Welcome back to PHY 3305

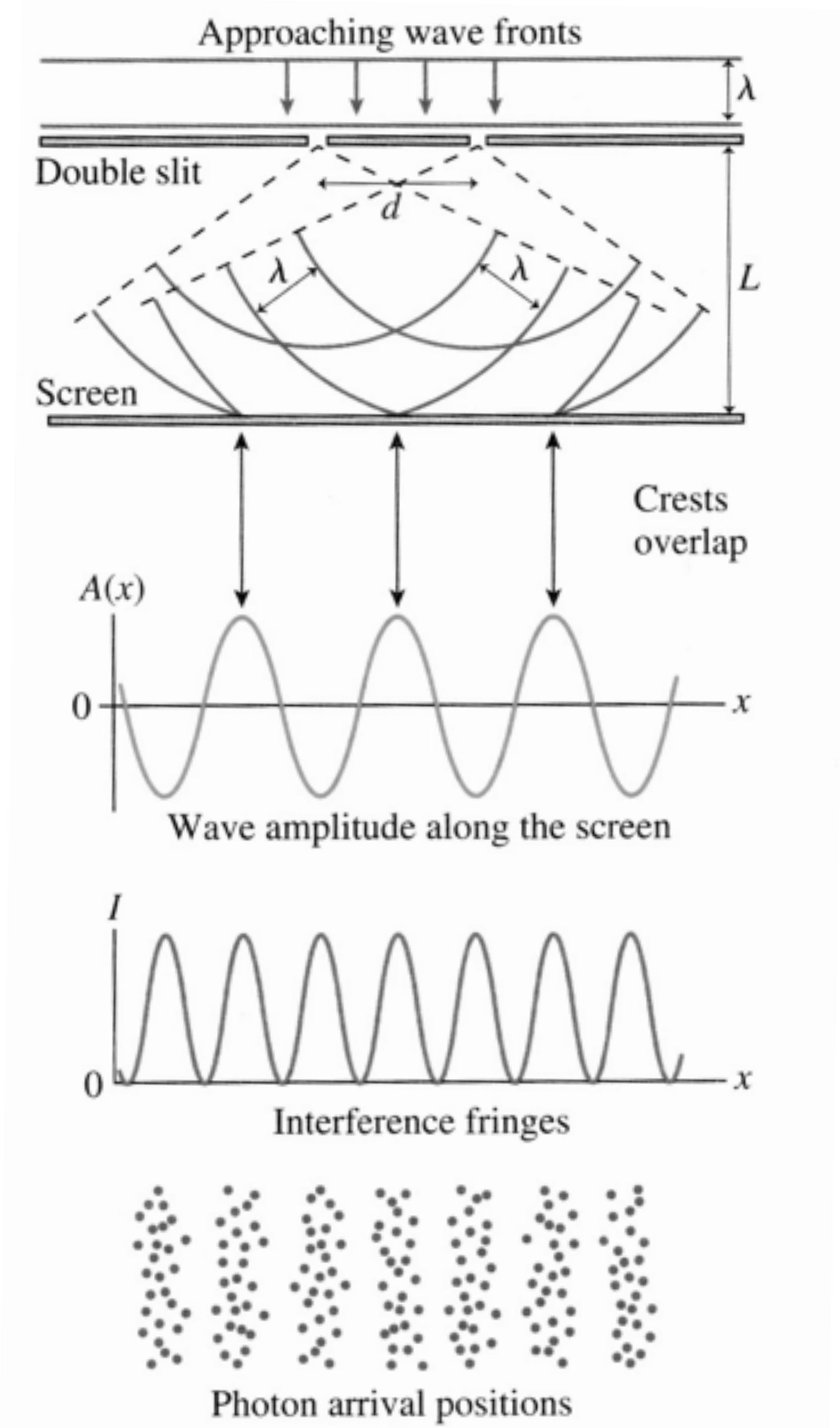
Today's Lecture:

Double Slit Experiment

Matter Waves

Louis-Victor-Pierre-Raymond, 7th duc de Broglie 1892-1987

DOUBLE-SLIT EXPERIMENT



- Photons pass through the double-slit apparatus.
- Crests of the two waves overlap, creating maxima amplitudes in the wave function.
- Fringes are observed at the maxima and minima in the wave amplitude.
- The observed fringes are the light's intensity (not amplitude).

$$I \propto A^2$$

$$P \propto A^2$$

What do we learn from the double slit experiment?

The probability of detecting a particle in a given location on the screen is related to the number of particles detected in that location.

more particles in a given place on the screen = higher chance of detecting a particle at that place

The density particles follows exactly the prediction for the intensity of light waves on the screen.

Light intensity is given mathematically by the SQUARE OF THE WAVE AMPLITUDE.

Putting it together:

When a phenomenon is detected as particles, we can not say for certain where it will be detected. We can only determine the probability of finding it in a certain location.

$$P \propto A^2$$

Probability is proportional to the square of the Amplitude of the associated wave.

- if the wave in question is an EM wave, its "associated particle " is the photon.
- if the particle in question is the photon, its "associated wave" is the EM field
- In E&M we learn that EM waves exert forces on electric charges, now we claim these waves also measure the probability of finding the photon.

Question: Which slit did the 17th photon pass through?

Detecting photons enroute to the slits or screen interferes with the wave/particle nature. In our case the single photons have to be thought of as passing the slits at the same time - wavelike behavior. Changing the experiment forces the photon to interact with something, exposing its particle nature and changing the outcome of the experiment.

Phet Demonstration illustrates this!

What is matter?

Light

1. Radiation emitted by moving electric charges.
2. No mass.
3. Is waves.

Matter

1. Has mass.
2. Is particles.

If light can exhibit both wave and particle properties, what about matter?

To see the dual nature of light we had to expose it to an apparatus with dimensions comparable to its wavelength. (Sodium street light, λ=0.6 μm passes through crack in door w/o diffracting. Diffracts easily though a slit of 1 μm.) No comparable apparatus for matter. ($\lambda_{matter} << \lambda_{light}$.)

EXAMPLE: ATOMS

- Electrons in atoms can only attain certain energies.
- Standing waves in a confined space can also only attain certain frequencies. (for dual natured situations this means discrete energies)
  - Think of a guitar string.
- Discrete behavior of atomic energy levels can be described as what happens to electrons when they are confined to something about the size of an atom (1 nm).
- Their wave nature becomes apparent and discretizes their allowed energy.

DOUBLE-SLIT EXPERIMENT FOR ELECTRONS

In light, we speak of the wave behavior as oscillating EM fields. What is oscillating in matter?

No one has ever 'seen' a matter wave. So, the only candidate of what is oscillating is probability.

(more on this later)

De Broglie's Hypothesis

- The relationships

  $f = E/h$ & $\lambda = h/p$

  apply to ALL particles, i.e. even those with mass

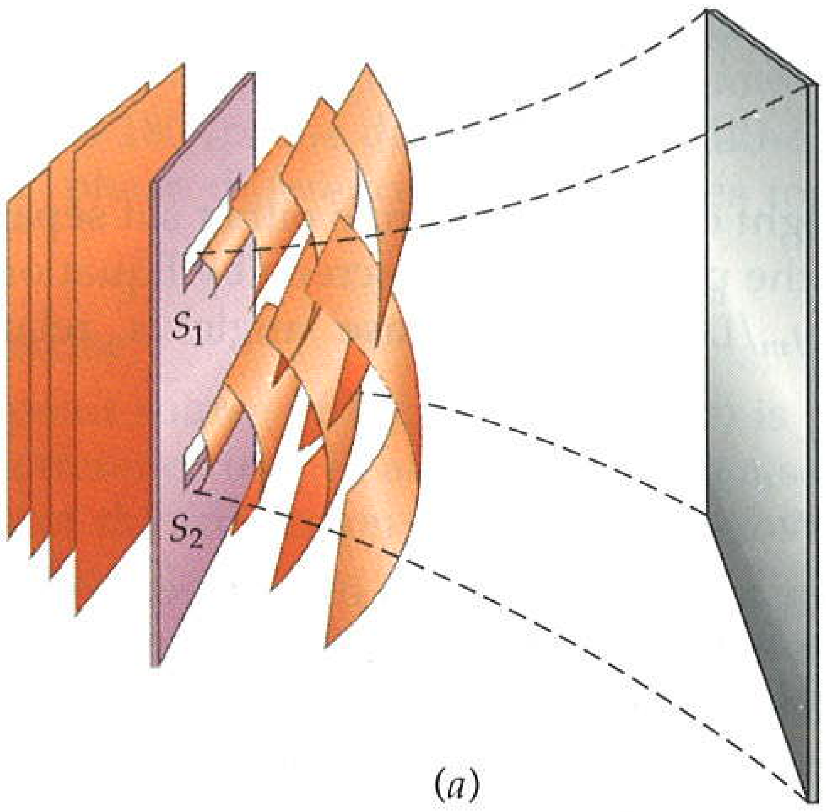
  ★ This was de Broglie's PhD thesis (1924), Nobel Prize (1929), λ in matter is called de Broglie wavelength

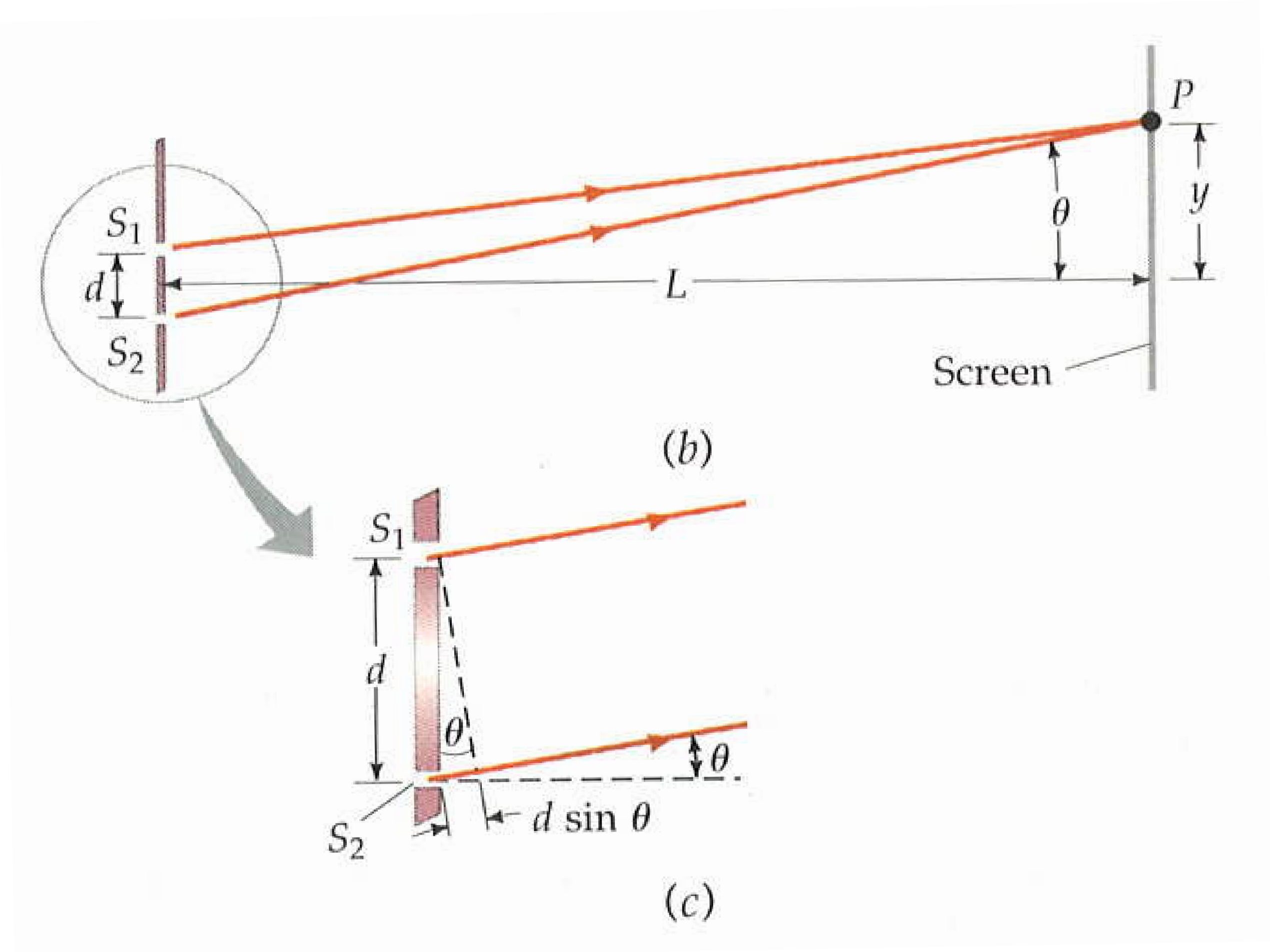
- **Why did de Broglie suggest this?**

  - There was no experimental evidence showing this behavior (yet)

  - The answer partly lies in relativity, e.g.
    $E^2 = (pc)^2 + (m_0c^2)^2$. There is nothing special about massless particles: $m_0=0$ is just another number.
    Unlike in classical mechanics $m_0=0 \Rightarrow$ there is no energy, or momentum, the particle doesn't "exist"

DOUBLE SLIT EXPERIMENT





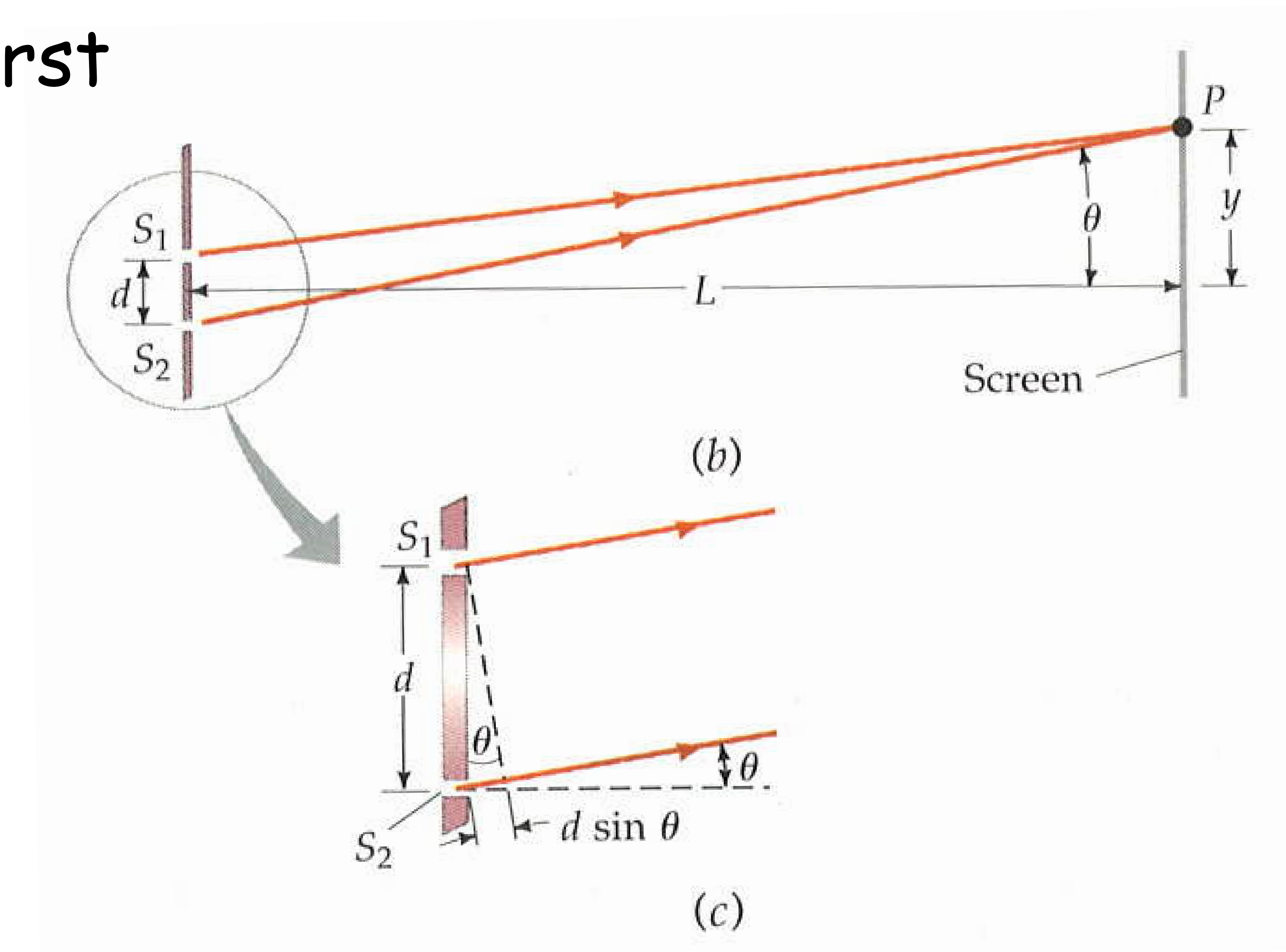
The minima occur at

$$\boxed{d \sin \theta = n\lambda}$$

DOUBLE SLIT CONTINUED

The distance from the central point to the first fringe is given by

$$\tan\theta = \frac{y}{L}$$



For small θ

$$\tan\,\theta \approx \sin\,\theta$$

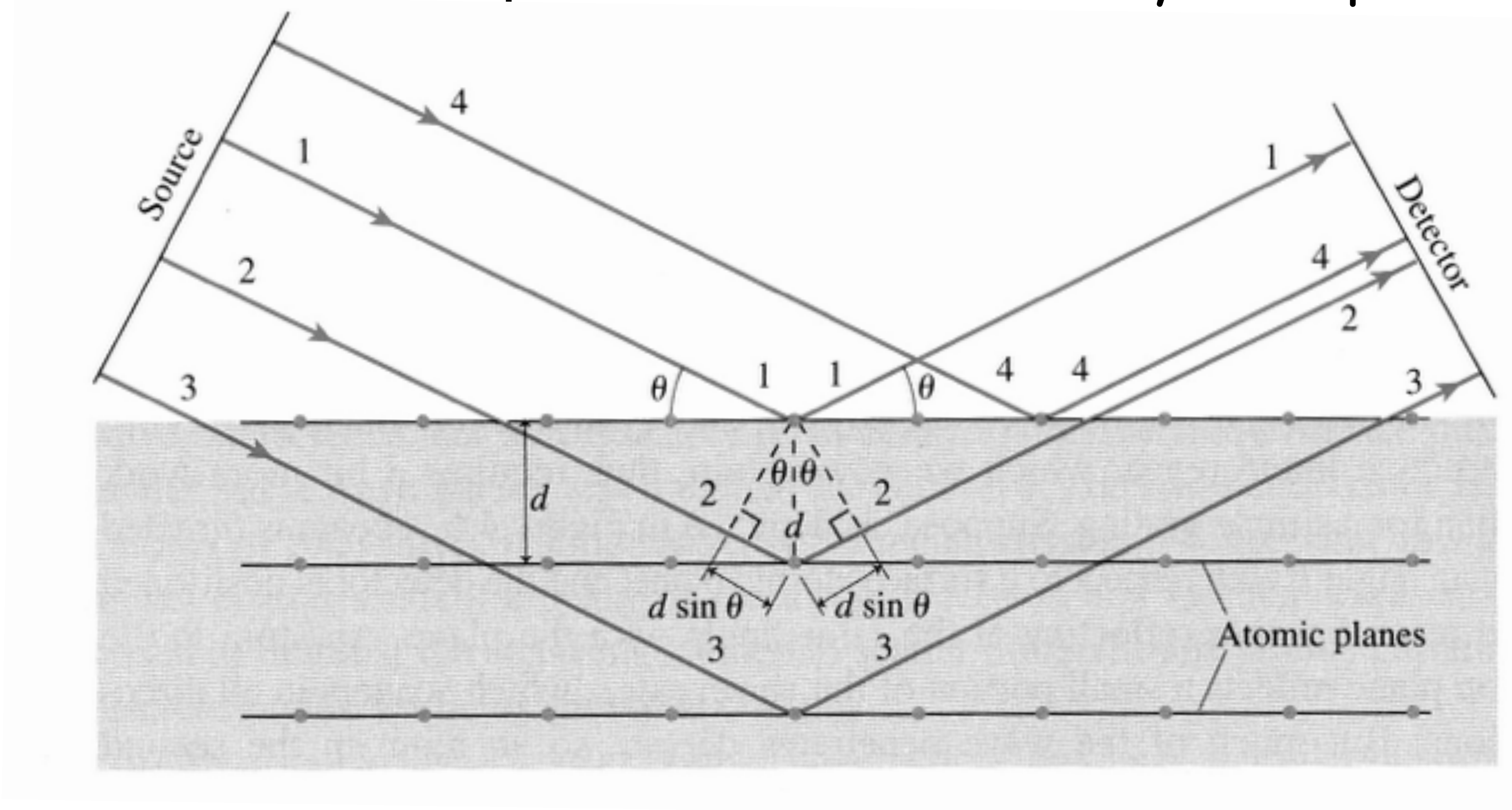
Thus,

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

$$d\sin\theta = d\frac{y}{L} \longrightarrow d\frac{y}{L} = n\lambda \longrightarrow \boxed{y = n\frac{\lambda L}{d}}$$

BRAGG SCATTERING

Father-son team of W. H. Bragg and W. L. Bragg were the first to show the interference pattern in matter. They used photons on a crystal.



Received the Nobel Prize in Physics (1915).

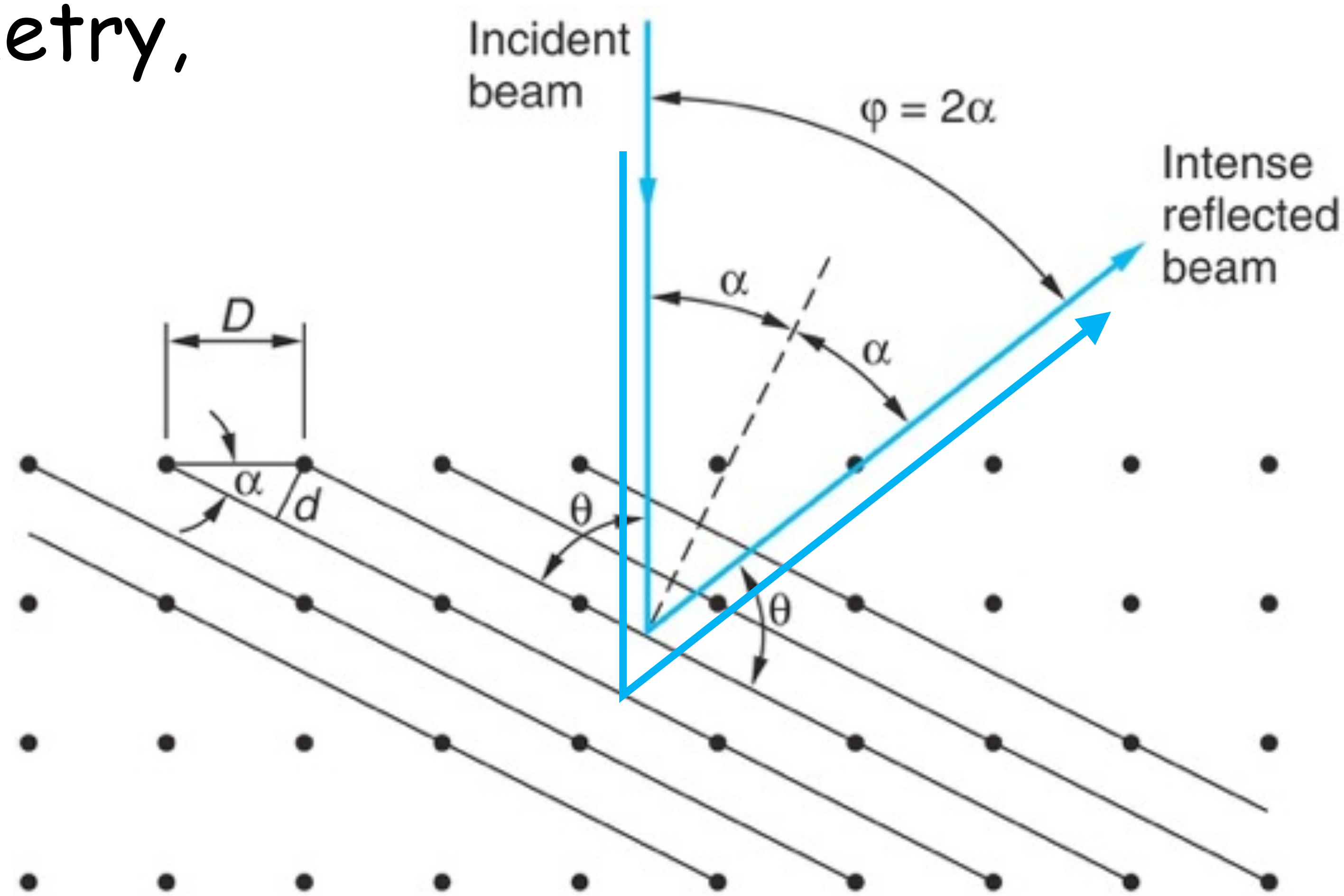
Notice that ray 2 has to travel 2dsinθ further than ray 1. Similarly ray 3 has to travel 2dsinθ further than ray 2.

$$\boxed{2d\sin\theta = n\lambda}$$ constructive interference

DAVISSION-GERMER EXPERIMENT

Crystal diffraction:

- Davission and Germer where the first to show that this interference also occurs using electrons.
- With a bit of trigonometry, one can also show constructive interference is seen when $n\lambda = D \sin\varphi$



Davission-Germer Experiment

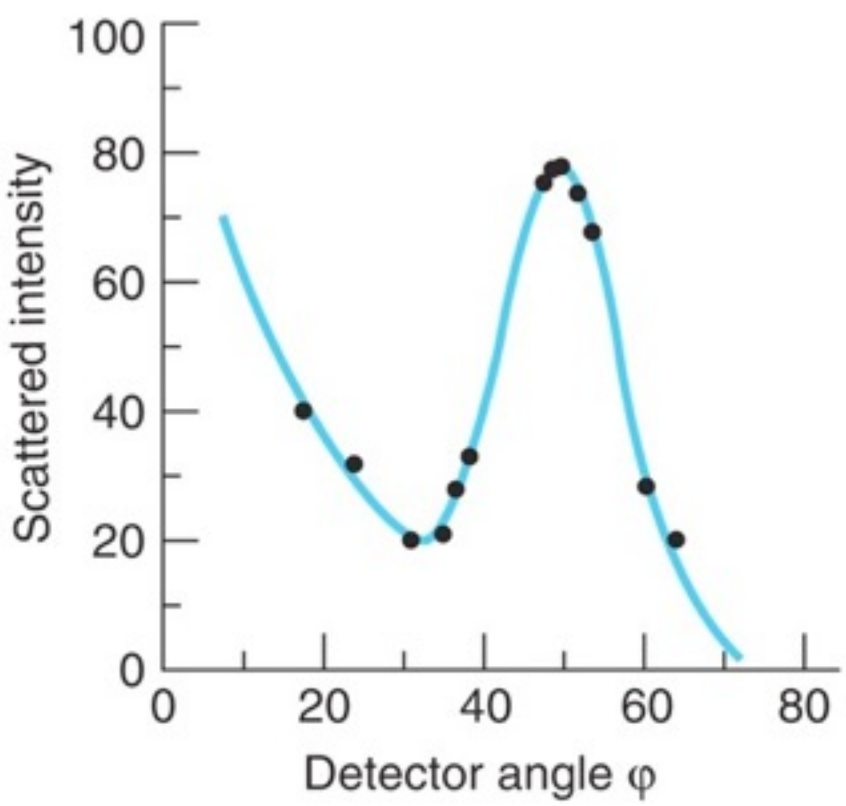
In 1927 Davisson & Germer did an experiment of shooting electrons with a kinetic energy of 54 eV at a Ni crystal. They observed a strong constructive interference peak at φ = 50° . Calculate the wavelength from the first interference peak.

D = crystal spacing of Ni = 0.215 nm

$$n\lambda = D \sin \phi$$

$$\lambda = (0.214nm) \sin 50$$

$$\boxed{\lambda = 0.165nm}$$



DAVISSION-GERMER EXPERIMENT

What does de Broglie's equation give us?

$$\lambda = \frac{h}{p}$$

Need to get p of the electron.

$$E^2 = (m_e c^2)^2 + (pc)^2$$

$$(pc)^2 = E^2 - (m_e c^2)^2 = (E_{KE} + m_e c^2)^2 - (m_e c^2)^2$$

$$= E_{KE}^2 + (m_e c^2)^2 + 2E_{KE} m_e c^2 - (m_e c^2)^2$$

$$= E_{KE}(E_{KE} + 2m_e c^2)$$

$$(pc)^2 = E_{KE}(E_{KE} + 2m_ec^2)$$

We were given E_{KE} = 54 eV and we know the rest mass of the electron, mc^2 = 511 keV.

$$E_{KE} + 2m_ec^2 \approx 2m_ec^2$$

since $E_{KE} \ll mc^2$

Thus,

$$(pc)^2 = 2E_{KE}(m_ec^2) \longrightarrow p = \sqrt{2E_{KE}m_e}$$

Substitute into the de Broglie equation.

note: $hc = 1240\ \text{eV} \cdot \text{nm}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2E_{KE}m_e}} = \frac{hc}{\sqrt{2E_{KE}m_ec^2}}$$

$$= \frac{1240eV \cdot nm}{\sqrt{2 \times 54eV \times (511 \times 10^3)eV}} \boxed{= 0.167nm}$$

Matches exp. value!

DAVISSION-GERMER EXPERIMENT

Experiment was repeated with different energies.

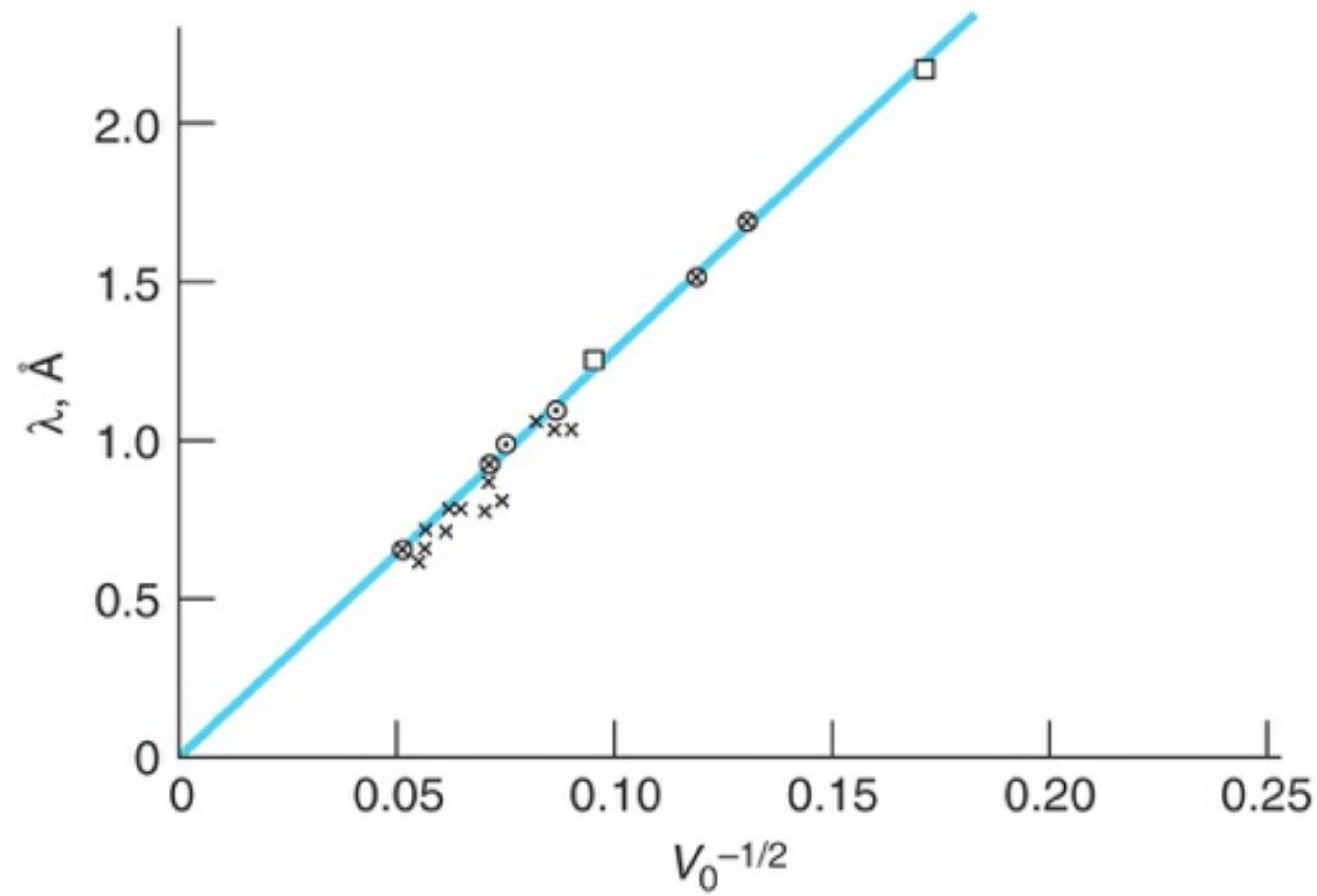
$$E_{KE} \propto V \quad \text{and} \quad \lambda = \frac{h}{p}$$

We just showed

$$p \propto \sqrt{E_{KE}}$$

So,

$$\lambda \propto \sqrt{\frac{1}{V}}$$



Davisson & Germer observe a linear relation between λ and $V^{-1/2}$

What happens to the wave properties of matter as you increase its speed?

The wavelength becomes shorter as the speed increases.

Classical physics $$v = \frac{p}{m} = \frac{h}{m\lambda}$$ de Broglie

The smaller the wavelength, the smaller the structure you can probe

Is it better to use matter or light to probe small structures?

matter - it has smaller wavelengths

ACCELERATING POTENTIAL

To put the wave nature of electrons to use, an accelerating potential is often the start. An accelerating potential gives the electron kinetic energy.

$$qV = \frac{1}{2}mv^2$$

V = accelerating potential for a particle of mass m and charge q

We know from classical physics and the de Broglie formula

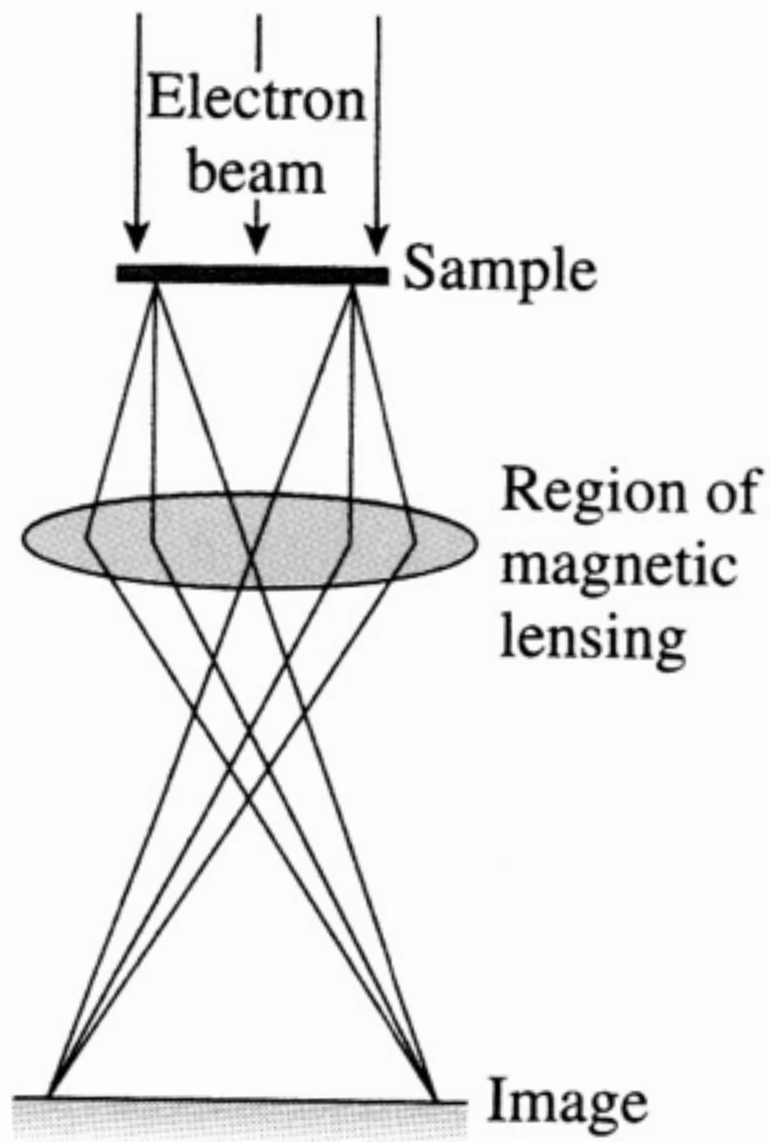
Classical physics $v = \dfrac{p}{m} = \dfrac{h}{m\lambda}$ de Broglie

Substituting,

$$V = \frac{h^2}{2mq\lambda^2}$$

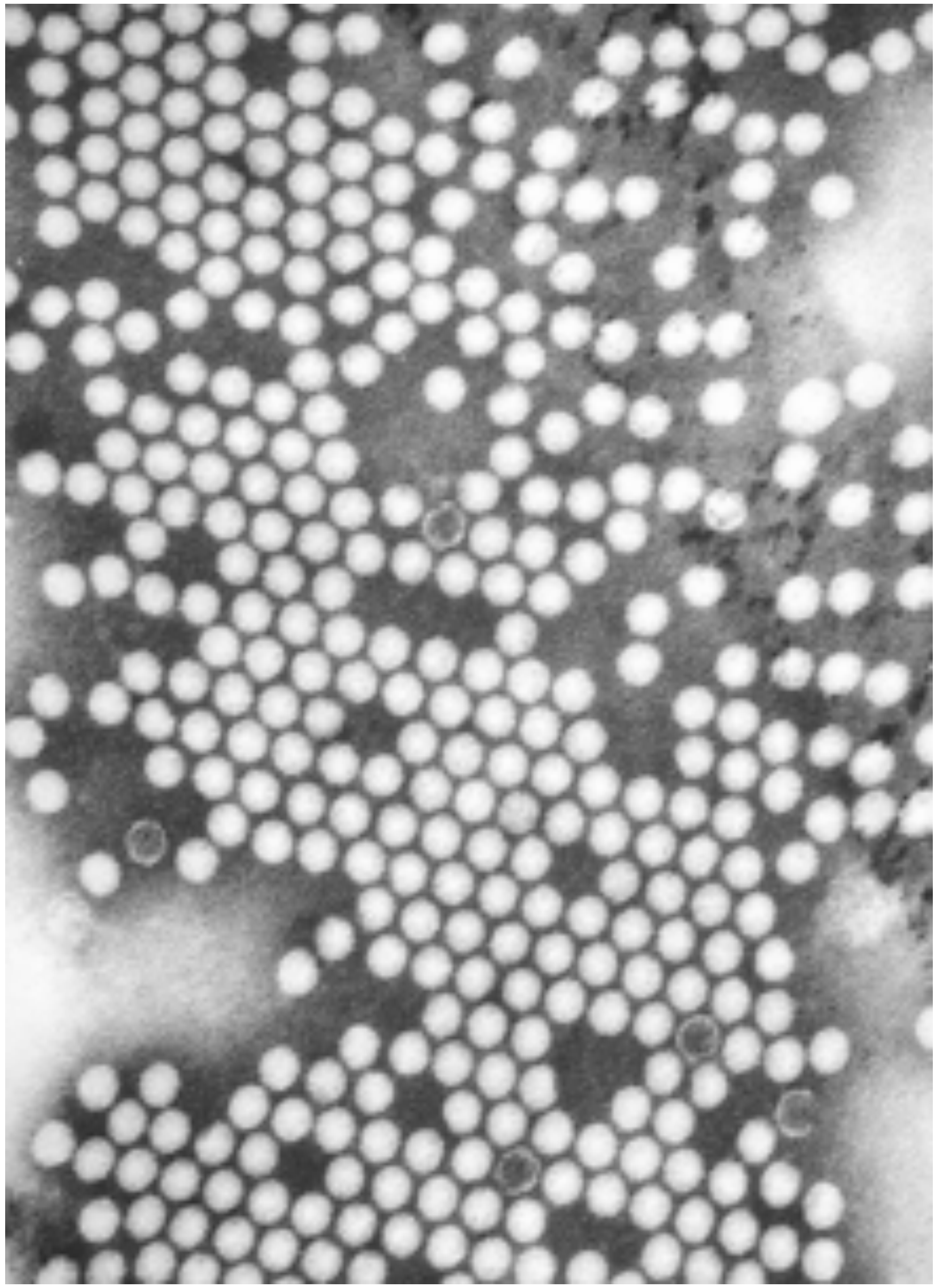
Transmission Electron Microscope (TEM):

An accelerating potential is one of the principles behind TEM - a tool of biology, engineering and surface science.



Basic principle: A beam of electrons is accelerated through a potential difference. After passing through a series of "lenses", the electron beam produces an image on a screen.

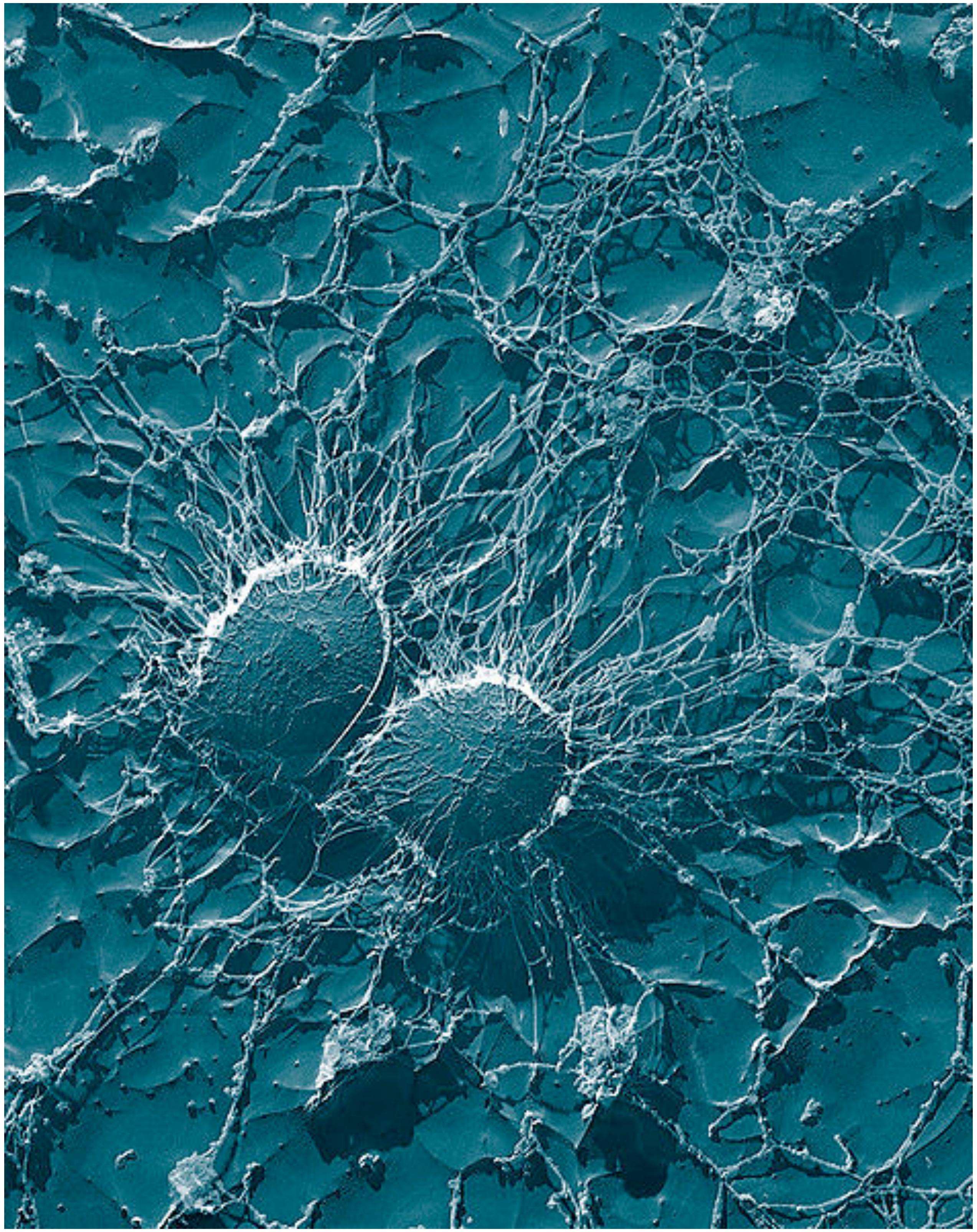
A microscope's resolution is limited by diffraction. Since photons have longer wavelengths than electrons, the electrons diffract less and hence reveal more detail.



Polio virus under TEM

- This virus is 30 nm in size.

http://en.wikipedia.org/wiki/Transmission_electron_microscopy



http://en.wikipedia.org/wiki/Transmission_electron_microscopy

Staphylococcus aureus
(Staff)
50,000x resolution

Low-Energy Electron Diffraction (LEED):

Another application that relies on accelerating potentials. LEED uses low accelerating potentials. Thus, the electrons do not penetrate as far.

Used to study the geometric structure of the atoms on the surface of an object.

EXAMPLE: LEED

To produce a good diffraction pattern an incident beam should have a wavelength comparable to the separation between the "slits" (the atoms that scatter the beam). A typical atomic spacing in a crystal is 0.2 nm. Approximately what potential difference do we need?

$$V = \frac{h^2}{2mq\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2(9.11 \times 10^{-31}kg)(1.6 \times 10^{-19}C)(0.2 \times 10^{-9}m)^2}$$

$$\boxed{V = 38V}$$

Hints: m_e = 9.11 x 10^{-31} kg, q_e = 1.6 x 10^{-19} C

THE END
(FOR TODAY)

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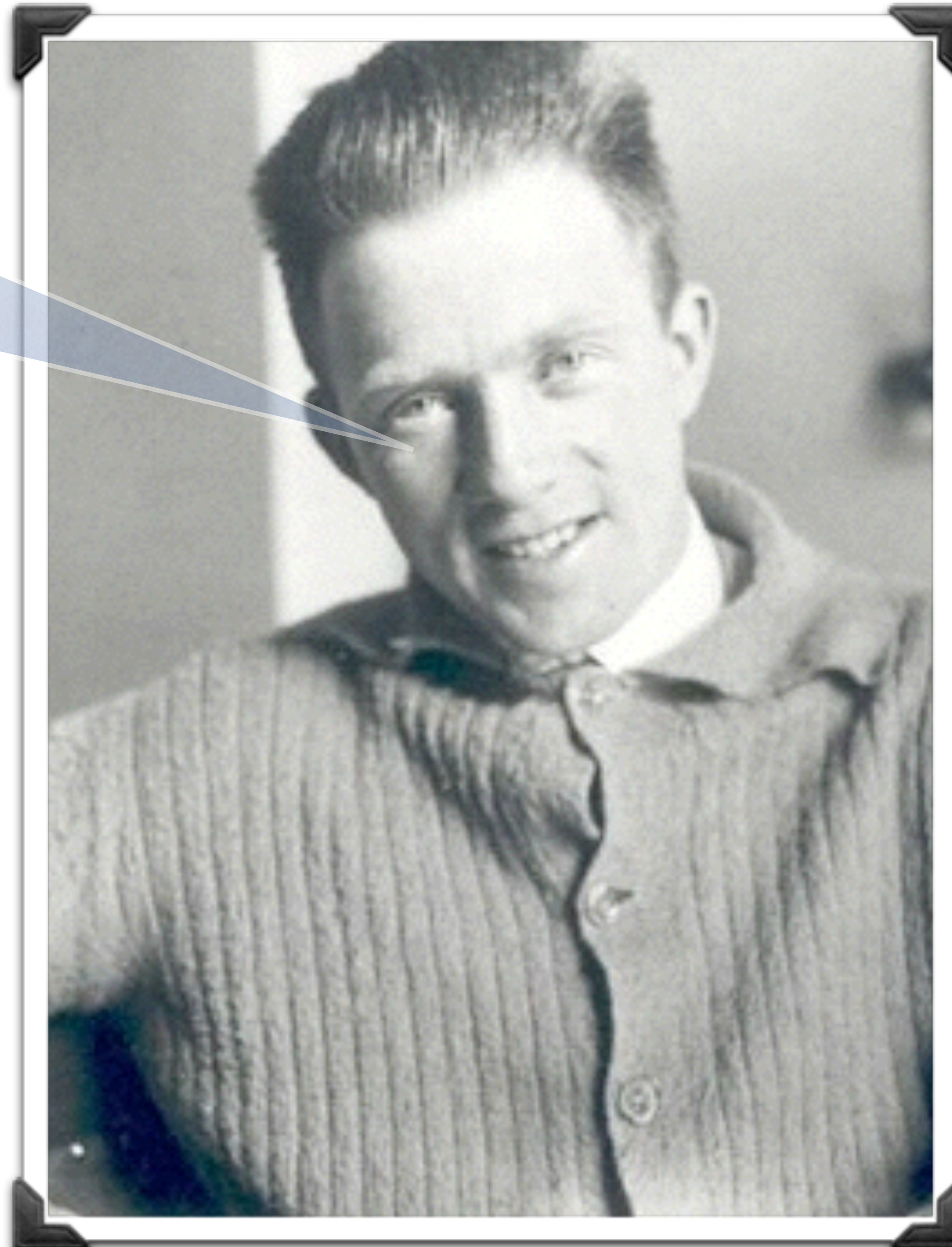


Today's Lecture:

Uncertainty Principle

Werner Heisenberg
1901-1976

When you perform an experiment, do you get the exact same result every time?

No. There is a fundamental uncertainty about the exact properties of a system.

How do we measure uncertainty in physics (and other disciplines)?

Mean:

$$\bar{Q} = \frac{\sum_i Q_i n_i}{\sum_i n_i}$$

Standard Deviation:

$$\Delta Q = \sqrt{\frac{\sum_i (Q_i - \bar{Q})^2 n_i}{\sum_i n_i}}$$

In quantum mechanics, we use de Broglie waves to describe particles.

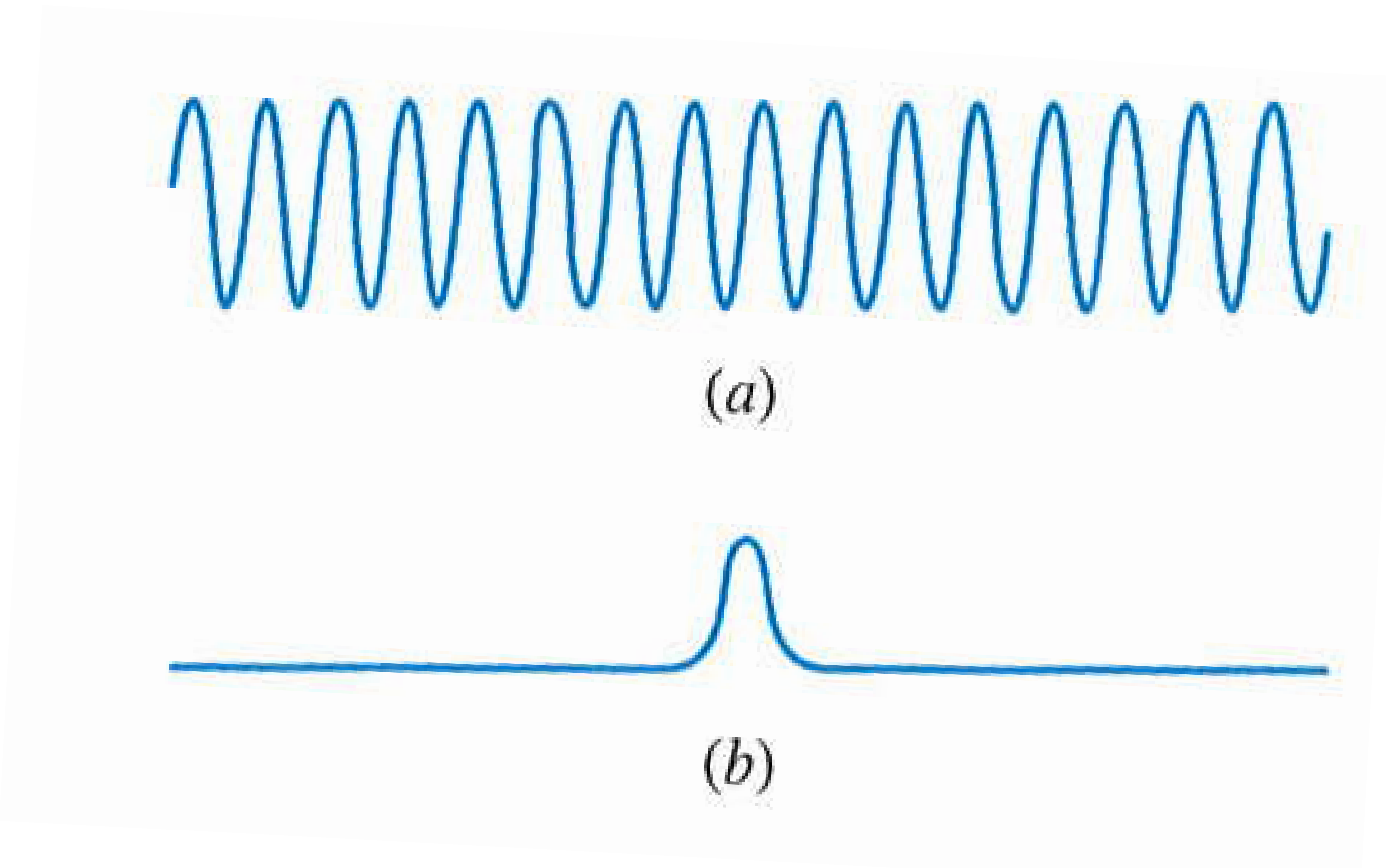
The wavelength tells us about the momentum of the particle.

$$\lambda = \frac{h}{p}$$

In quantum mechanics, the better we know a particle's position the less we know about it's momentum. The more we know about it's momentum, the less we know about its position.

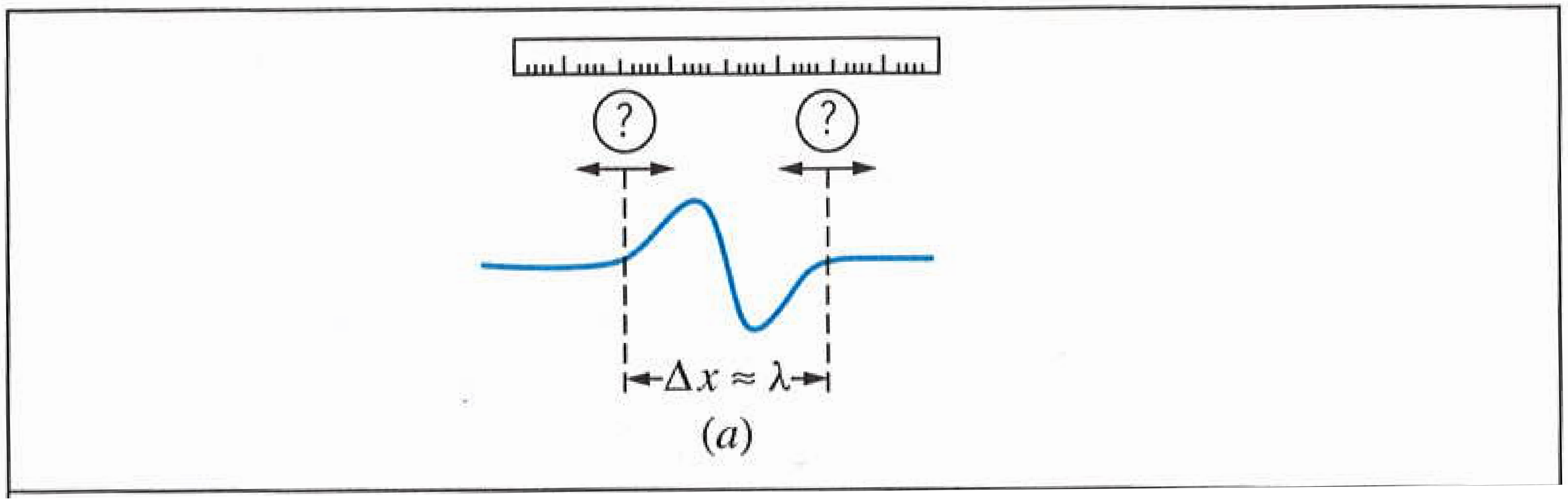
Which wave is better at telling us the location of the particle?

Which wave is better at telling us the wavelength of the particle?



Better for wavelength.

Better for position.



Measure the wavelength of this wave packet.

We may have difficulty finding the exact ends of the wave. Thus, we have an uncertainty $\Delta\lambda$.

$$\Delta\lambda \sim \epsilon\lambda$$

ε is a fraction of the wavelength

We want to examine the product of the size of the wave packet and the uncertainty in wavelength. In this case $\Delta x \sim \lambda$, so

$$\Delta x \Delta\lambda \sim \epsilon\lambda^2$$

Note the inverse relationship between size of the wave packet and the uncertainty in wavelength. As Δx gets smaller $\Delta\lambda$ gets larger.

What if we make the wave packet larger?

There are N cycles of the wave - thus

$$\Delta x \sim N\lambda$$

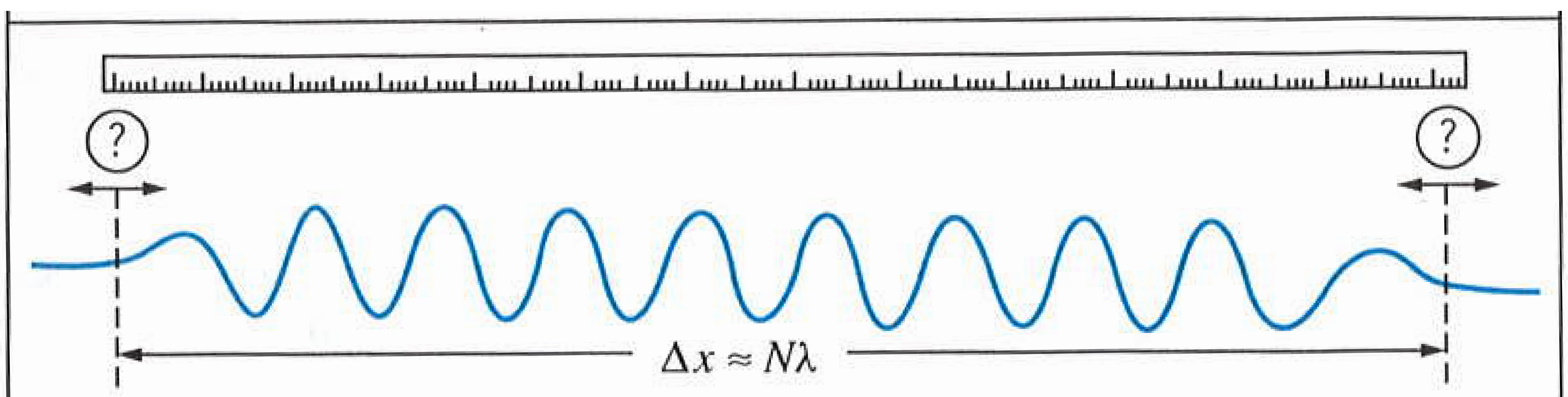
Uncertainty in the endpoints

$$\Delta\lambda \sim \frac{\epsilon\lambda}{N}$$

Combine together

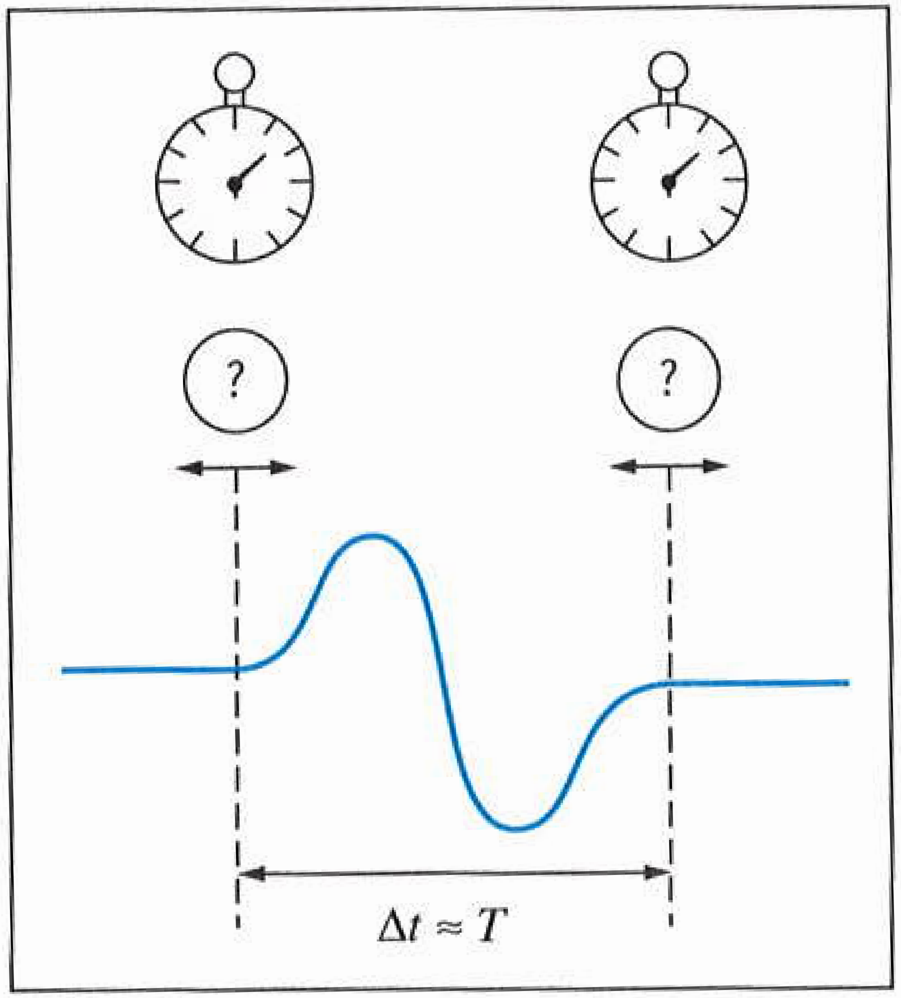
$$\Delta x \Delta\lambda \sim N\lambda\frac{\epsilon\lambda}{N} = \epsilon\lambda^2$$

Same result as case of smaller wave packets!



(b)

What if we measure period instead of wavelength?



The "size" of the wave packet is now a duration in time (one period).

$$\Delta t \approx T$$

We still have the difficulty of locating the start and end of the wave.

$$\Delta T \sim \epsilon T$$

ε is a fraction of the period

We want to examine the relationship between the duration of the wave packet and our ability to measure its period.

$$\Delta t \Delta T \sim \epsilon T^2$$

For a wave packet of a given period, the smaller the duration of the wave packet, the larger the uncertainty in our measurement of the period.

What if we want to write it in terms of frequency and not period?

$$f = \frac{1}{T} \xrightarrow{?} \cancel{\Delta f = \frac{1}{\Delta T}}$$

Do the calculus:

$$df = -\frac{1}{T^2} dT$$

Now convert. Note we can ignore (-) because we are interested in the magnitude of the uncertainties.

$$\Delta f = \frac{1}{T^2} \Delta T$$

Combine with $\Delta t \Delta T \sim \epsilon T^2$

$$\Delta f \Delta t \sim \epsilon$$

The longer the duration of the wave packet, the more precisely we can measure its frequency.

$$\Delta x \Delta \lambda \sim \epsilon \lambda^2$$

Apply to de Broglie Waves

$$p = \frac{h}{\lambda}$$

Take differential

$$dp = -\frac{h}{\lambda^2} d\lambda \longrightarrow \Delta p = \frac{h}{\lambda^2} \Delta \lambda$$

Combine with our equation relating λ and x.

$$\Delta p \sim \frac{h}{\lambda^2} \frac{\epsilon \lambda^2}{\Delta x}$$

$$\Delta p \Delta x \sim \epsilon h$$

The smaller the size of the wave packet, the larger the uncertainty in its momentum.

Last details:

There is a formal procedure for calculating Δx and Δp. The outcome of these calculations gives the wave packet with the smallest possible value of the product $\Delta x\, \Delta p$ as $h/4\pi$. (section 4.7 of your book).

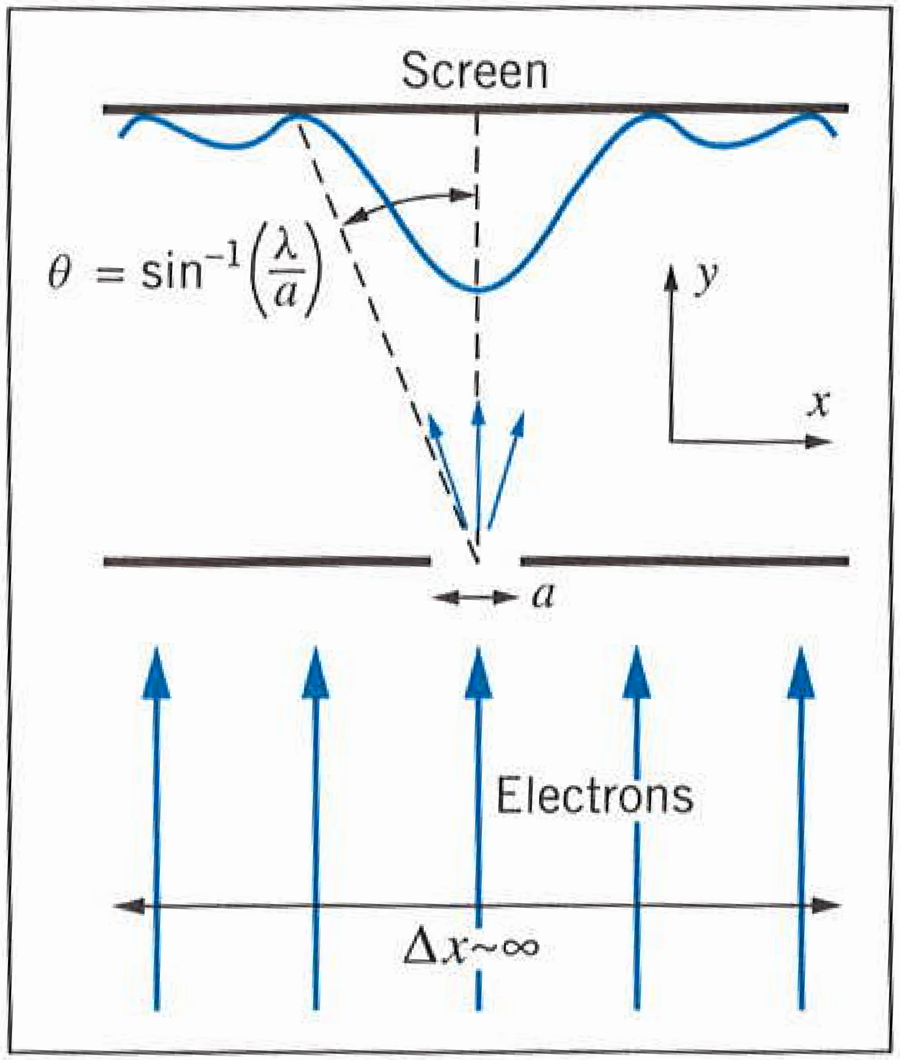
$$\boxed{\Delta p_x \Delta x \geq \frac{\hbar}{2}}$$

Nobel Prize: 1932

Heisenberg Uncertainty Principle:

Because of a particle's wave nature, it is theoretically impossible to know precisely both its position along an axis and its momentum component along that axis; Δx and Δp can not be zero simultaneously. There is a strict theoretical lower limit on their product.

EXAMPLE: SINGLE-SLIT DIFFRACTION



Initial:

$\Delta p_x = 0$, all momentum in y-direction

Δx = infinity, we know nothing about position.

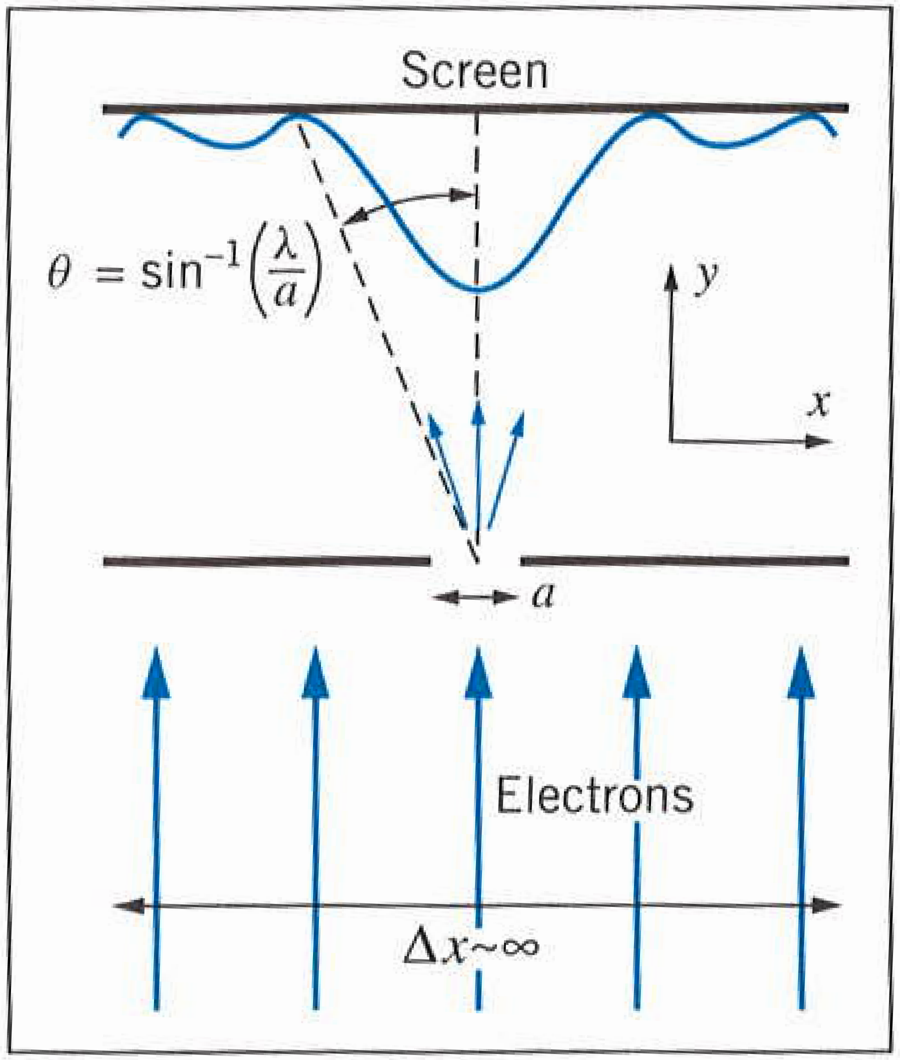
Pass through slit:

We know that their x location is no larger than a. $\Delta x = a$

$$\Delta p_x \geq \frac{\hbar}{2a}$$

The first minima in the diffraction pattern is given by

$$\sin\theta = \frac{\lambda}{a}$$



Find the angle θ that specifies where a particle with this value of lands on the screen.

for small θ $\quad \sin\theta \approx \tan\theta = \dfrac{p_x}{p_y} = \dfrac{\frac{\hbar}{2a}}{p_y}$

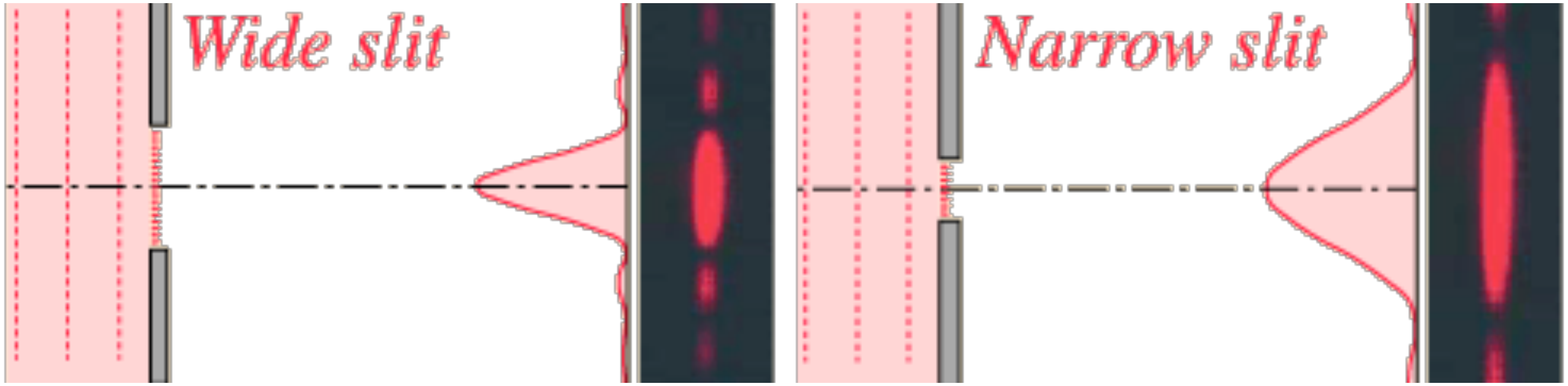
Use λ = h/p_y, the de Broglie wavelength of the electrons.

$$\sin\theta \approx \frac{\lambda}{4\pi a}$$

The first minima in the diffraction pattern is given by

$$\sin\theta = \frac{\lambda}{a}$$

transverse momentum given by uncertainty principle is roughly equivalent to spreading of the beam.



The diffraction (spreading) of the beam is an effect of the uncertainty principle. As the slit becomes narrow, p_x increases and the beam spreads even more.

There is a trade off in knowing the position (x) and the momentum (x-dir) of the particle.

What about our second relationship?

$$\Delta f \Delta t \sim \epsilon$$

Use the energy-frequency relationship for light

$$E = hf$$

$$\Delta E = h \Delta f$$

Substitute

$$\Delta E \Delta t \sim \epsilon h$$

Again there is a formal procedure for these calculations. Here I give the result.

$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$$

INTERPRETATION

If a state or particle exists for only a limited span of time, it's energy is uncertain.

- life span of some subatomic particles is quite short (10^{-20} s) which leads to considerable uncertainty in their mass/energy
- state temporarily occupied by an electron as it jumps down an energy level in an atom - since the state is occupied for a finite time, its energy is uncertain by some amount. This gives rise to broadening of spectral lines.

UNCERTAINTY PRINCIPLE AND DARK ENERGY

Observations of the universe indicate that not only is the universe expanding, the expansion of the universe is accelerating. This acceleration is said to be caused by the dark energy in the universe. One leading candidate for dark energy is <u>vacuum energy</u>.

The idea here is that particle-antiparticle pairs would spontaneously appear and then annihilate in an otherwise empty vacuum. The total energy and lifetime of these particles must satisfy the uncertainty principle.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

It has been suggested that the vacuum energy density is the Plank energy density.

$$\mathcal{E} \sim \frac{E_P}{\mathcal{L}_P^3} = \frac{1.2 \times 10^{28} eV}{(1.6 \times 10^{-34} m)^3} = 3 \times 10^{133} \frac{eV}{m^3}$$

$$\mathcal{L}_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

$$E_P = \sqrt{\frac{\hbar c^5}{G}} = 1.2 \times 10^{28} eV$$

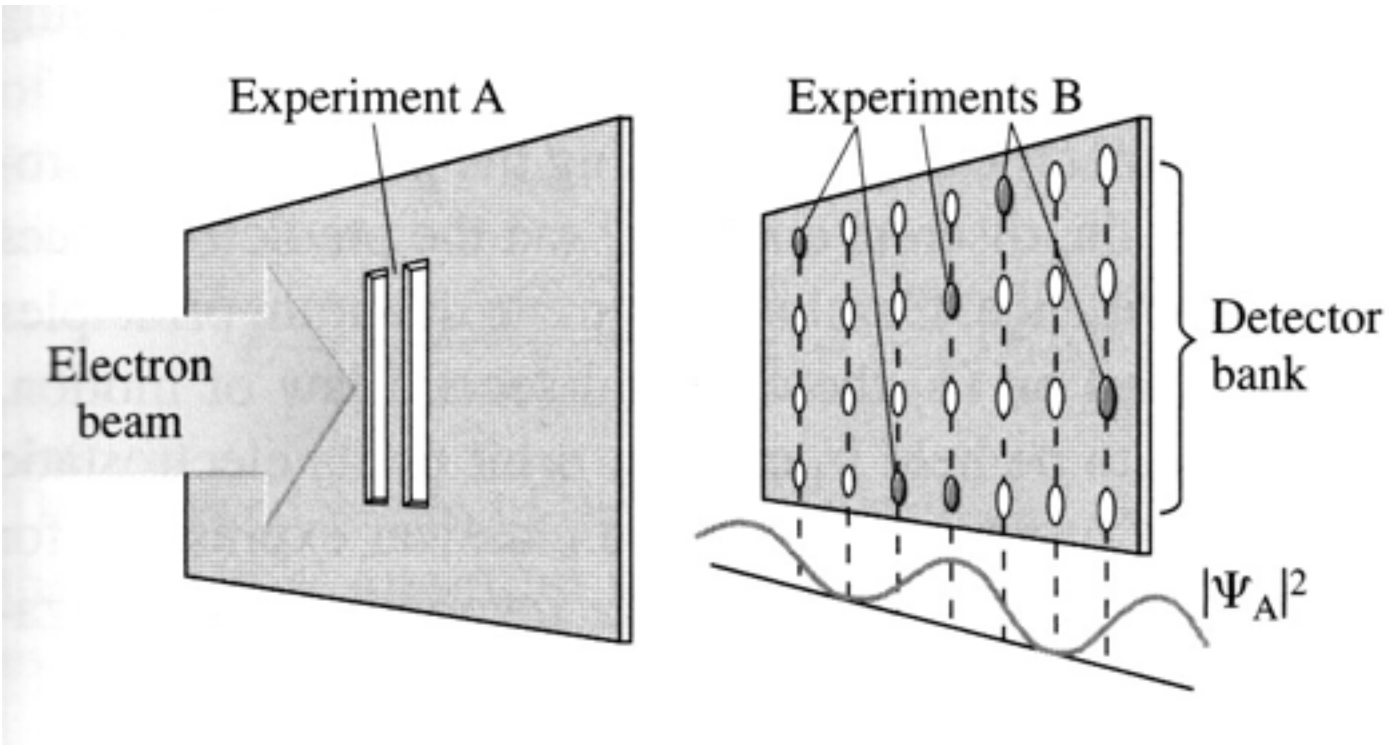
This is 124 orders of magnitude larger than the current critical energy density (required for flatness) of the universe!

LIMITATION OF KNOWLEDGE

Summary of what we know so far:

- The equations describing particles and forces can be very precisely stated.
- The wave function encodes all properties of matter
- The wave function, by its nature, prevents us from knowing both momentum and position (or energy and time) precisely at the same time

Revisit Double Slit:



Experiment A, "the slit" establishes an initial wave function Ψ_A

Experiment B "screen" detects the particle.

Where Ψ_A is large, many particles are registered, where it is zero, no particles are registered.

What happens if we conduct the intermediate experiment to determine through which slit the particle passes?

This experiment alters the result - it alters the wave function. To observe interference we must allow the particles wave function to pass through both slits simultaneously.

COPENHAGEN INTERPRETATION

If we can not know the location of a particle until we actually look for it, how can we justify the claim it has a location?

The modern interpretation of this is know as the **"Copenhagen Interpretation"**.

Until the experiment actually localizes the particle, it does not have a location.

SUMMARY

- Classical physics gave us the ideas of position and velocity.
- Quantum mechanics allows us to only know probabilities and corresponding uncertainties passed on the most recent observation of the "particle".
- A determination of one property is liable to alter another property.

THE END
(FOR TODAY)