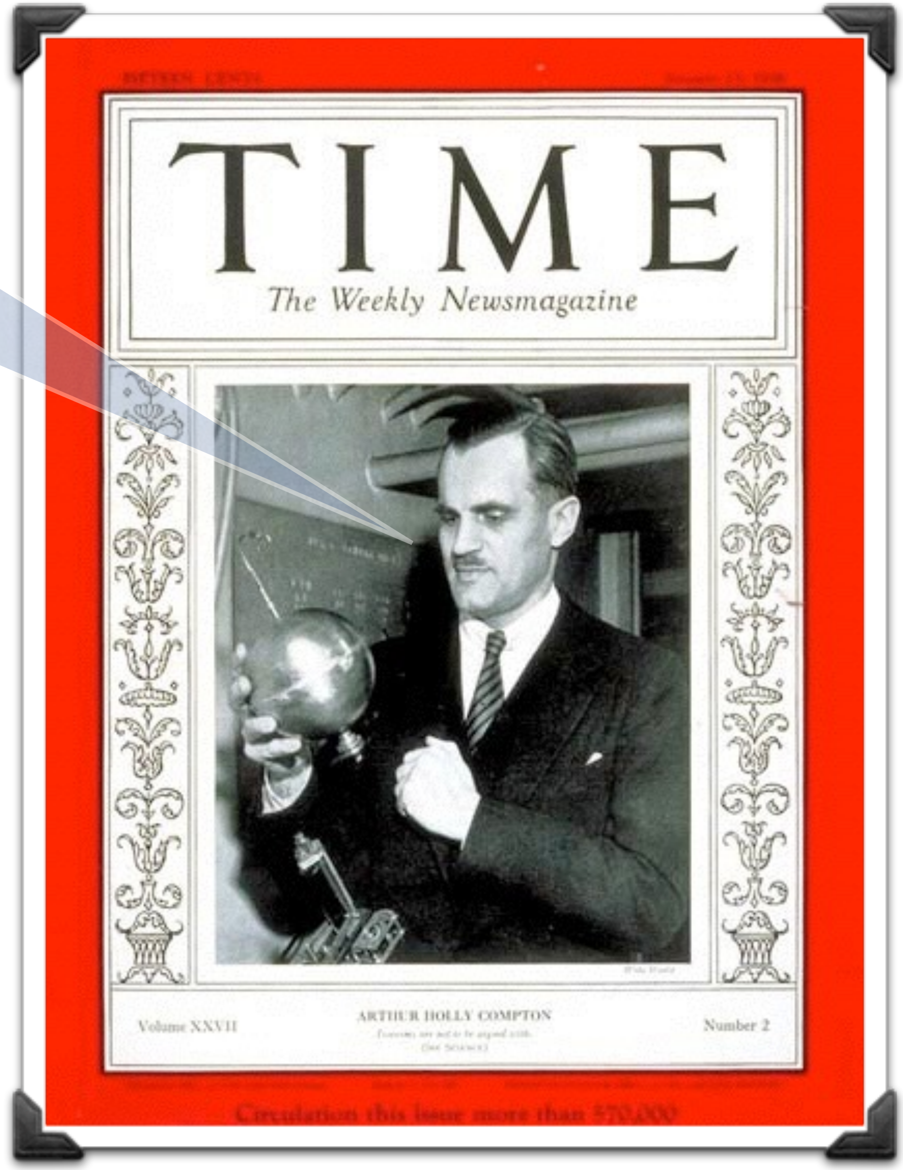
Welcome back to PHY 3305

Today's Lecture:

X-ray Production

Compton Scattering

Dual Nature of Light



Arthur Compton

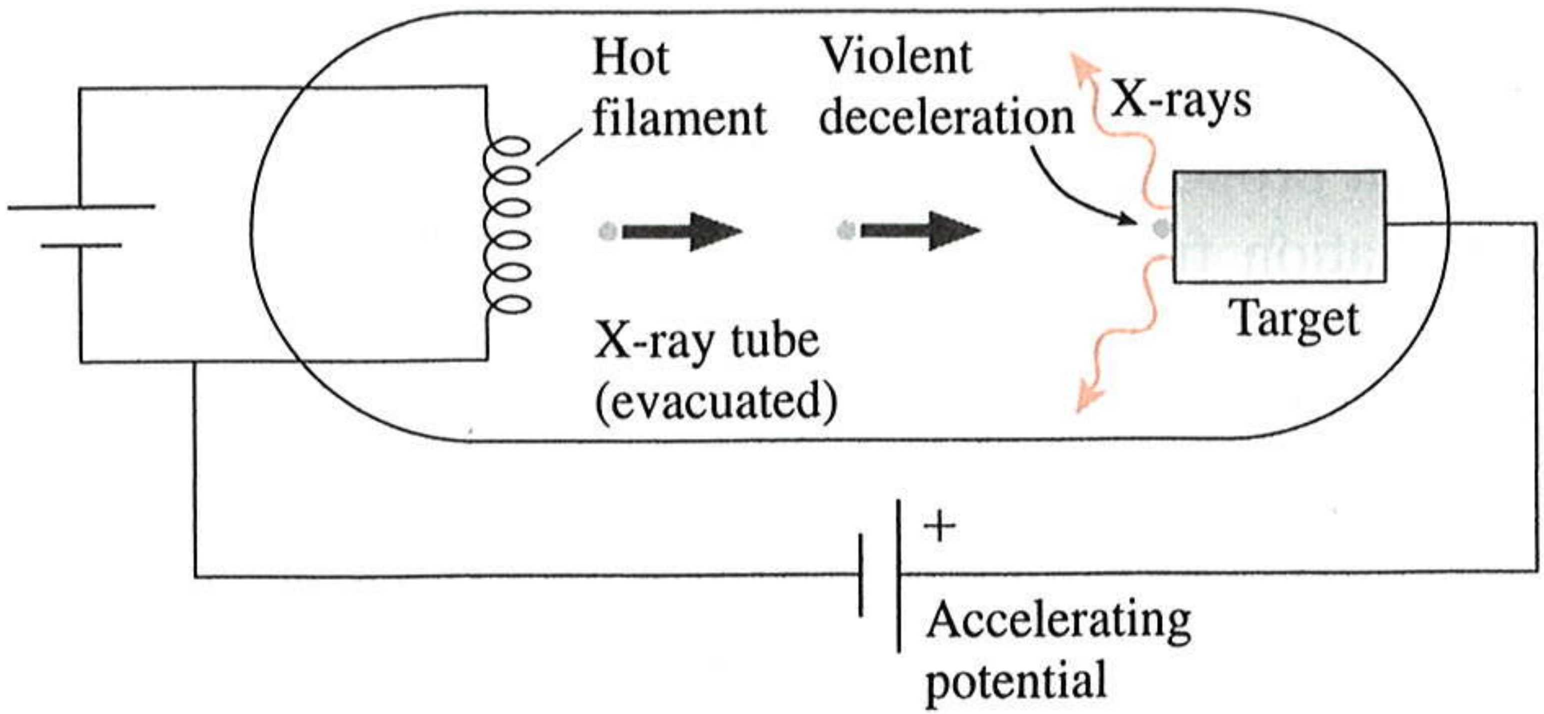
1892 - 1962

THE PRODUCTION OF XRAYS

First "medical" x-ray image (taken of Mrs. Roentgen's hand).

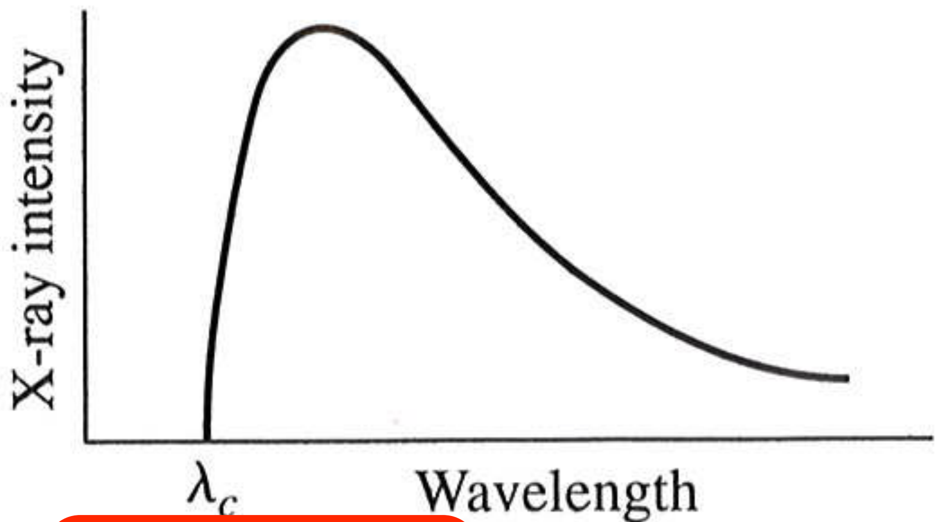
- X-rays were discovered in 1895 by German physicist Wihelm k. Roentgen. (First Nobel Prize in Physics 1901.)
  - "rays" originating from the point where cathode rays (electrons) hit the glass tube (or target) could pass through materials opaque to light and activate a fluorescent screen or photographic film.

THE PRODUCTION OF XRAYS



Xrays can be produced by smashing high speed electrons into a metal target.
Bremsstrahlung is radiation produced by the decelerating charges.

X-RAY SPECTRUM



0.050nm

error in textbook

Intensity and wavelength for electrons of KE 25 keV striking a molybdenum target.

Classical

- no explanation for the cutoff

Nonclassical

- radiation is quantized
- the minimum energy allowed at a frequency f that of a single photon (E = hf).
- can not produce half a γ.
- no photon could ever be produced with energy greater than that of a single electron.

Does this make sense?

$$KE_{max} = \frac{hc}{\lambda} \longrightarrow \lambda = \frac{hc}{KE_{max}} = \frac{1240eV \cdot nm}{25 \times 10^3 eV} = 0.050nm$$

THE COMPTON EFFECT

Prediction from classical physics:

- x-rays incident on a material containing charges should cause charges in a solid to oscillate.
- Oscillating charges will make EM waves radiate in all directions with a frequency equal to the incident waves.

Experiment:

- some waves come back toward the source of the x-rays with much lower frequencies.

Hypothesis:

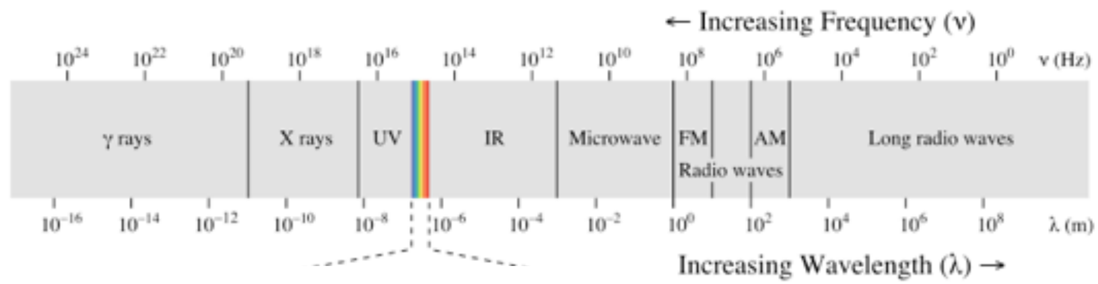
- light, a particle and not a wave, collides with an electron and in ejecting the electron scatters backward with less energy

The Compton Effect:

$m_e = 9.1 \times 10^{-31}$ kg

$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

Why is this effect observed in x-rays and not radio waves or visible light?



Visible light (500 nm):

$$\lambda_{after} = 500nm + \frac{h}{m_e c}(1 - \cos 180) = 500nm + \frac{2h}{m_e c}$$

$$\lambda_{after} = 500nm + \frac{2(6.63 \times 10^{-34} J \cdot s}{(9.109 \times 10^{-31} kg)(3 \times 10^{8})m/s}$$

$$\lambda_{after} = 500nm + 0.005nm$$

The effect is very small (0.1%)!

$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

What is the effect for x-rays which have $\lambda = 5 \times 10^{-2}$ nm (microwave).

$$\lambda_{after} = 5 \times 10^{-2} nm + \frac{2h}{m_e c}$$

$$\lambda_{after} = 5 \times 10^{-2} nm + 5 \times 10^{-3} nm$$

This is more like a 10% effect!

Answer:

For long wavelengths the classical description of light is very accurate. For short wavelengths, the classical description is not accurate.

$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

At what angle do we see a maximum change in wavelength?

Answer: 180 degrees --> maximum $\Delta\lambda$ = 0.00495 nm

If instead of scattering off an electron, suppose the photon scatters off a proton. What is the maximum possible change in wavelength? (The mass of a proton is 1.67×10^{-27} kg.)

$$\Delta\lambda = \frac{2h}{m_p c} = \frac{2(6.63 \times 10^{-34} J \cdot s)}{(1.67 \times 10^{-27} kg)(3 \times 10^8 \frac{m}{s})}$$

$$\Delta\lambda = 0.00000265 nm = 2.65 \times 10^{-6} nm$$

The collision off the electron more clearly demonstrates the particle like nature of EM radiation.

$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

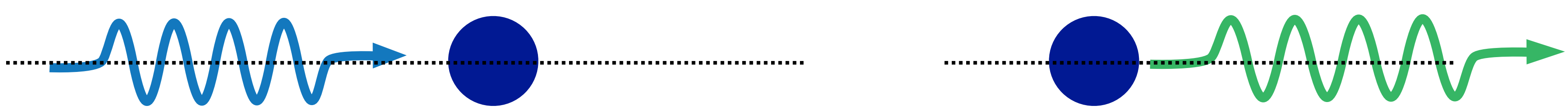
What is the minimum energy a photon can lose in a Compton Scatter?

Minimum energy loss occurs when $\Delta\lambda$ is minimum.

$$E = hf = \frac{hc}{\lambda} \longrightarrow \Delta E = hc\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right)$$

$$\Delta E = hc\left(\frac{1}{\lambda_b + \frac{h}{m_e c}(1 - \cos\theta)} - \frac{1}{\lambda_b}\right)$$

No energy loss occurs if $\lambda_a = \lambda_b$. This occurs if $\Theta = 0$ degrees



photon passes straight through, no energy loss

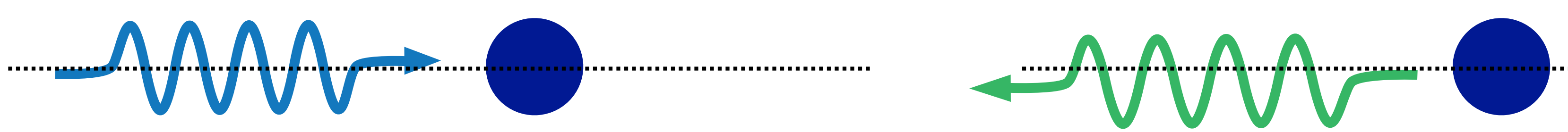
$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

What is the maximum energy a photon can lose in a Compton Scatter?

$$\Delta E = hc\left(\frac{1}{\lambda_b + \frac{h}{m_e c}(1 - \cos\theta)} - \frac{1}{\lambda_b}\right)$$

The maximum value of $\Delta\lambda$ happens when $\Theta = 180$.

$$\Delta E = hc\left(\frac{1}{\lambda_b + \frac{2h}{m_e c}} - \frac{1}{\lambda_b}\right)$$



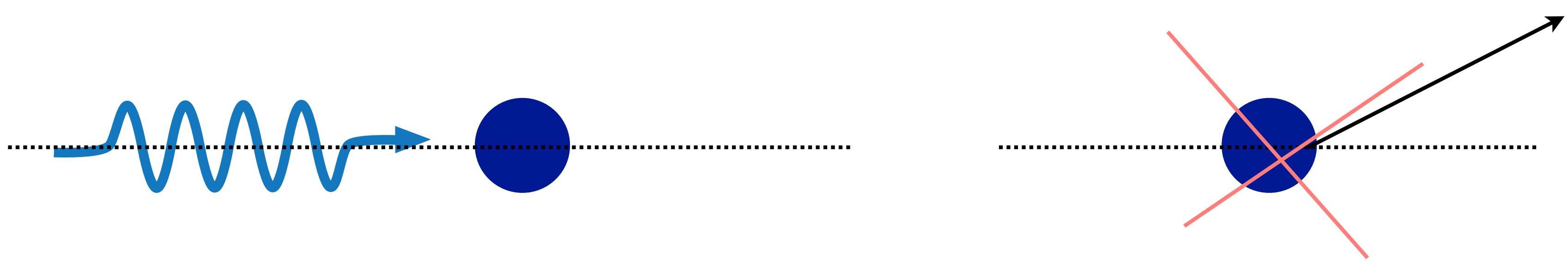
When the photon scatters straight back it gives max energy to the electron.

$$\lambda_{after} - \lambda_{before} = \frac{h}{m_e c}(1 - \cos\theta_{scatter})$$

Is it possible for a photon to be fully absorbed in a Compton Scatter?

This corresponds to a situation where E = 0. That implies that λ_a = infinity. We just saw that $\Delta\lambda$ is a finite quantity. Thus, λ_a can not equal infinity and the photon can not be fully absorbed.

In addition, a case where the photon is fully absorbed violates momentum and energy conservation.



Correspondence Principle:

The classical behavior of radiation is restored when the wavelength is sufficiently long.

Note: This is a guideline not a law.

There is a continuity between what is classical physics and what is a more general description of radiation.

MOMENTUM OF LIGHT

Einstein's work on the photoelectric effect:

$$E_{light} = hf$$

Special Relativity:

$$E_{light} = pc$$

Combine these ideas together - **momentum of light**

$$pc = hf \xrightarrow{f = c/\lambda} \boxed{p = h\frac{f}{c} = \frac{h}{\lambda}}$$

Final remarks:

- The Compton effect demonstrates the electromagnetic radiation interacts with matter as a particle.

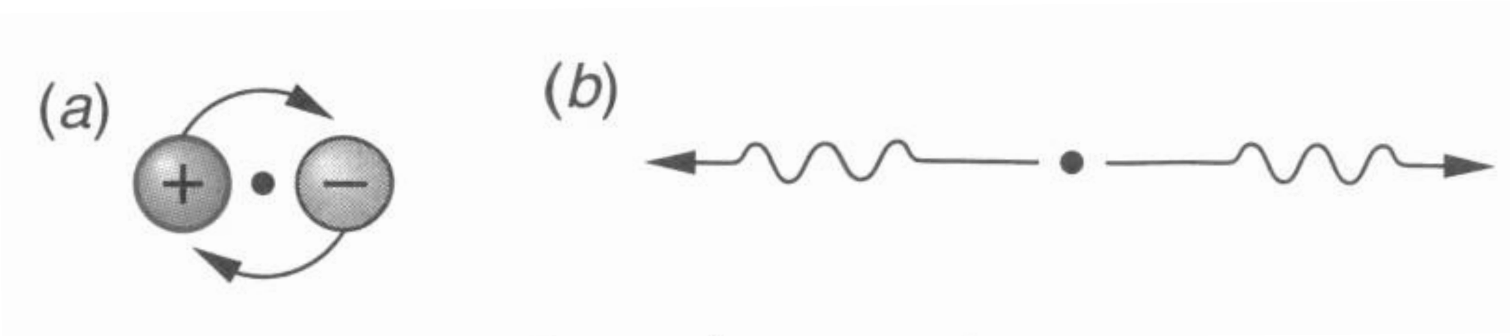
- Compton received the 1927 Nobel prize "for his discovery of the effect named after him"

- We now have 4 pieces of evidence demonstrating the particle nature of light - photoelectric effect, blackbody radiation, x-ray production and the Compton effect.

- We now have an expression for the momentum of light.

ANNIHILATION OF PARTICLES

Elementary particles with mass can combine with their antiparticles, the masses of both being completely converted to energy in a process called **annihilation**. (Creation of energy from mass.)



A positron orbits an electron about their center of mass.

After a short time (0.1 ns) the positron and electron annihilate producing 2 photons.

To make the calculation clear, assume that the speeds of the electron and positron are u<<c (rest mass). Each particle has a total energy of 0.511 MeV.

$$E_{tot} = 1.022 MeV$$

$$2mc^2 = 1.022 MeV$$

The particles' momenta are always equal and opposite in direction. Thus, the total momentum of the photons produced must be zero.

We know that for photons, E = pc. So each photon must have equal energy. Thus, from conservation of energy -

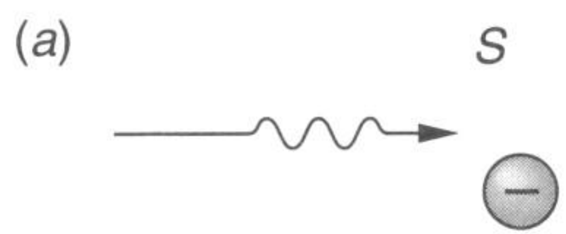
$E_\gamma = 0.511 MeV$ for each photon.

CREATION OF PARTICLES

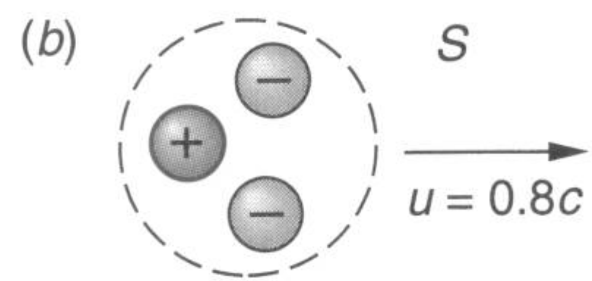
The creation of mass from energy can also occur. The energy to create mass can be provided by either

1. kinetic energy of another massive particle
2. "pure" energy of a photon

Assuming the appropriate conservation laws are satisfied. (restricts certain processes)



Photon travels through space, "hits" an electron.



Produces an electron-positron pair which as a group move off together.

Let's suppose that the details of the interaction are such that the three particles (e, e, e+) all move off together with the same velocity, u = 0.8c - they are all at rest in S'. Also assume S' moves to the right with a speed u relative to S. What must the energy E_γ of the photon be in order that this particular electron-positron pair is created?

Step 1: Consider the conservation of Energy and Momentum.

Before Pair Creation

$$E_i = E_\gamma + E_e = E_\gamma + mc^2$$

$$p_i = p_\gamma = \frac{E_\gamma}{c}$$

After Pair Creation

$$E_i = E_f = E_\gamma + mc^2$$

$$p_i = p_f = \frac{E_\gamma}{c}$$

Step 2: Examine the energy after pair creation.

$$E_i = E_f = E_\gamma + mc^2$$

$$p_i = p_f = \frac{E_\gamma}{c}$$

Rest Energy   Momentum

$$E^2 = m^2c^4 + p^2c^2$$

$$(mc^2)^2 = E^2 - (pc)^2$$

Rest Energy

The rest energy of the system equals the rest energies of the constituent particles. Thus, the final system rest energy is $3mc^2$. We need to modify the above equation.

$$(3mc^2)^2 = E^2 - (pc)^2$$

$$9(mc^2)^2 = (E_\gamma + mc^2)^2 - (\frac{E_\gamma}{c}c)^2$$

$$9(mc^2)^2 = \cancel{E_\gamma^2} + 2E_\gamma mc^2 + m^2c^4 - \cancel{E_\gamma^2}$$

$$2E_\gamma = 8mc^2$$

$$\boxed{E_\gamma = 4mc^2}$$

Initial photon needs 4 times the electron's rest energy in order to create 2 new electron rest masses.

Why does the initial photon need the "extra energy"?

The three particles in the final system share momentum. Thus, they must also share KE.

$$E_{KE} = E_i - E_f$$

$$E_{KE} = E - 3mc^2 = (E_\gamma + mc^2) - 3mc^2$$

$$E_{KE} = 4mc^2 + mc^2 - 3mc^2$$

$$\boxed{E_{KE} = 2mc^2}$$

The photon must provide both the $2mc^2$ to create the pair and the $2mc^2$ of kinetic energy that the pair and the existing electron share as a result of momentum conservation.

What is the nature of light? Is light a particle or a wave?

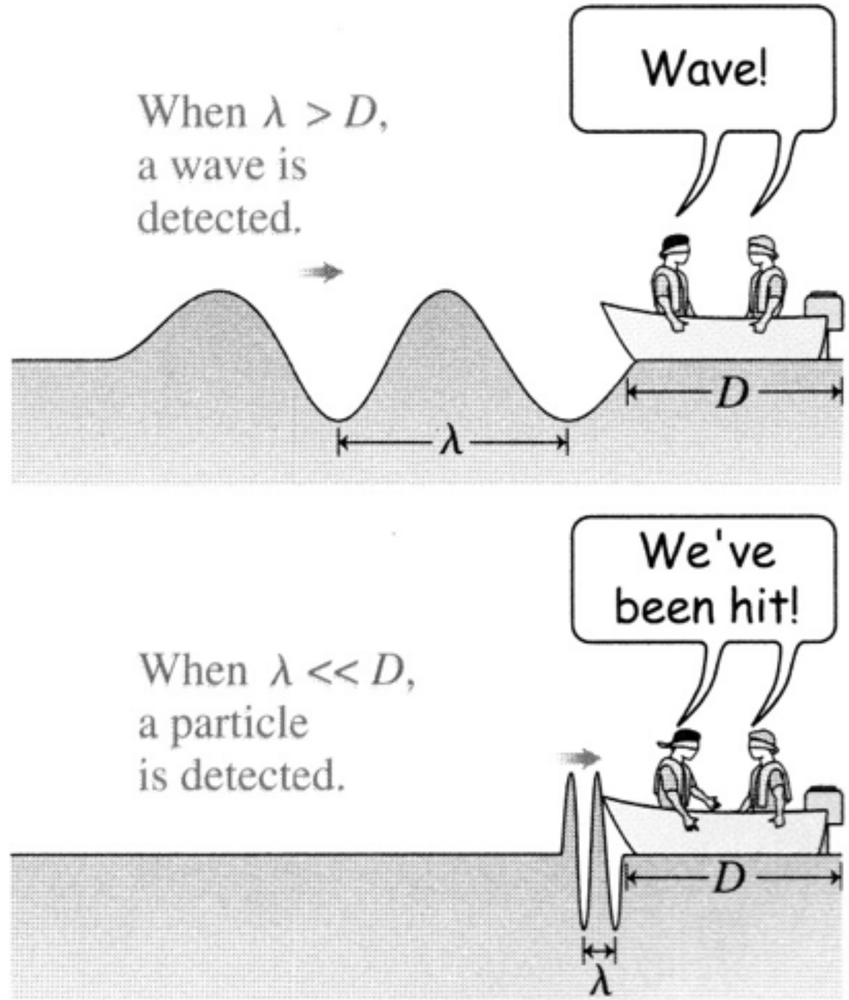
There is no predetermined wave-ness or particle-ness. The "phenomenon" that we observe as exhibiting both wave-like and particle-like properties.

The behavior depends on the comparison between the wavelength of the phenomenon and the relevant dimensions of the experimental apparatus.

What is meant by relevant dimensions?

Consider the Compton Effect: when the wavelength was small (x-rays) compared to the particle-behavior effects we noticed them. Relevant dimension in this case are the sizes of the atoms in the metal.

Is the phenomenon a particle or a wave?



$\lambda \ll D$: particle

$\lambda \gtrsim D$: wave

DUAL NATURE

The dual nature is not particle vs. wave, but rather complimentary between the two.

(particle) $E_{light} = hf$ (wave)

(particle) $p = \frac{h}{\lambda}$ (wave)

The relationship between wavelength + frequency and energy + momentum reflect this duality.

THE END
(FOR TODAY)