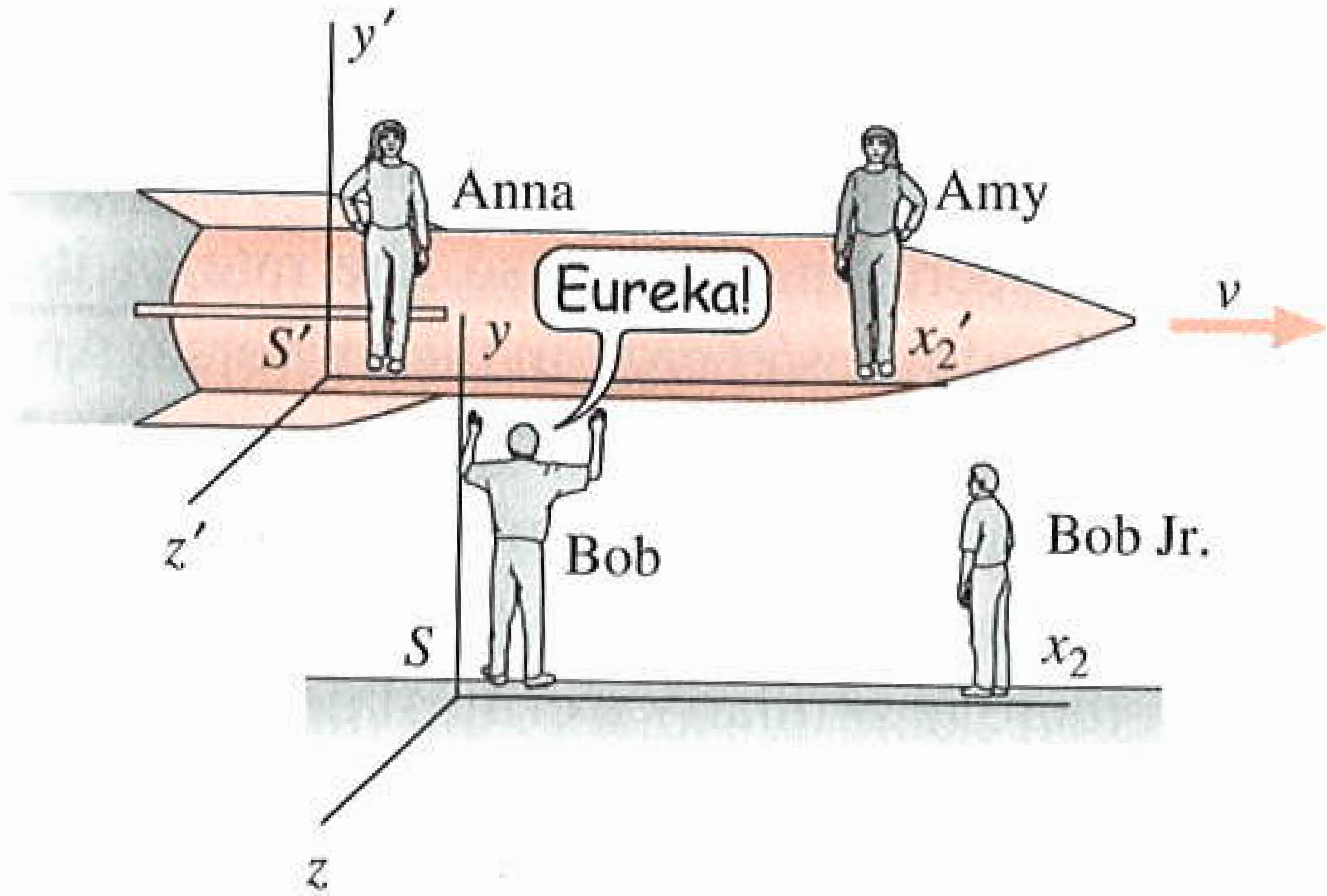
Lorentz Equations - No information travels faster than light

Imagine the following scenario:

Event 1:
Bob and Anna pass each other.
Bob sends a symphony to Bob Jr.

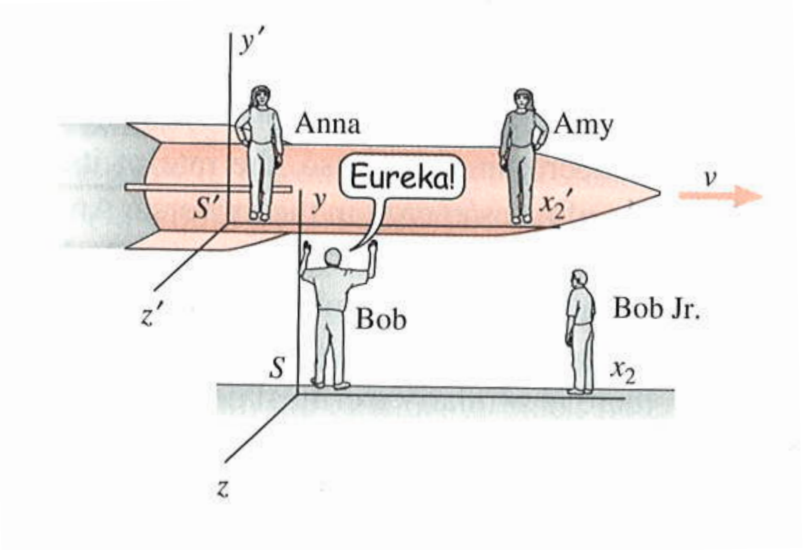
Event 2:
Symphony arrives at Bob Jr.
and is transmitted to Amy
instantaneously (Bob Jr. & Amy
receive symphony at the exact
same time).

Event 3:
Symphony reaches Anna.



Calculate x/x' and t/t' for Event 1:

Event 1:
Bob and Anna pass each other.
Bob sends a symphony to Bob Jr.



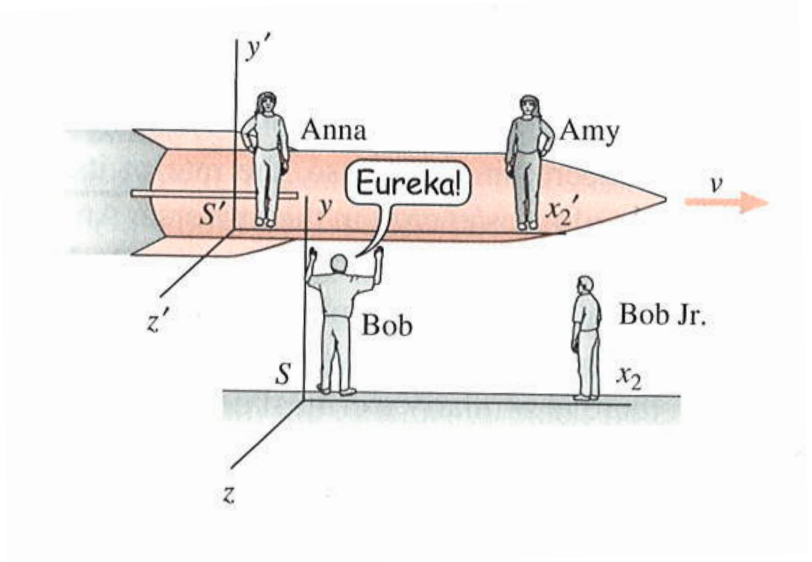
How are x and t in the primed and unprimed frames related to each other?

$$t_1 = t_1' = 0$$

$$x_1 = x_1' = 0$$

Calculate t/t' and x/x' for Event 2:

Event 2:
Symphony arrives at Bob Jr. and is transmitted to Amy instantaneously (Bob Jr. & Amy receive symphony at the exact same time).



Find an expression for t_2.

$$t_2 = \frac{x_2}{u}$$

Note: u is the speed of the information

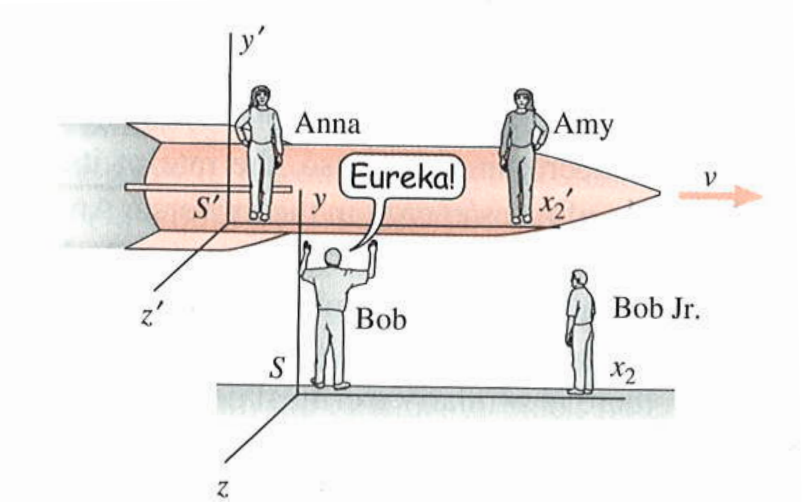
The Lorentz equation for x'_2 in terms of unprimed variables is

$$x'_2 = \gamma(x_2 - v\frac{x_2}{u}) = \gamma(1 - \frac{v}{u})x_2$$

The Lorentz equation for t'_2 in terms of unprimed variables is

$$t'_2 = \gamma(-\frac{v}{c^2}x_2 + \frac{x_2}{u}) = \gamma(-\frac{v}{c^2} + \frac{1}{u})x_2$$

Calculate t/t' and x/x' for Event 3:



Event 3:

Symphony reaches Anna.

First find an expression for the time it takes for the symphony to be transmitted from Amy to Anna.

$$t'_{A\to A} = \frac{x'_2 - x'_1}{u} = \frac{x'_2}{u}$$

Then we can find t'₃ in terms of the unprimed frame.

$$t'_3 = t'_2 + t_{A\to A} = t'_2 + \frac{x'_2}{u}$$

$$= \gamma(-\frac{v}{c^2} + \frac{1}{u})x_2 + \frac{1}{u}(\gamma(1 - \frac{v}{u})x_2)$$

$$= \gamma x_2(-\frac{v}{c^2} + \frac{1}{u} + \frac{1}{u} - \frac{v}{u^2}) = \frac{\gamma x_2}{u}(-\frac{vu}{c^2} + 2 - \frac{v}{u})$$

Write t'_3 in a more suggestive form:

$$t'_3 = \frac{\gamma x_2}{u}(-\frac{vu}{c^2} + 2 - \frac{v}{u})$$
$$= \gamma x_2 \frac{2}{u}(\frac{uv}{2c^2} + 1 - \frac{v}{2u})$$

$$\boxed{t'_3 = \gamma x_2 \frac{2}{u}(1 - \frac{v}{c}(\frac{u}{2c} - \frac{c}{2u}))}$$

What if u<c?

t'_3 is positive, event 1 happens before event 3, Bob sends the new symphony before Anna hears it.

What if u>c?

t'_3 is negative, event 3 happens before event 1, Anna is whistling the symphony as she passes Bob which is before Bob conceives it. This violates cause and effect. Violates Einstein's 2nd postulate.

Events simultaneous in one reference frame occur in different orders in another reference frame.

Is it possible that the effect can precede the cause?

NO!

The Lorentz transformations forbid it! Events can only be reversed if they are not causally related.

Consider what happens if Δt' is negative.

$$\Delta t' = \gamma(-\frac{v}{c^2}\Delta x + \Delta t)$$

$$\frac{\Delta t'}{\Delta t} = \gamma(-\frac{v}{c^2}\frac{\Delta x}{\Delta t} + 1)$$

If the time intervals are of opposite sign, $\frac{\Delta t'}{\Delta t} < 0$.

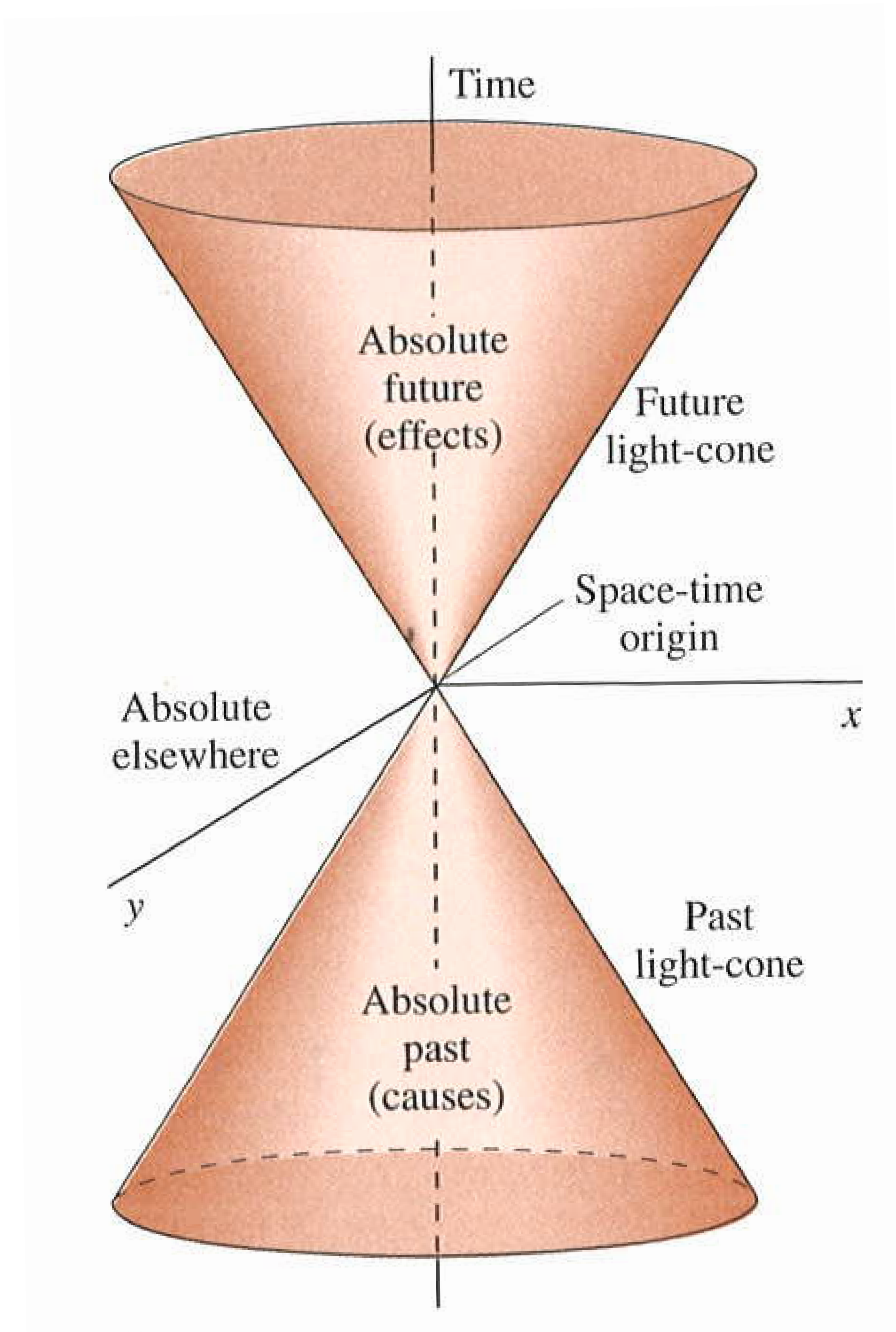
$$-\frac{v}{c^2}\frac{\Delta x}{\Delta t} + 1 < 0$$

$$-\frac{v}{c^2}\frac{\Delta x}{\Delta t} < -1$$

$$\frac{\Delta x}{\Delta t} > \frac{c^2}{v} > c$$

Speed needed is greater than that of light!

Light-cone:



- One way to visualize causality is the light-cone.
- An event happens (now) at the origin.
- Only events that happen within the cone (absolute past) and effect the event at the origin (now).
- Only events within the cone (absolute future) can be effected by the event at the origin (now).
- The sides of the cone are determined by the speed of light.

THE END
(FOR TODAY)