Today's Lecture:

Matter Waves

Schrodinger Equation

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Alexander Schrödinger
1887-1961

ANNOUNCEMENTS

- Reading Assignment for Tuesday, October 3rd: Chapter 4.4 - 4.5.
- Problem set 6 is due Tuesday, October 3rd at 12:30 pm.
- Regrade for problem set 5 is due Tuesday, October 3rd at 12:30 pm.
- Dr. Cooley will be out of town Thursday, October 5th. Mr. Thomas will be in class to lead the lecture discussion in her place. Problem set 6 is due Tuesday, October 4th at 12:30 pm.

REVIEW QUESTION 1

Interference of sub-atomic particles is evidence that:

A) the particles have very small mass

B) the particles exhibit wavelike behavior

C) the particles interact electromagnetically

D) the particles are moving relativistically

E) the particles have very large mass

From Video Lecture:

Now we can rewrite the particle/wave relationships in terms of the wave number and angular frequency.

$$k \equiv \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = 2\pi f$$

thus,

$$\boxed{E = hf = \hbar\omega} \quad \text{and} \quad \boxed{p = \frac{h}{\lambda} = \hbar k}$$

where

$$\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} J \cdot s$$

Notes on Velocity:

Need to be careful when dealing with matter waves.

$$v_{wave} = f\lambda = \frac{E}{h}\frac{h}{p} = \frac{E}{p}$$

This, however, is NOT necessarily the speed of the particle.

- This relationship confirms that E = pc for light.
- However, matter waves would need to travel at c if this equation were to hold for massive particles — it does not!
- It is also the case that E does NOT equal $pv_{particle}$.
- We will discuss later in the course wave (phase) and particle (group) velocities.

If E is not equal to $pv_{particle}$, what is the energy of a massive free particle?

For a nonrelativistic particle:

$$E = \frac{1}{2}mv_{particle}^2 = \frac{1}{2}pv_{particle}$$

$$\boxed{v_{particle} = \frac{2E}{p}}$$

For a matter wave:

$$\boxed{v_{wave} = \frac{E}{p}}$$

Consider relativistic effects for a massive particle.

$$p = \gamma m v_{particle}$$

$$E = \gamma m c^2$$

$$\frac{p}{E} = \frac{\gamma m v_{particle}}{\gamma m c^2}$$

$$\frac{p}{E} = \frac{v_{particle}}{c^2}$$

Even relativistically,
E is not equal to $pv_{particle}$.

$$\boxed{v_{particle} = \frac{pc^2}{E}}$$

Check this out ...

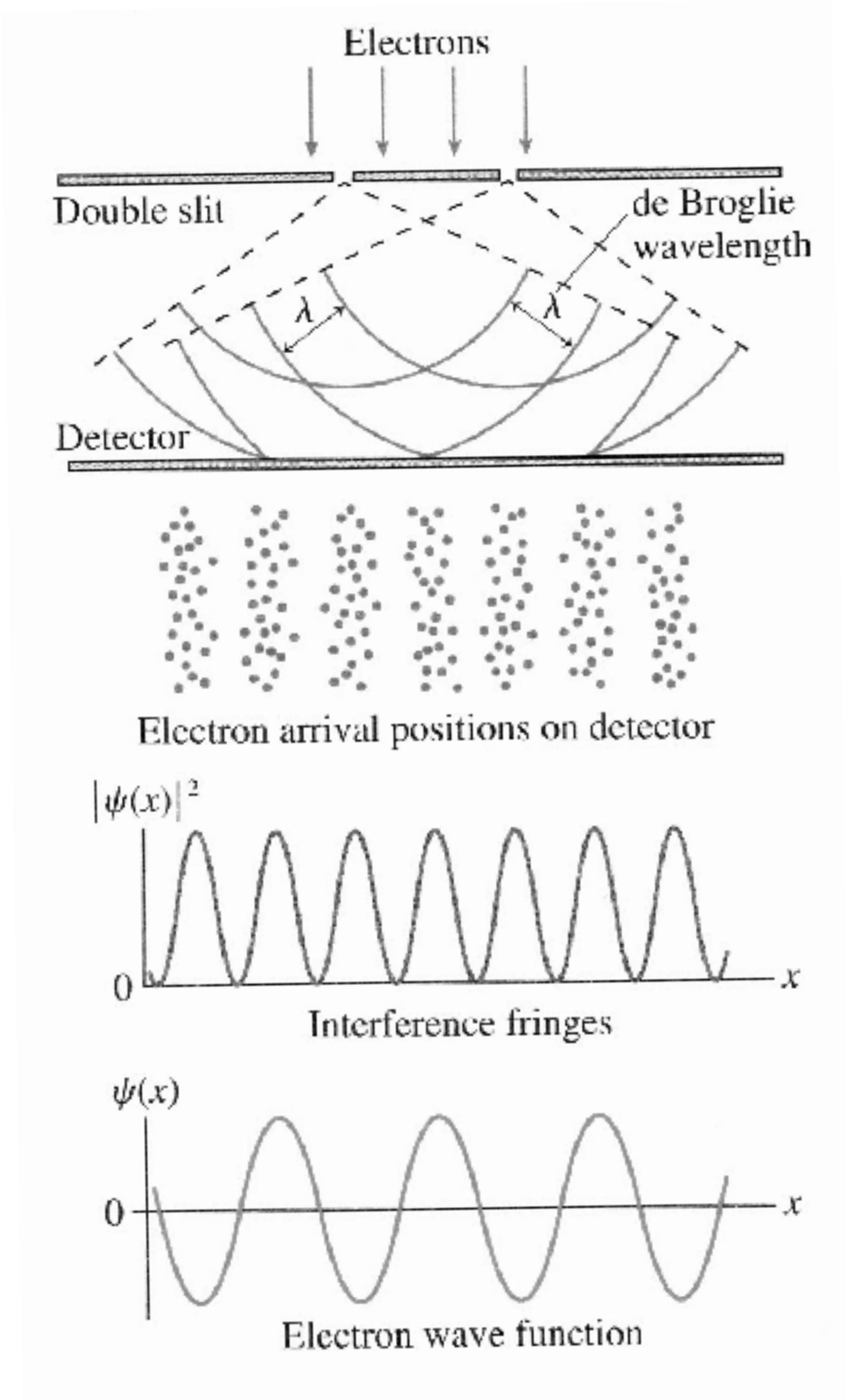
$$v_{wave} = \frac{E}{p} = \frac{c^2}{v_{particle}}$$

$v_{particle} < c$

$$\longrightarrow v_{wave} \geq c$$

The wave is superluminal!

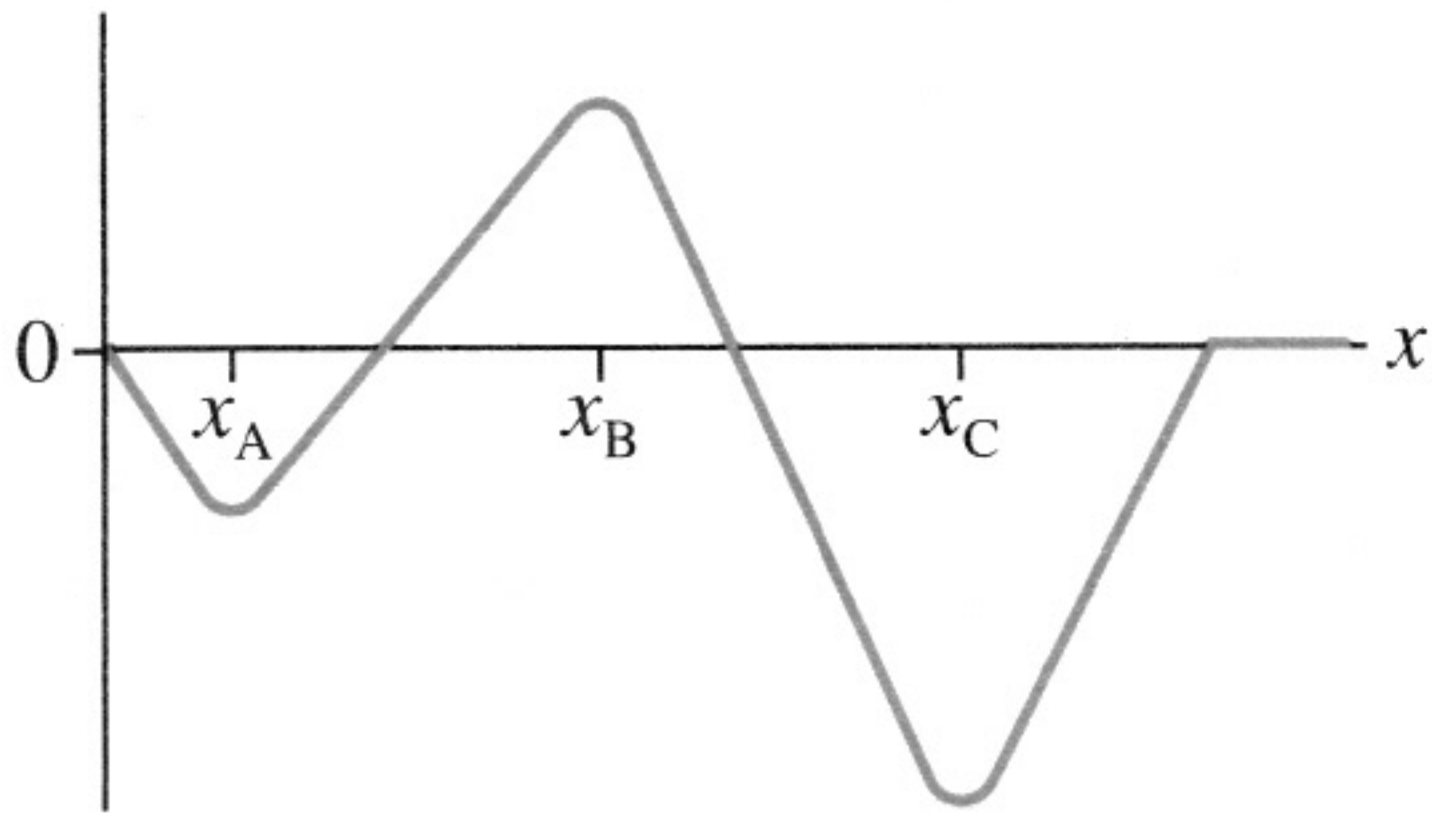
From Video Lecture:



- Denote the amplitude of a matter wave by ψ.
- The intensity of the wave is given by $|\psi|^2$.
- Hence, the probability for finding a particle at a given location is

$$\boxed{P \propto \psi^2}$$

QUESTION



This is the wave function of a neutron. At what value of x is the neutron most likely to be found?

Ans = C. The probability is largest at the point where the square of ψ^2 is largest.

From Video Lecture:

Schrodinger Equation for a free particle:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

The equation **IS** a law - like Newton's laws or Maxwell's Equations for E&M.

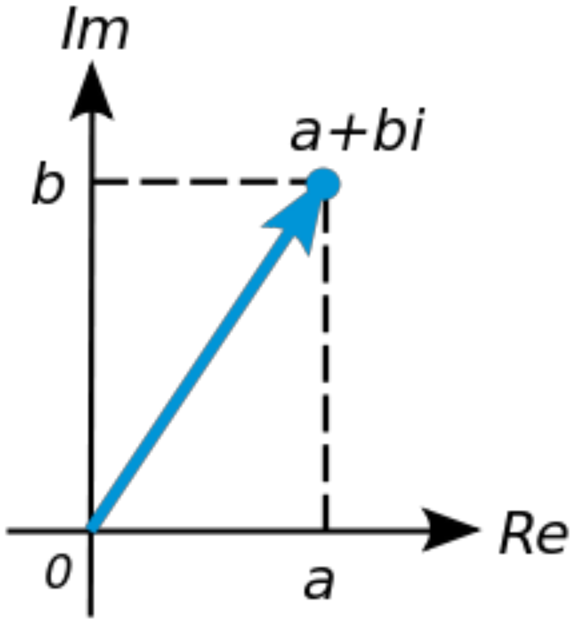
Furthermore, the wave function can be used to find the probability density (probability of finding a particle in a unit volume).

$$\text{probability density } = |\Psi(x,t)|^2$$

From Video Lecture:

Complex numbers:

What does it mean to square a complex number?



$$z = a + bi$$

Complex conjugate:

$$z^* = a - bi$$

$$|z|^2 = zz^* = a^2 + b^2$$

$$|z| = \sqrt{zz^*} = \sqrt{a^2 + b^2}$$

Consider the following complex function.

$$f(x) = ae^{ix}$$

What are it's real and imaginary parts?

$$e^{ix} = 1 + (ix) + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \dots$$

$$e^{ix} = 1 + (ix) - \frac{(x)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(x)^4}{4!} + \frac{(ix)^5}{5!} + \dots$$

$$e^{ix} = 1 - \frac{(x)^2}{2!} + \frac{(x)^4}{4!} + (ix) + \frac{(ix)^3}{3!} + \frac{(ix)^5}{5!} + \dots$$

$$e^{ix} = \cos(x) + i\sin(x)$$

Note: the real and imaginary parts are 90 degrees out of phase with one another: cos(x) = sin(x + π/2)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

The plane wave solution:

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

A is a constant

If we insert this into the SWE, we obtain

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega$$

Note: The functional dependence on time cancels. Thus, this function obeys Schoedinger's equation for all space and time.

$$\frac{\hbar^2 k^2}{2m} = \hbar\omega$$

Relate plane waves to particles.

$p = \hbar k$ and $E = \hbar\omega$

$k = \dfrac{p}{\hbar}$ and $\omega = \dfrac{E}{\hbar}$

Substitute

$$\frac{\hbar^2 p^2}{2m\hbar^2} = \frac{\hbar E}{\hbar}$$

$$\boxed{\frac{p^2}{2m} = E}$$

$$\frac{p^2}{2m} = E$$

What happens in the low velocity limit?

$$p = mv$$

Substitute,

$$\frac{m^2v^2}{2m} = E$$

$$\boxed{\frac{1}{2}mv^2 = E}$$

This is the energy of a particle whose velocity << c.

An electron moves along the x-axis with a well-defined momentum of 5.0×10^{-25} kg m/s. Write an expression describing the matter wave associated with this electron.

The matter wave is given by

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

Need to find k and ω.

$$p = \hbar k \longrightarrow k = \frac{p}{\hbar} = \frac{5.0 \times 10^{-25}\ kg \cdot m/s}{1.055 \times 10^{-34}\ J}$$

$$\boxed{k = 4.7 \times 10^{9}\ m^{-1}}$$

$$k = 4.7 \times 10^9\ m^{-1}$$

$$E = \hbar\omega \quad \longrightarrow \omega = \frac{E}{\hbar} \quad = \frac{p^2}{2m\hbar}$$

$$= \frac{(5.0 \times 10^{-25}\ kg \cdot m/s)^2}{2(9.11 \times 10^{-31}\ kg)(1.055 \times 10^{-34}\ J)}$$

$$\boxed{\omega = 1.3 \times 10^{15}\ s^{-1}}$$

Thus,

$$\boxed{\Psi(x,t) = Ae^{i((4.7\times 10^9\ m^{-1})x - (1.3\times 10^{15}\ s^{-1})t)}}$$

What is the probability of finding the particle at any given place in space?

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

The probability density (probability of finding a particle in a unit volume) is given by

$$|\Psi|^2 = \Psi^*(x,t)\Psi(x,t)$$
$$= A^2 e^{-i(kx-\omega t)} e^{i(kx-\omega t)}$$

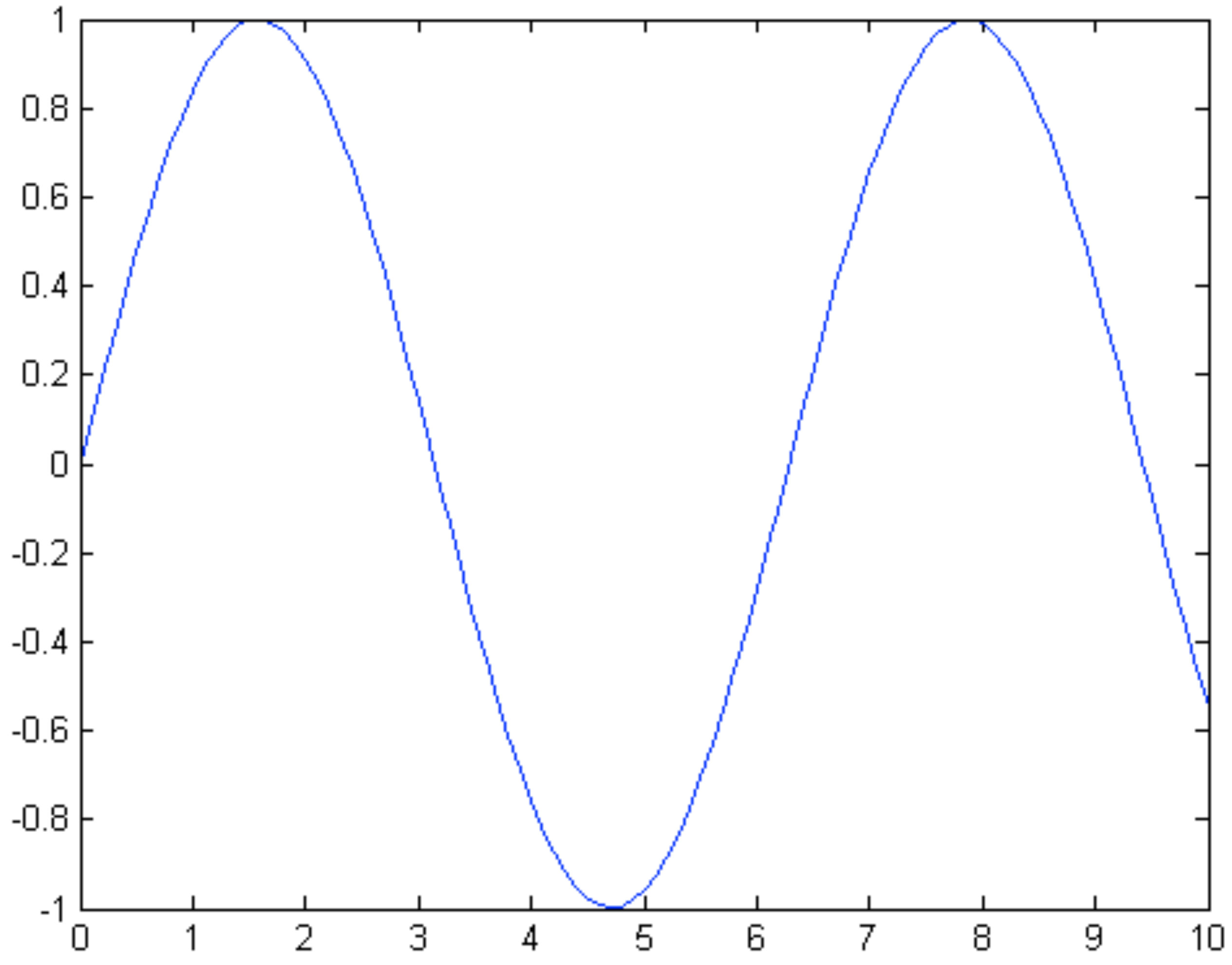
$$\boxed{|\Psi|^2 = A^2}$$

Probability Density = $|\Psi|^2$

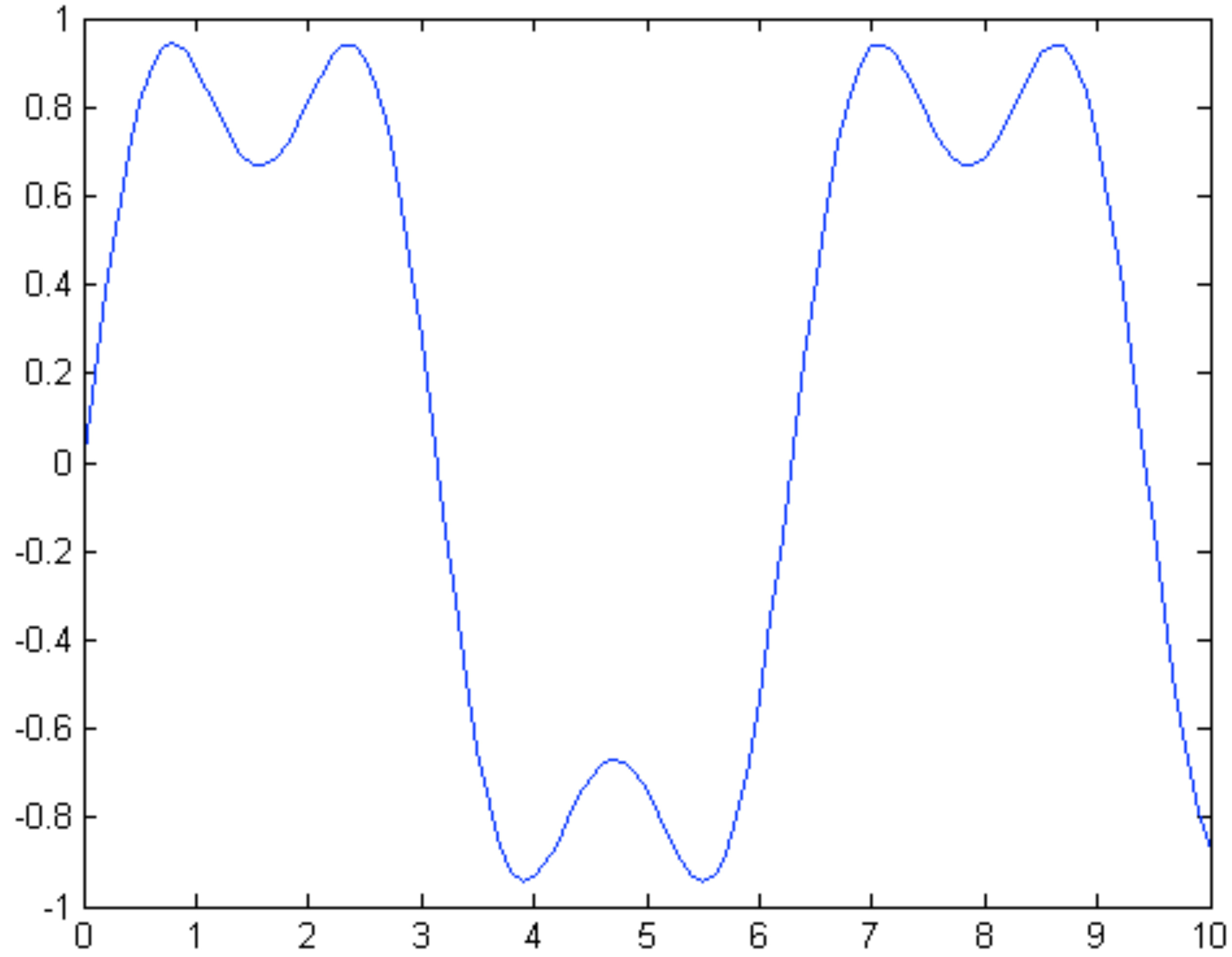
Since *A* is constant, this means that such a particle is likely to be found anywhere in time.

Final Notes:

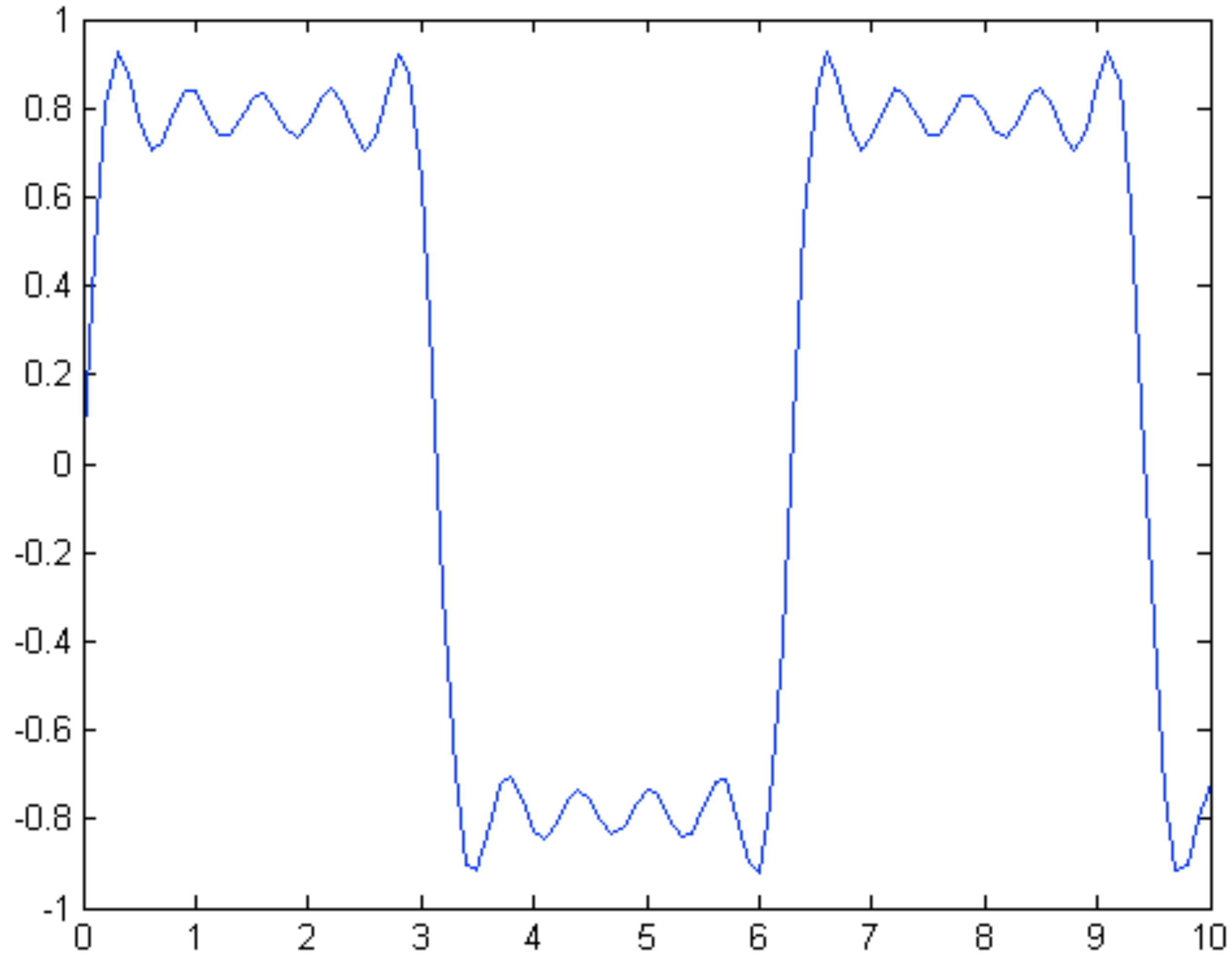
- The plane wave is not necessarily a useful description of a real situation, but a building block.
- We can add many plane waves together to obtain a description of a real situation.
- Use several sine waves to generate a square wave.



$$y = \sin(t)$$

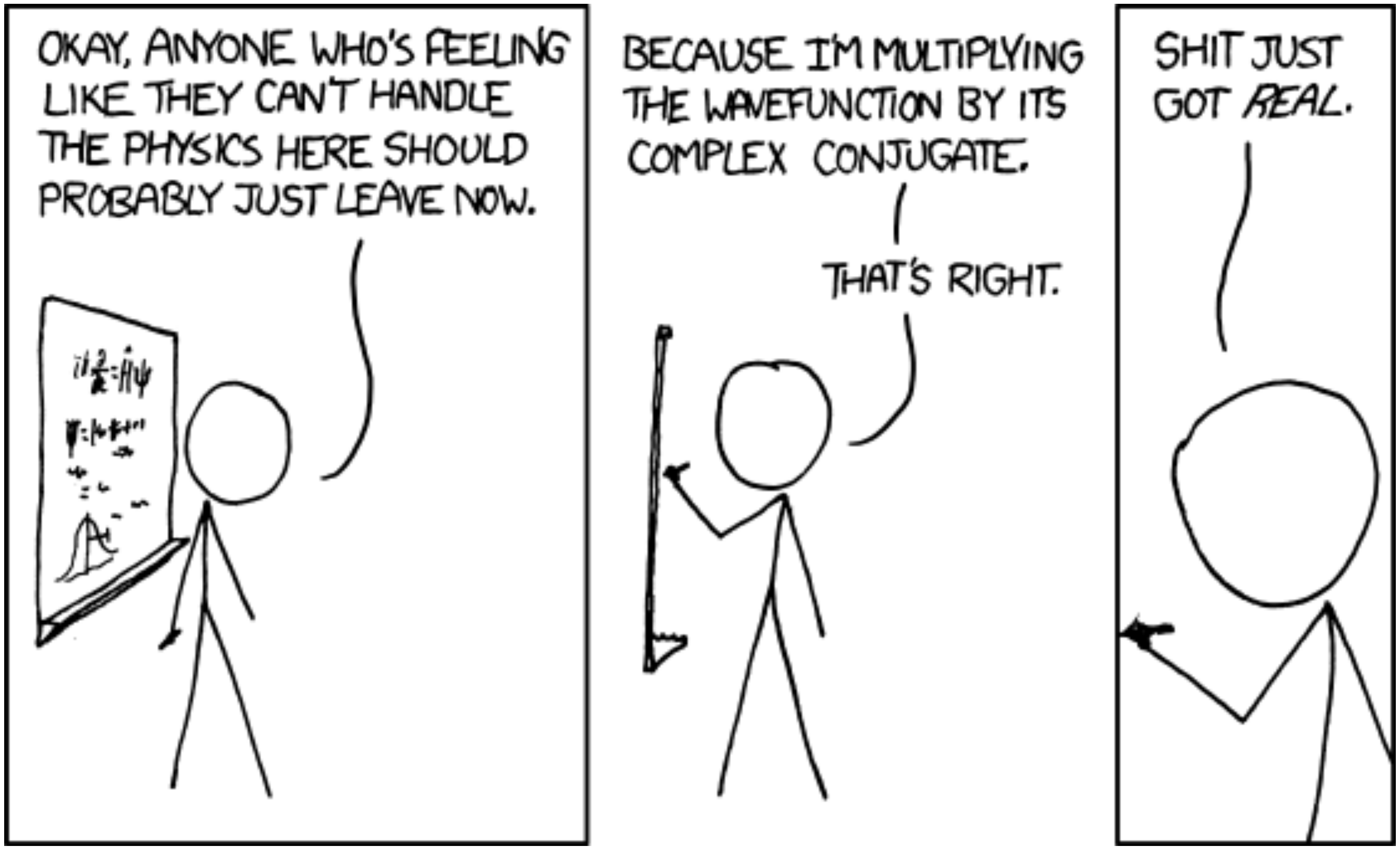


$$y = \sin(t) + \frac{1}{3}\sin(3t)$$



$$y = \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \frac{1}{7}\sin(7t) + \frac{1}{9}\sin(9t)$$

The End (for today)!



xkcd.com: comic 849