Welcome back to PHY 3305

Today's Lecture:

Uncertainty Principle

Werner Heisenberg
1901-1976

When you perform an experiment, do you get the exact same result every time?

No. There is a fundamental uncertainty about the exact properties of a system.

How do we measure uncertainty in physics (and other disciplines)?

Mean:

$$\bar{Q} = \frac{\sum_i Q_i n_i}{\sum_i n_i}$$

Standard Deviation:

$$\Delta Q = \sqrt{\frac{\sum_i (Q_i - \bar{Q})^2 n_i}{\sum_i n_i}}$$

In quantum mechanics, we use de Broglie waves to describe particles.

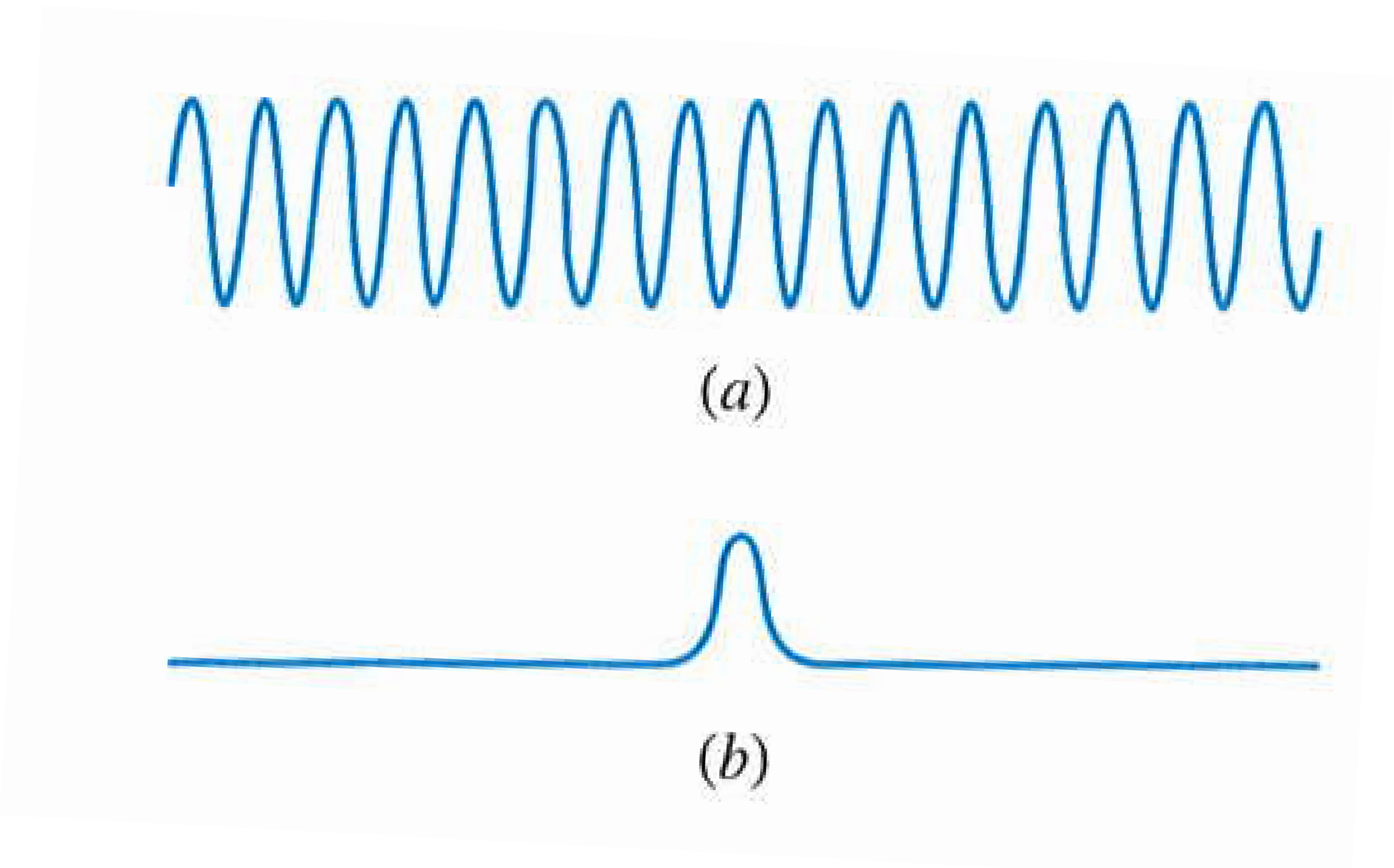
The wavelength tells us about the momentum of the particle.

$$\lambda = \frac{h}{p}$$

In quantum mechanics, the better we know a particle's position the less we know about it's momentum. The more we know about it's momentum, the less we know about its position.

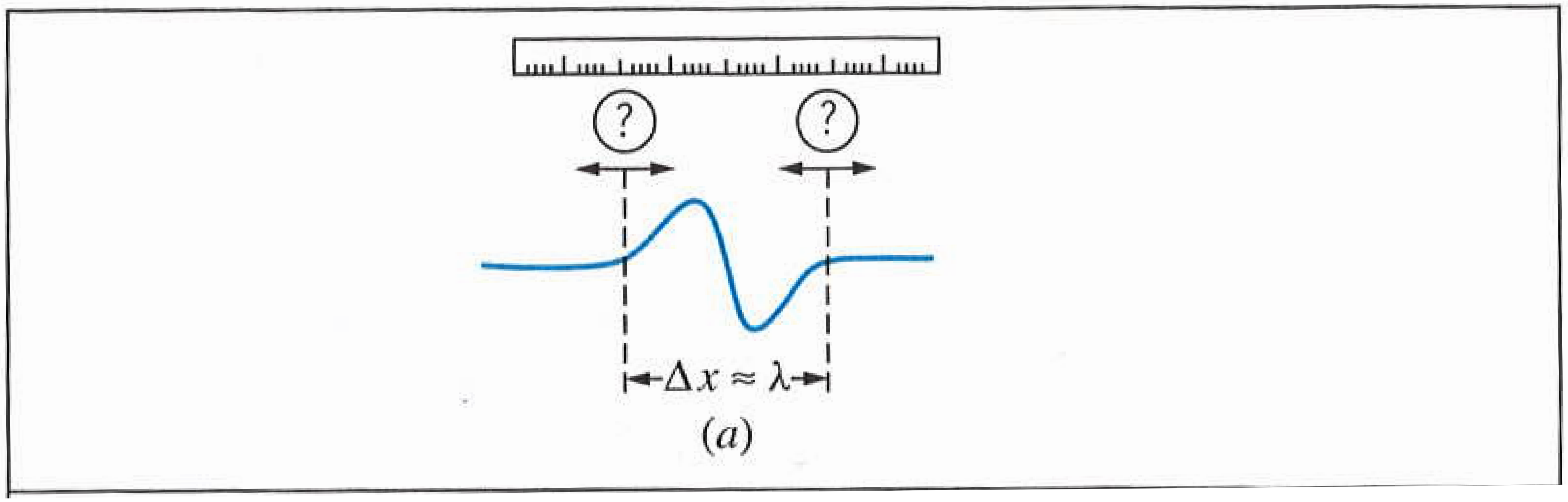
Which wave is better at telling us the location of the particle?

Which wave is better at telling us the wavelength of the particle?



Better for wavelength.

Better for position.



Measure the wavelength of this wave packet.

We may have difficulty finding the exact ends of the wave. Thus, we have an uncertainty Δλ.

$$\Delta\lambda \sim \epsilon\lambda$$

ε is a fraction of the wavelength

We want to examine the product of the size of the wave packet and the uncertainty in wavelength. In this case Δx ~ λ, so

$$\Delta x \Delta\lambda \sim \epsilon\lambda^2$$

Note the inverse relationship between size of the wave packet and the uncertainty in wavelength. As Δx gets smaller Δλ gets larger.

What if we make the wave packet larger?

There are N cycles of the wave - thus

$$\Delta x \sim N\lambda$$

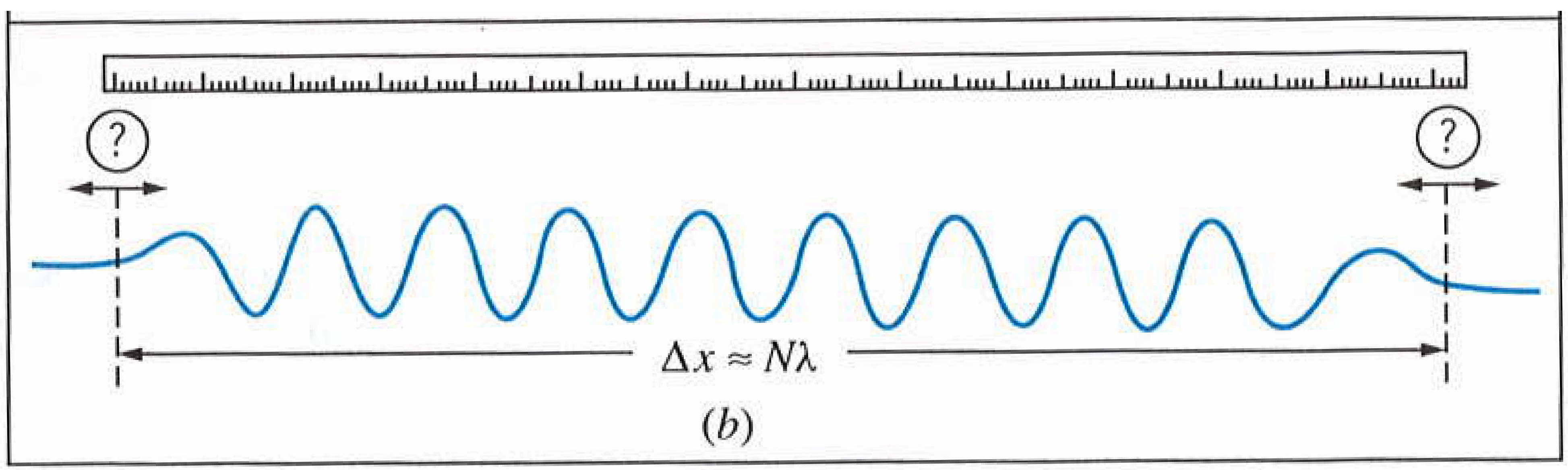
Uncertainty in the endpoints

$$\Delta\lambda \sim \frac{\epsilon\lambda}{N}$$

Combine together

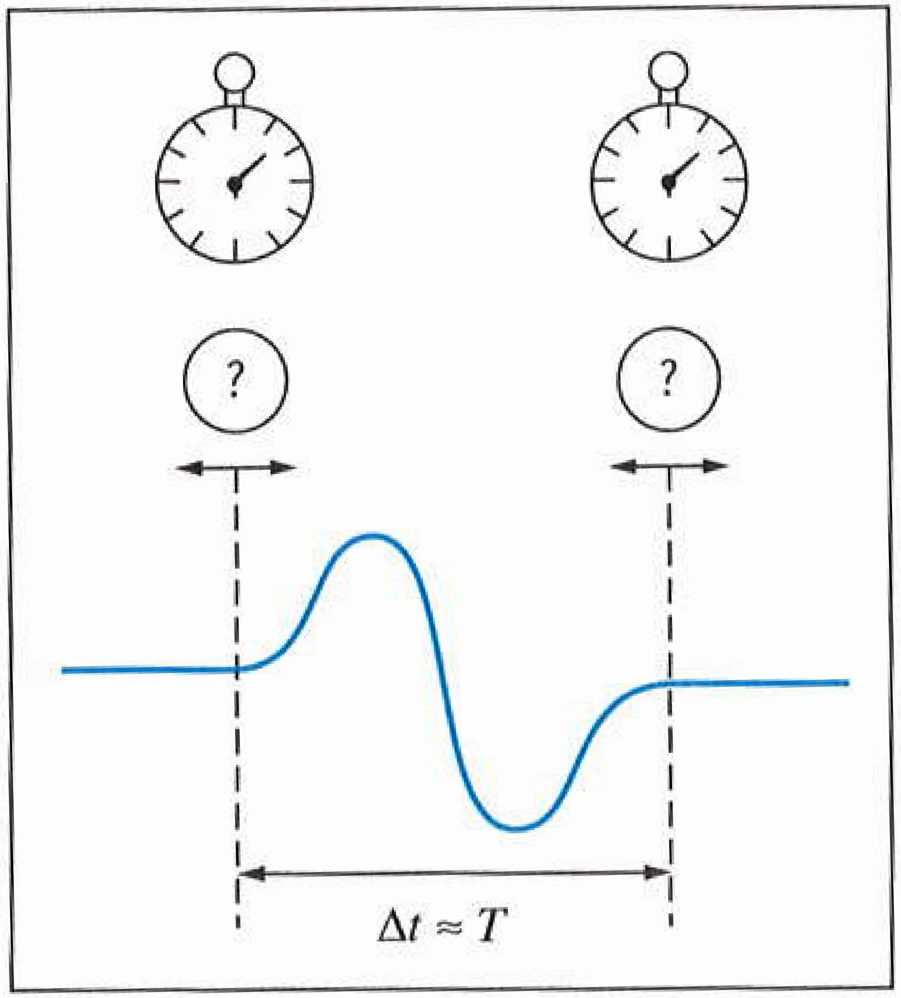
$$\Delta x \Delta\lambda \sim N\lambda\frac{\epsilon\lambda}{N} = \epsilon\lambda^2$$

Same result as case of smaller wave packets!



(b)

What if we measure period instead of wavelength?



The "size" of the wave packet is now a duration in time (one period).

$$\Delta t \approx T$$

We still have the difficulty of locating the start and end of the wave.

$$\Delta T \sim \epsilon T$$

ε is a fraction of the period

We want to examine the relationship between the duration of the wave packet and our ability to measure its period.

$$\Delta t \Delta T \sim \epsilon T^2$$

For a wave packet of a given period, the smaller the duration of the wave packet, the larger the uncertainty in our measurement of the period.

What if we want to write it in terms of frequency and not period?

$$f = \frac{1}{T} \xrightarrow{?} \cancel{\Delta f = \frac{1}{\Delta T}}$$

Do the calculus:

$$df = -\frac{1}{T^2}dT$$

Now convert. Note we can ignore (-) because we are interested in the magnitude of the uncertainties.

$$\Delta f = \frac{1}{T^2}\Delta T$$

Combine with $\quad \Delta t \Delta T \sim \epsilon T^2$

$$\Delta f \Delta t \sim \epsilon$$

The longer the duration of the wave packet, the more precisely we can measure its frequency.

Apply to de Broglie Waves

$$\Delta x \Delta \lambda \sim \epsilon \lambda^2$$

$$p = \frac{h}{\lambda}$$

Take differential

$$dp = -\frac{h}{\lambda^2} d\lambda \longrightarrow \Delta p = \frac{h}{\lambda^2} \Delta \lambda$$

Combine with our equation relating λ and x.

$$\Delta p \sim \frac{h}{\lambda^2} \frac{\epsilon \lambda^2}{\Delta x}$$

$$\Delta p \Delta x \sim \epsilon h$$

The smaller the size of the wave packet, the larger the uncertainty in its momentum.

Last details:

There is a formal procedure for calculating Δx and Δp. The outcome of these calculations gives the wave packet with the smallest possible value of the product $\Delta x\, \Delta p$ as $h/4\pi$. (section 4.7 of your book).

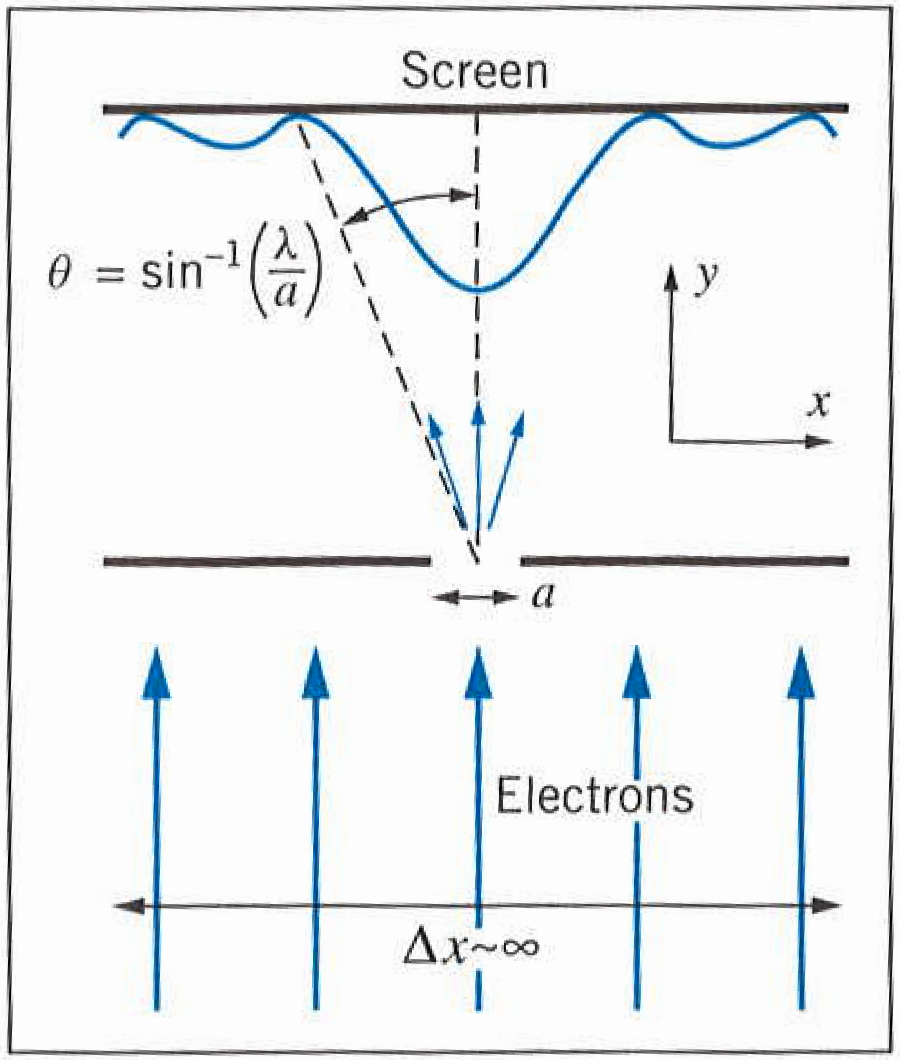
$$\boxed{\Delta p_x \Delta x \geq \frac{\hbar}{2}}$$

Nobel Prize: 1932

Heisenberg Uncertainty Principle:

Because of a particle's wave nature, it is theoretically impossible to know precisely both its position along an axis and its momentum component along that axis; Δx and Δp can not be zero simultaneously. There is a strict theoretical lower limit on their product.

EXAMPLE: SINGLE-SLIT DIFFRACTION



Initial:

$\Delta p_x = 0$, all momentum in y-direction

Δx = infinity, we know nothing about position.

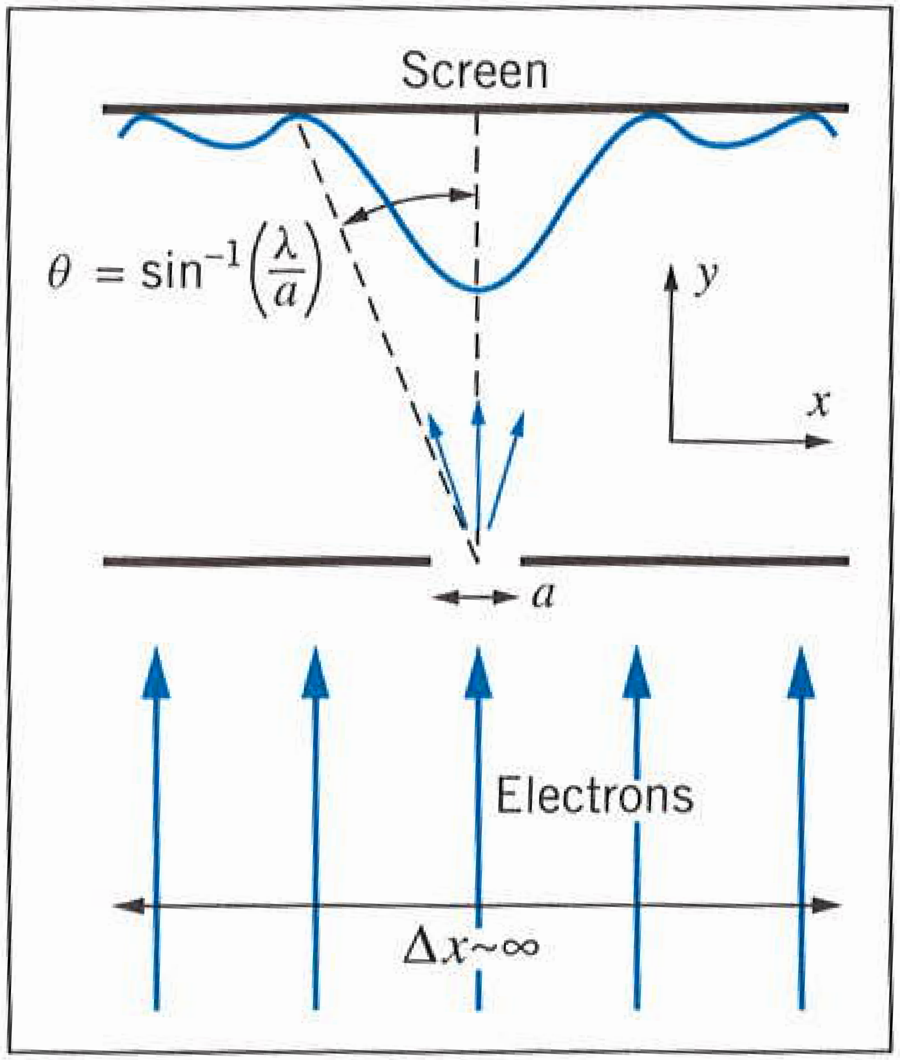
Pass through slit:

We know that their x location is no larger than a. $\Delta x = a$

$$\Delta p_x \geq \frac{\hbar}{2a}$$

The first minima in the diffraction pattern is given by

$$\sin\theta = \frac{\lambda}{a}$$



Find the angle θ that specifies where a particle with this value of lands on the screen.

for small θ $\sin\theta \approx \tan\theta \quad = \dfrac{p_x}{p_y} \quad = \dfrac{\frac{\hbar}{2a}}{p_y}$

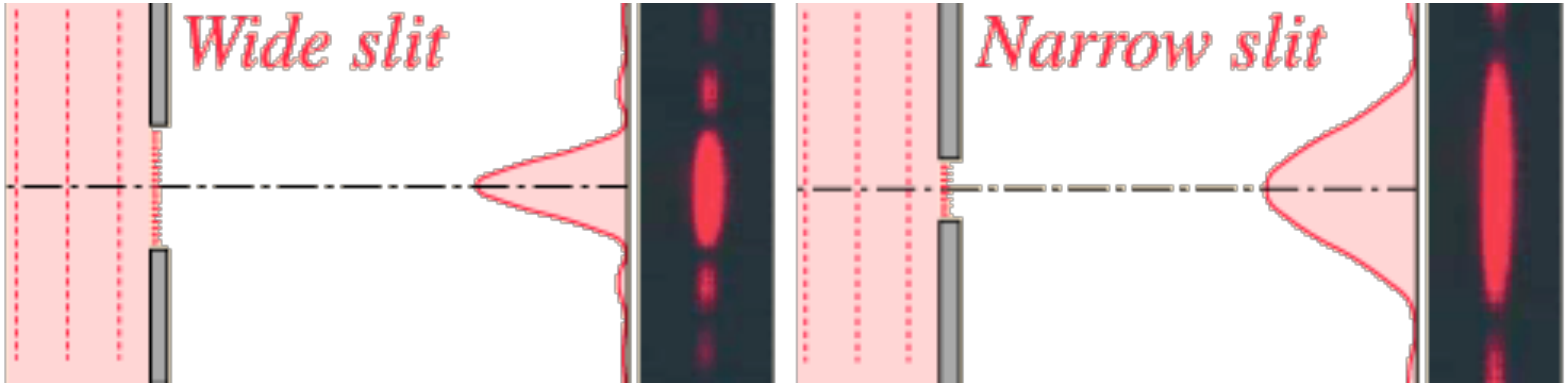
Use λ = h/p_y, the de Broglie wavelength of the electrons.

$$\sin\theta \approx \frac{\lambda}{4\pi a}$$

The first minima in the diffraction pattern is given by

$$\sin\theta = \frac{\lambda}{a}$$

transverse momentum given by uncertainty principle is roughly equivalent to spreading of the beam.



The diffraction (spreading) of the beam is an effect of the uncertainty principle. As the slit becomes narrow, p_x increases and the beam spreads even more.

There is a trade off in knowing the position (x) and the momentum (x-dir) of the particle.

What about our second relationship?

$$\Delta f \Delta t \sim \epsilon$$

Use the energy-frequency relationship for light

$$E = hf$$

$$\Delta E = h \Delta f$$

Substitute

$$\Delta E \Delta t \sim \epsilon h$$

Again there is a formal procedure for these calculations.
Here I give the result.

$$\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$$

INTERPRETATION

If a state or particle exists for only a limited span of time, it's energy is uncertain.

- life span of some subatomic particles is quite short (10^{-20} s) which leads to considerable uncertainty in their mass/energy
- state temporarily occupied by an electron as it jumps down an energy level in an atom - since the state is occupied for a finite time, its energy is uncertain by some amount. This gives rise to broadening of spectral lines.

UNCERTAINTY PRINCIPLE AND DARK ENERGY

Observations of the universe indicate that not only is the universe expanding, the expansion of the universe is accelerating. This acceleration is said to be caused by the dark energy in the universe. One leading candidate for dark energy is <u>vacuum energy</u>.

The idea here is that particle-antiparticle pairs would spontaneously appear and then annihilate in an otherwise empty vacuum. The total energy and lifetime of these particles must satisfy the uncertainty principle.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

It has been suggested that the vacuum energy density is the Plank energy density.

$$\mathcal{E} \sim \frac{E_P}{\mathcal{L}_P^3} = \frac{1.2 \times 10^{28} eV}{(1.6 \times 10^{-34} m)^3} = 3 \times 10^{133} \frac{eV}{m^3}$$

$$\mathcal{L}_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

$$E_P = \sqrt{\frac{\hbar c^5}{G}} = 1.2 \times 10^{28} eV$$

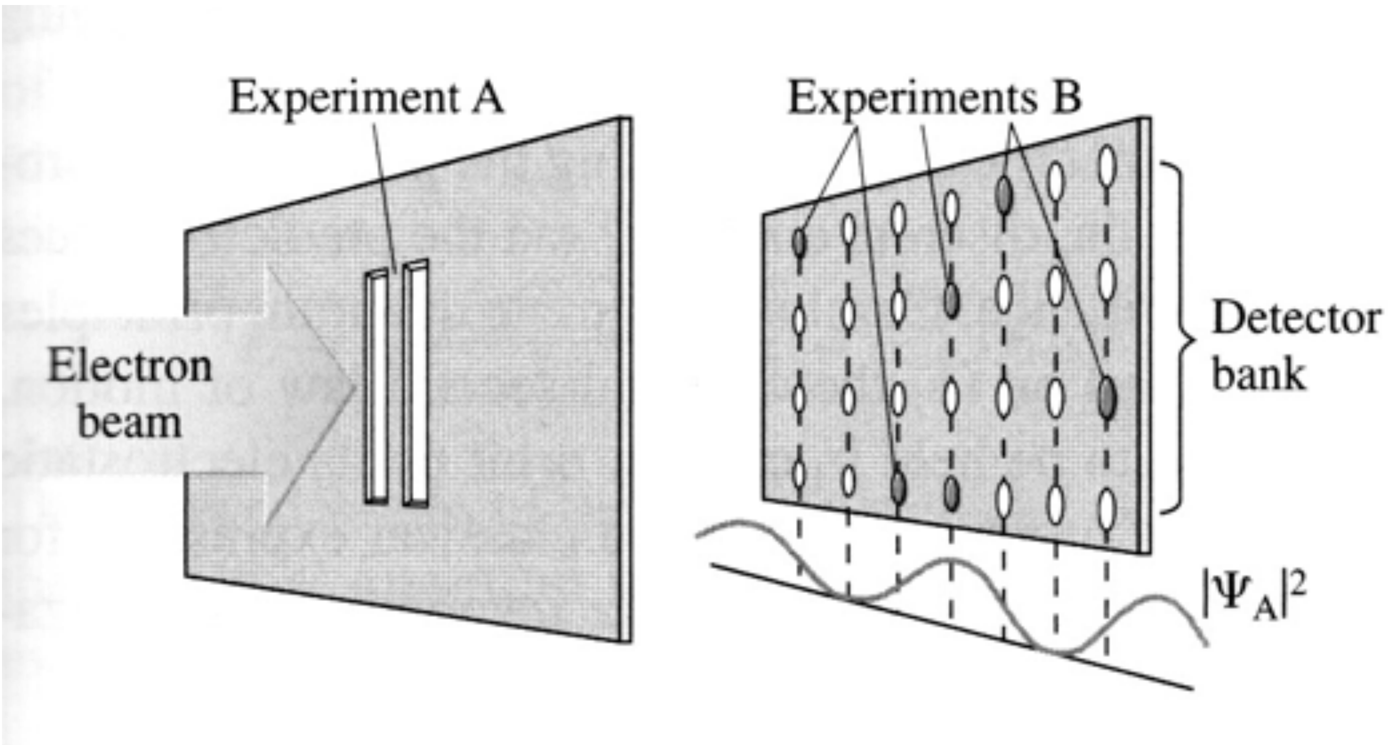
This is 124 orders of magnitude larger than the current critical energy density (required for flatness) of the universe!

LIMITATION OF KNOWLEDGE

Summary of what we know so far:

- The equations describing particles and forces can be very precisely stated.
- The wave function encodes all properties of matter
- The wave function, by its nature, prevents us from knowing both momentum and position (or energy and time) precisely at the same time

Revisit Double Slit:



Experiment A, "the slit" establishes an initial wave function Ψ_A

Experiment B "screen" detects the particle.

Where Ψ_A is large, many particles are registered, where it is zero, no particles are registered.

What happens if we conduct the intermediate experiment to determine through which slit the particle passes?

This experiment alters the result - it alters the wave function. To observe interference we must allow the particles wave function to pass through both slits simultaneously.

COPENHAGEN INTERPRETATION

If we can not know the location of a particle until we actually look for it, how can we justify the claim it has a location?

The modern interpretation of this is know as the **"Copenhagen Interpretation"**.

Until the experiment actually localizes the particle, it does not have a location.

SUMMARY

- Classical physics gave us the ideas of position and velocity.
- Quantum mechanics allows us to only know probabilities and corresponding uncertainties passed on the most recent observation of the "particle".
- A determination of one property is liable to alter another property.

THE END
(FOR TODAY)