Welcome back to PHY 3305

Today's Lecture:

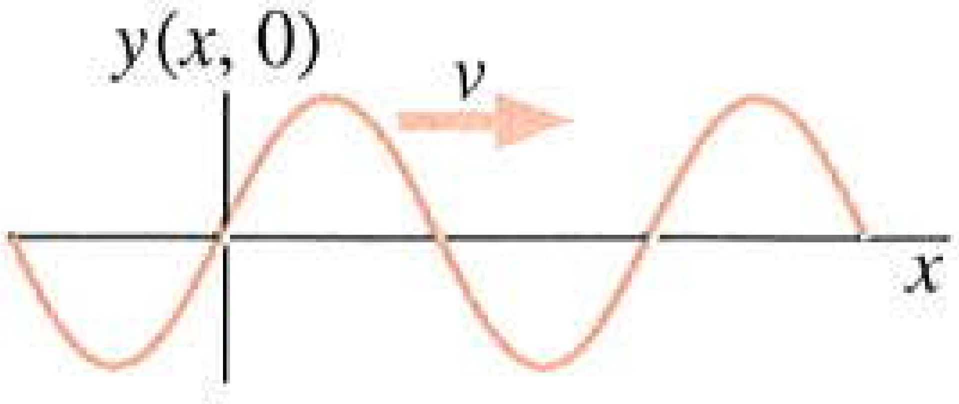
Matter Waves

Schrodinger Equation

Erwin Rudolf Josef Alexander Schrödinger

1887-1961

WAVE MOTION



Quantify the behavior of a perfect wave.

Consider describing the wave as a sine function.

$$A\sin\left(\frac{2\pi x}{\lambda}\right)$$

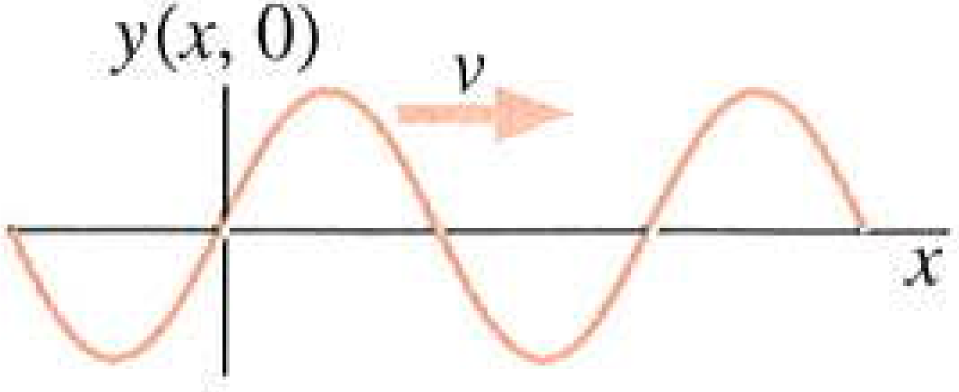
x = 0 --> zeroth wave

x = λ --> end of the first wave

x = 2λ --> end of second wave

Wave number = the spacial frequency of the wave.

$$k \equiv \frac{2\pi}{\lambda}$$



After one wave cycle, the cycle repeats. The repetition rate is the frequency.

$$A \sin\left(2\pi f t\right)$$

Note: The cycle ends at t = 1/f, the second cycle ends at t = 2/f, etc. This leads us to define angular frequency - how long it takes to complete one cycle.

$$\omega = 2\pi f$$

Now we can rewrite the particle/wave relationships in terms of these variables.

$$E = hf = h\frac{2\pi}{2\pi}f = \frac{h}{2\pi}\omega$$

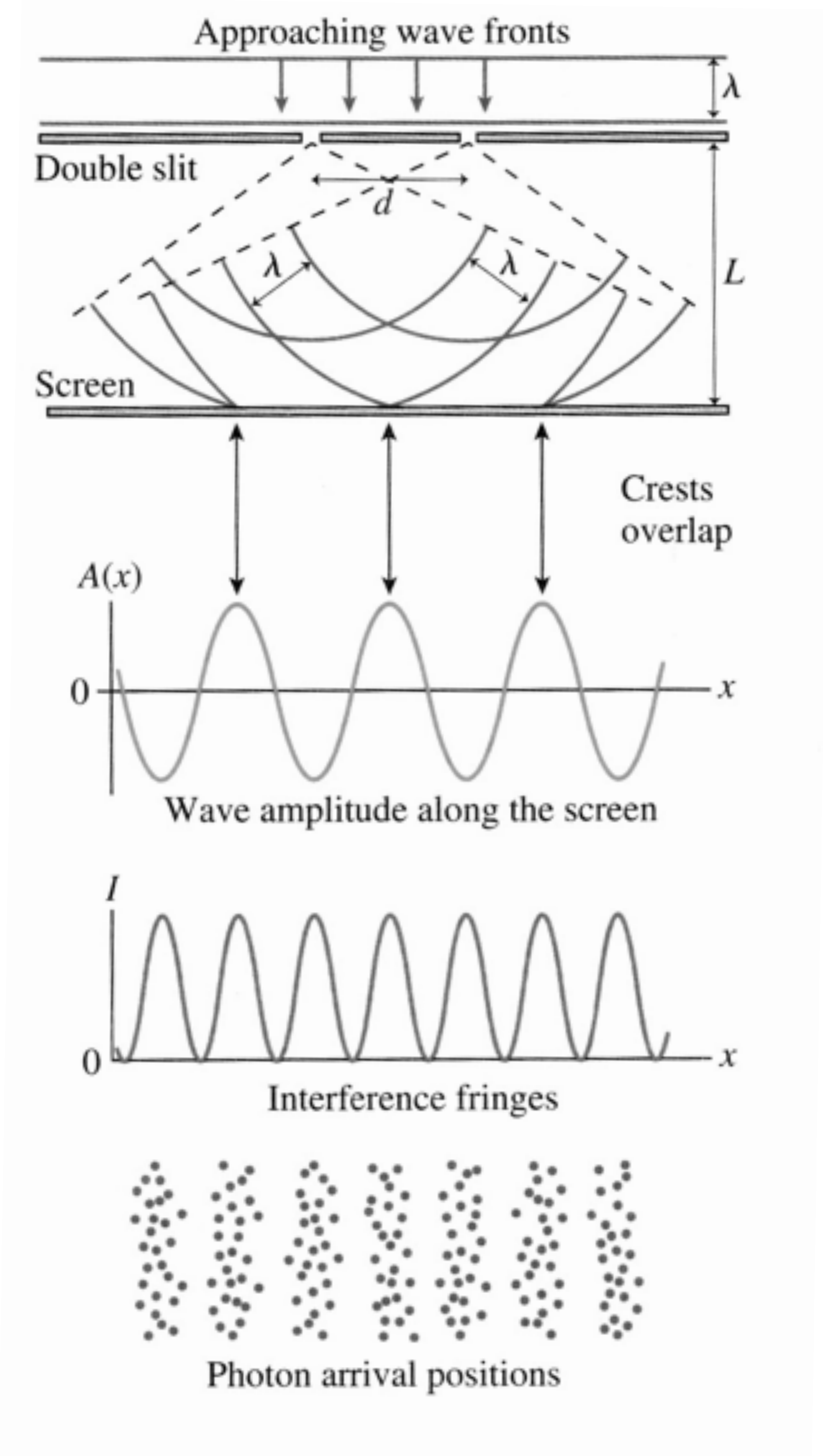
$$E = \hbar\omega$$

where $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} J \cdot s$

Similarly,

$$p = \hbar k$$

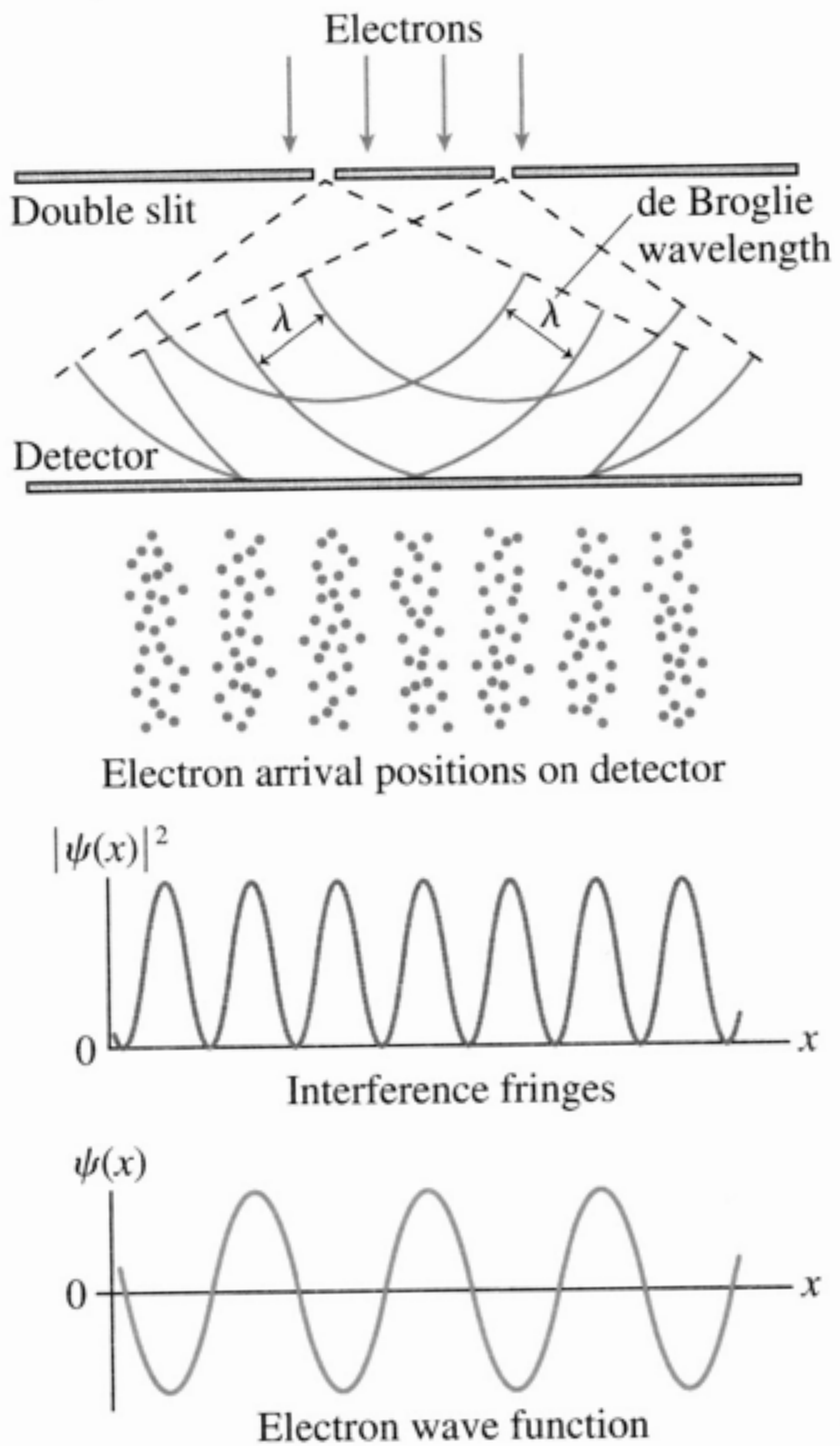
REVISIT DOUBLE-SLIT



- Photons pass through the double-slit apparatus.
- Crests of the two waves overlap, creating maxima amplitudes in the wave function.
- Fringes are observed at the maxima and minima in the wave amplitude.
- The observed fringes are the light's intensity (not amplitude).

$$I \propto A^2$$

$$P \propto A^2$$



- Denote the amplitude of a matter wave by ψ.
- The intensity of the wave is given by ψ
- Hence, the probability for finding a particle at a given location is

$$P \propto \psi^2$$

MECHANICAL WAVES

Goal: Find the wave equation for a matter wave of which the wave function must be a solution.

Transverse wave on a string.

$$v^2 \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$$

v = speed of the wave
y(x,t) = wave function, tells you the amplitude of the wave as a function of position and time.

This equation is derived from Newtons law.

$$\vec{F} = m\vec{a}$$

$$v^2 \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$$

The solution to this equation is a wave function.

$$y(x,t) = |A| \sin(kx - \omega t)$$

where

$$\frac{\omega}{k} = v$$

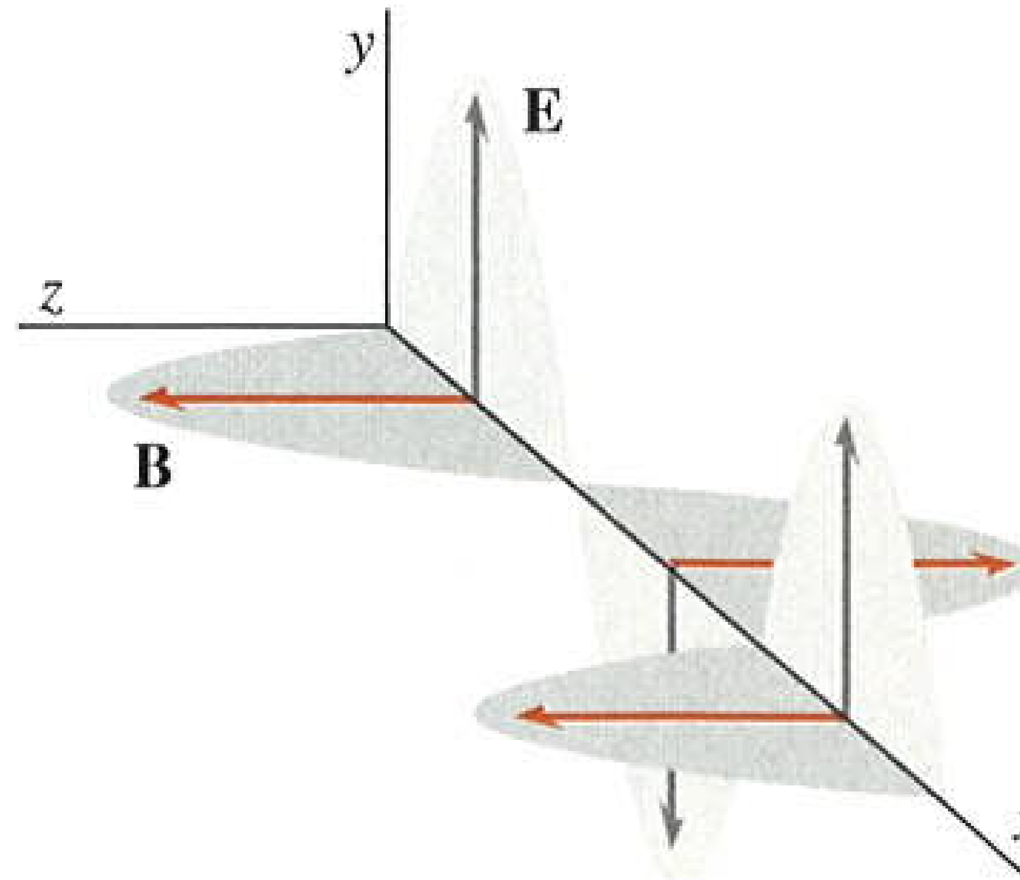
ELECTROMAGNETIC WAVES

The wave nature of light is described in terms of oscillating strengths of the E&M fields that comprise light.

Maxwell's equations can be used to derive the form of the wave equation for light. The solutions to that equation are PLANE WAVES, whose properties are

1. They move in one direction at constant velocity
2. They do not spread out in space.
3. They have two parts - an electric oscillation and a magnetic oscillation.

The solution looks like



$$\vec{E}(x,t) = Asin(kx - \omega t)\hat{y}$$

$$\vec{B}(x,t) = \frac{1}{c}Asin(kx - \omega t)\hat{z}$$

where

$$\frac{\omega}{k} = c$$

MATTER WAVES

- Matter obeys the **Schroedinger Wave Equation**.
- **Free-Particle Case** - matter is free of the action of external forces.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Notes:

- m is the mass of the particle
- the equation is complex, so the wave functions are not necessarily real - they may be complex functions containing real and imaginary components
- the equation is second-order in space and first-order in time (differs from standard mechanical wave)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

The form of the Schoedinger equation can not be derived from first principles. It is built on the recognition that it makes correct predictions for the outcome of experiments.

The equation **IS** a law - like Newton's laws or Maxwell's Equations for E&M.

What is oscillating?

The interpretation of experiments is that the PROBABILITY AMPLITUDE is oscillating. What changes in time in space is not energy or mass, but rather the probability of finding the matter.

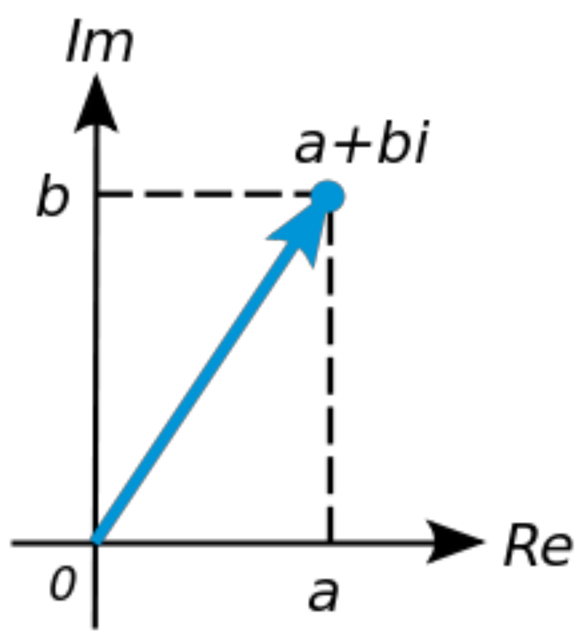
What does it mean when a function is complex?

It does not mean that the function does not represent a real thing. It means that the thing in question can not be described by a real-valued function.

Properties:

$$z = a + bi$$

real ↗ ↖ imaginary



$$i^2 = -1$$

Complex conjugate:

$$z^* = a - bi$$

To square a complex number you must multiply by the conjugate.

$$|z|^2 = zz^* = a^2 + b^2$$

Complex numbers:

Things become a little "sticky" here because the probability amplitude $\Psi(x,t)$ is a wave function.

This wave function is complex.

Furthermore, the wave function can be used to find the probability density (probability of finding a particle in a unit volume).

$$\text{probability density } = |\Psi(x,t)|^2$$

What does it mean when a function is complex?

Example: Consider E&M. We can treat E and B as components of a complex vector.

$$\vec{E} + i\vec{B}$$

The mathematical results of the application are the same, even though we treat the B-field as imaginary.

B-Fields are REALLY real!

Does it matter that we can't ascribe "reality" to the wave function itself?

No, you can make something measurable from a complex function/number. As an example, think of damping in mechanical and electrical systems.

THE END
(FOR TODAY)