Welcome back to PHY 3305

Today's Lecture:

Schrödinger Equation

Erwin Rudolf Josef Alexander Schrödinger

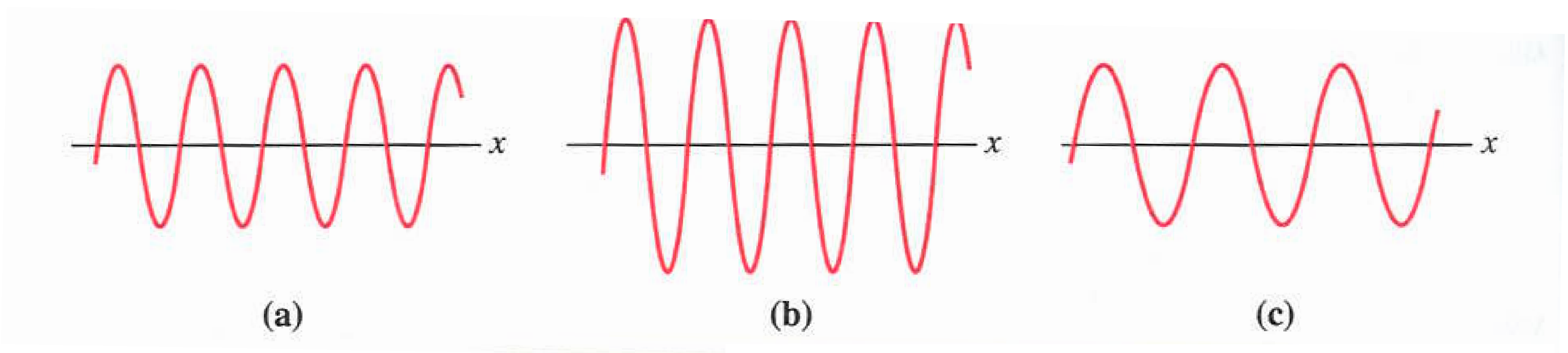
1887-1961

ANNOUNCEMENTS

- Reading Assignment for Thursday, October 12th: Chapter 5.5.
- Problem set 7 is due **Thursday, October 12th** at 12:30 pm.
- Regrade for problem set 6 is due **Thursday, October 12th** at 12:30 pm.
- Exam 2 is in class, Thursday, October 19th at 12:30 pm. It will directly cover chapters 3 and 4. This does not mean you can forget everything from the beginning of the course as the material builds.
- Dr. Cooley will be out of town October 16th - 18th. Her office hours on October 16th and 17th are canceled. She will be available via email and can arrange one-on-one remote office hours via ZOOM if you need extra help.

REVIEW QUESTION

Three de Broglie waves are shown for particles of equal mass. Rank in order from fastest to slowest, the speeds of particles a, b, and c. Assume the x-axis is the same length in each figure.



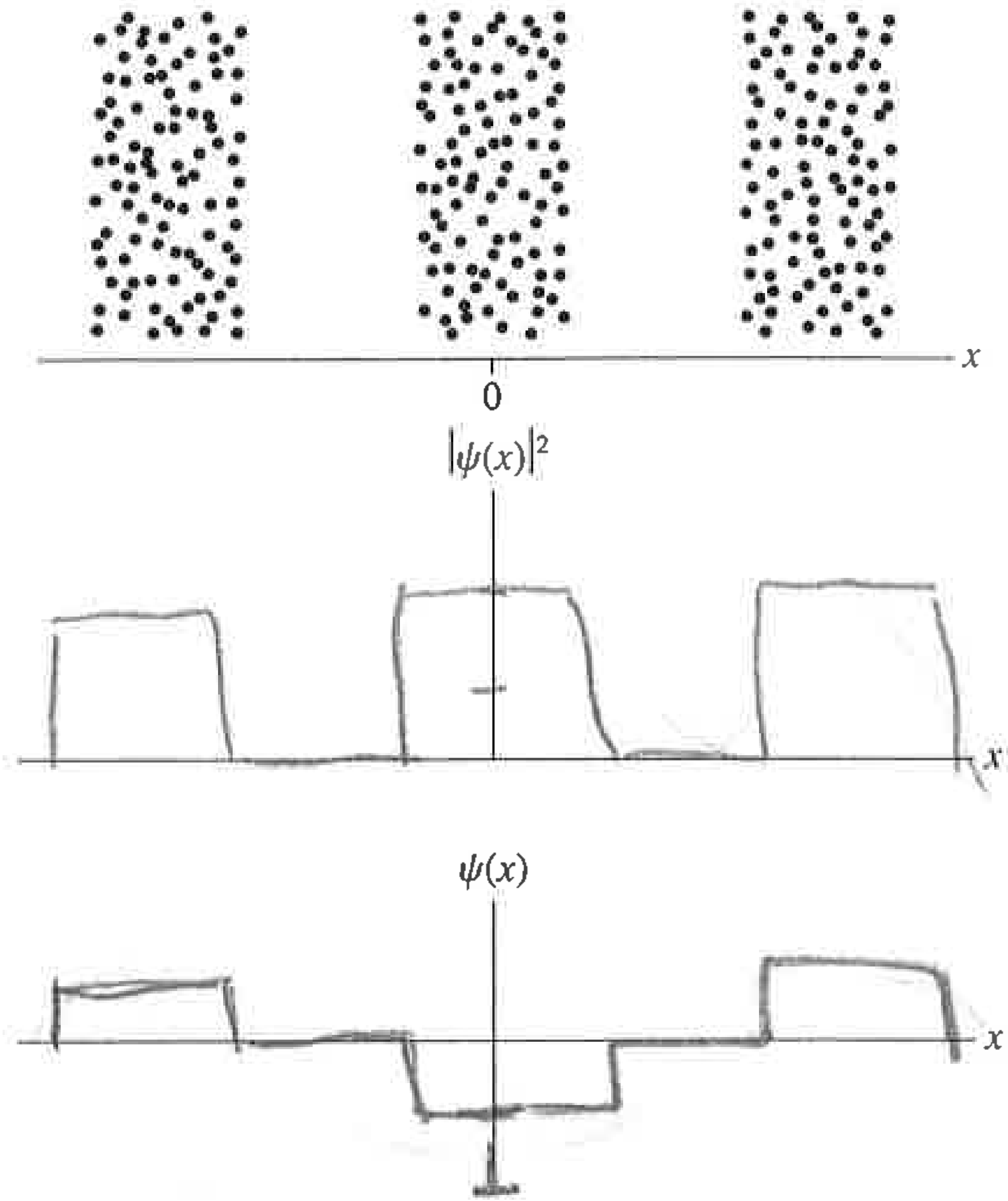
Particles with longer wavelengths are moving slower.

a = b > c

$$\lambda = \frac{h}{p}$$

REVIEW QUESTION

The figure shows the dot patterns landing on a screen.

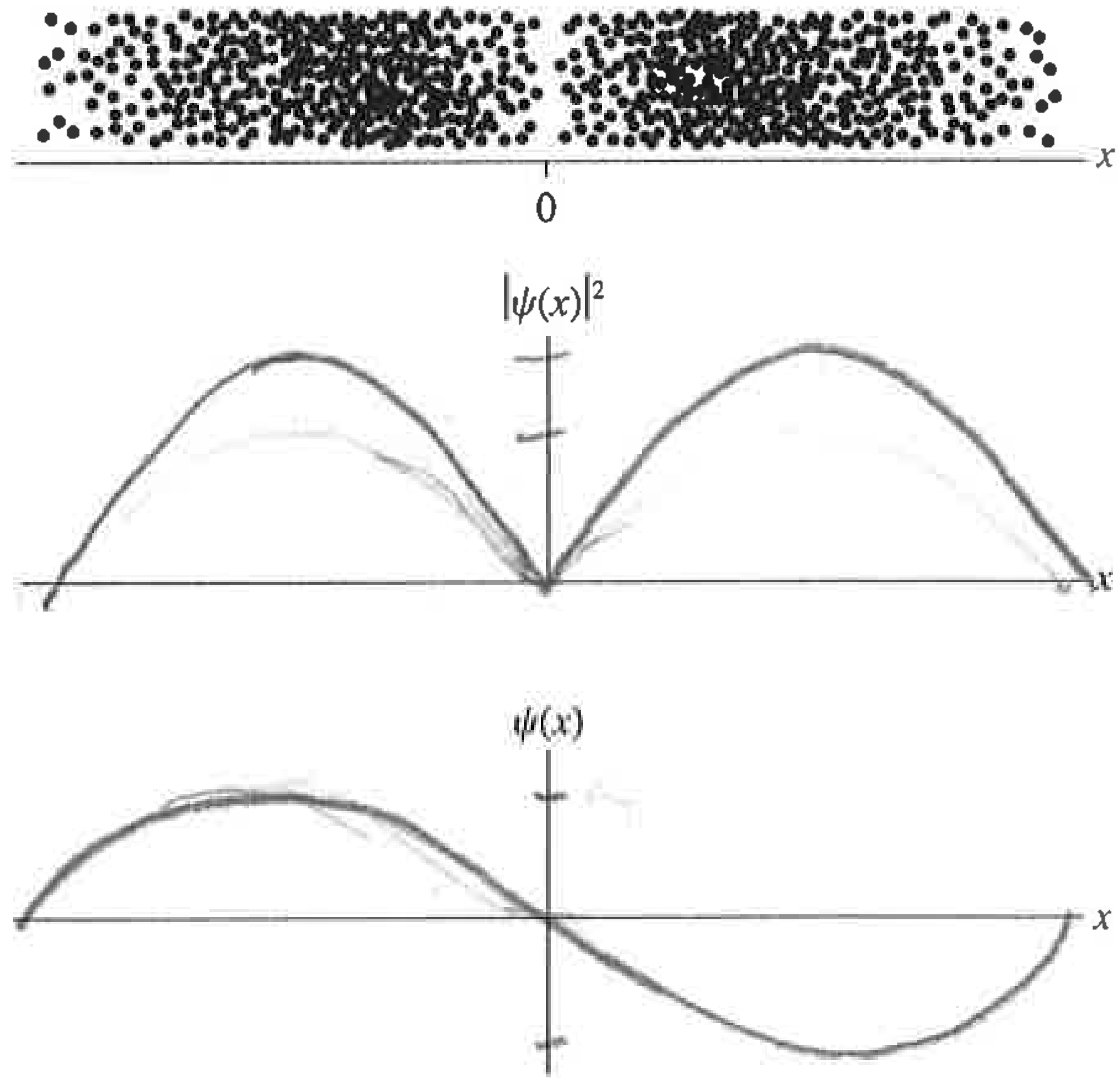


a) Draw the probability density as a function x.

b) Draw the wave function.

REVIEW QUESTION

The figure shows the dot patterns landing on a screen.



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b) Draw the wave function.

SUBATOMIC PARTICLE ρ^0

The ρ^0 is a subatomic particle of fleeting existence. Data tables don't usually quote a its lifetime. Rather they quote a "width" meaning energy uncertainty of 150 MeV. Roughly, what is its lifetime?

Solution:

$$\Delta t \Delta E \geq \frac{1}{2}\hbar$$

$$\Delta t \geq \frac{\hbar}{2\Delta E}$$

$$\Delta t \geq \frac{6.58 \times 10^{-16}\ eV \cdot s}{2(150 \times 10^{6}\ eV)}$$

$$\Delta t \geq 2.2 \times 10^{-24}\ s$$

From video lecture:

Time-Dependent Schroedinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Remember, we started by assuming our wave function could be written as

$$\Psi(x,t)=\psi(x)\phi(t)$$

Solving for time we found that our total wave function can be written as

$$\Psi(x,t)=\psi(x)e^{-i(\frac{E}{\hbar})t}$$

$$\Psi(x,t) = \psi(x)e^{-i(\frac{E}{\hbar})t}$$

What is the probability density for this wave function?

$$\Psi^*(x,t)\Psi(x,t) = [\psi^*(x)e^{i\frac{E}{\hbar}t}][\psi(x)e^{-i\frac{E}{\hbar}t}]$$

$$\Psi^*(x,t)\Psi(x,t) = \psi^*(x)\psi(x)$$

Is there a time dependence in the probability?

1. No, it disappears under the case we can separate space and time components of the wave function.
2. The properties do not change in time - **"stationary states"**

From video lecture:

Time-Independent SWE

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Physical Conditions:

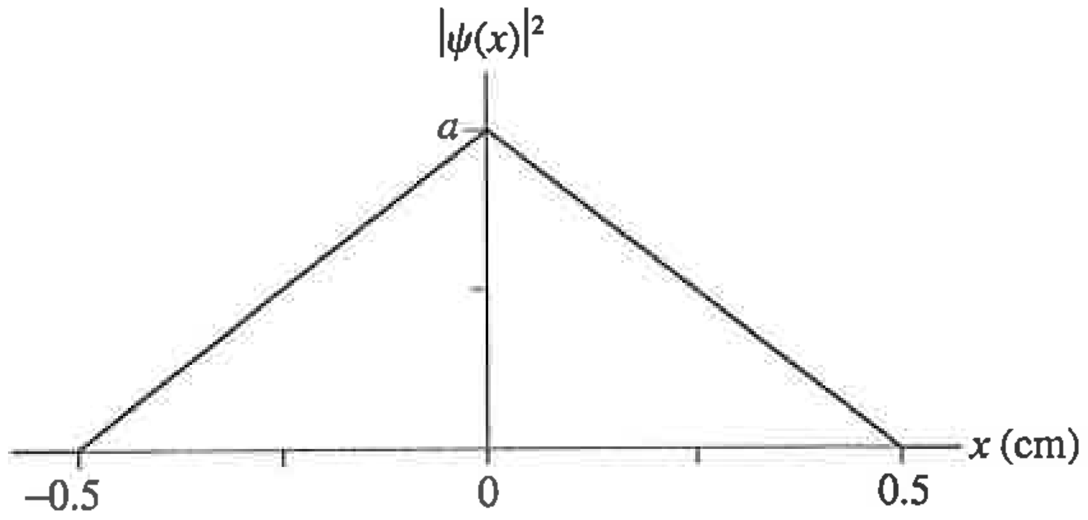
- Each case we will consider will have it's own wave function ψ(x) to describe the particle.
- Certain physical conditions must be met.
  - The total probability of finding the particle must be one. (**Normalization**)
  - The particle must have a definite charge with a strict value.
  - The wave function must be **smooth**.

From video lecture:

To be physically acceptable, a wave function must be normalizable.

$$\int |\Psi(x,t)|^2 dx = 1$$

What is the value of the constant a that makes this a normalized wave function?



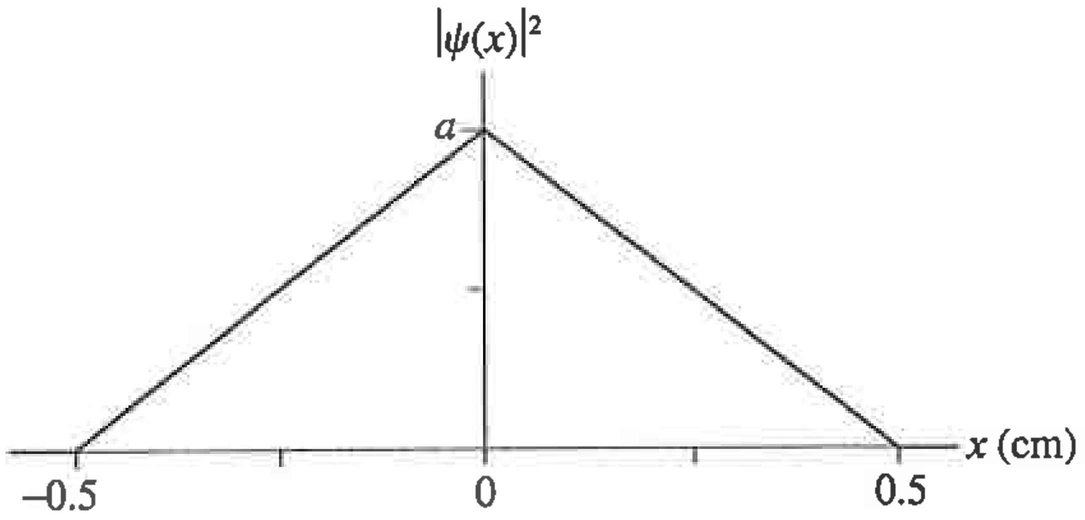
The area under the curve needs to be equal to one for normalization.

$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}ba$$

$$a = \frac{2A}{b} = \frac{2(1)}{1\ cm}$$

$$a = 2\ cm^{-1}$$

What is the probability that any particular electron lands somewhere between x = 0 cm and x = 0.5 cm?



Intuitively, due to the symmetry of this object we know

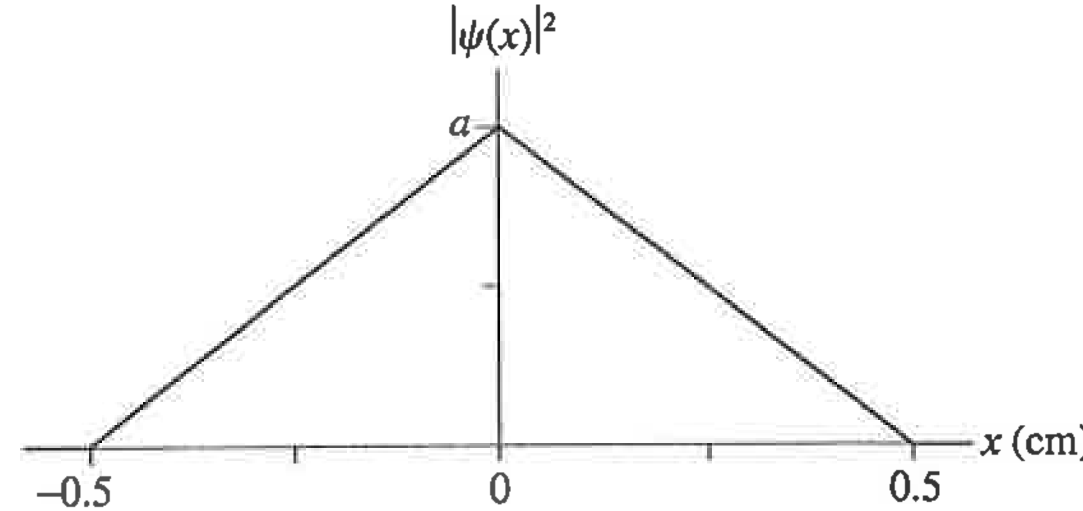
$$P(0 > x > 0.5) = 50\%$$

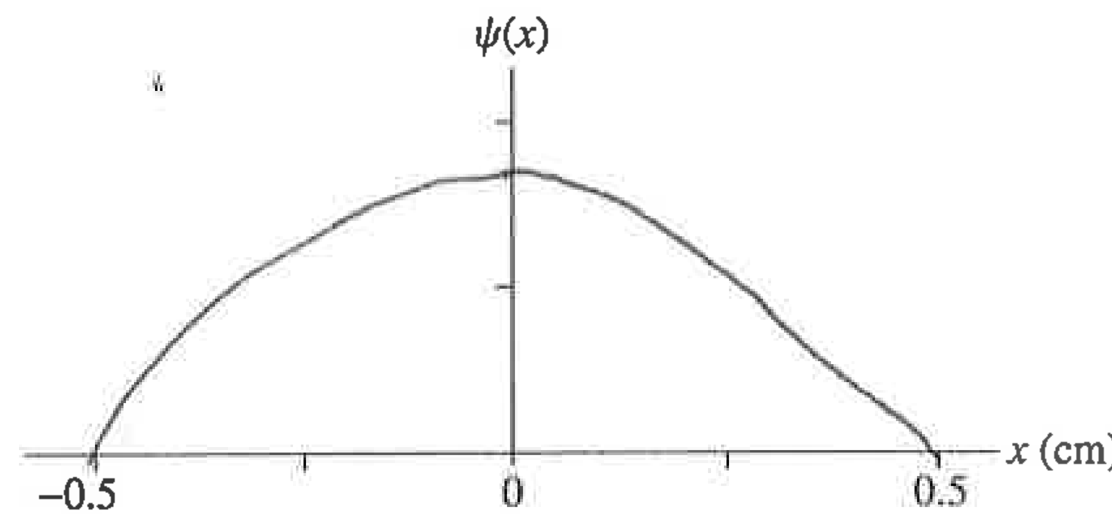
However, we can show this as well:

$$A = \frac{1}{2}bh \quad = \frac{1}{2}(0.5\ cm)(2\ cm^{-1})$$

$$\boxed{P \propto A = 50\%}$$

Draw the graph of the wave function ψ(x). Add the appropriate vertical scale to your graph.





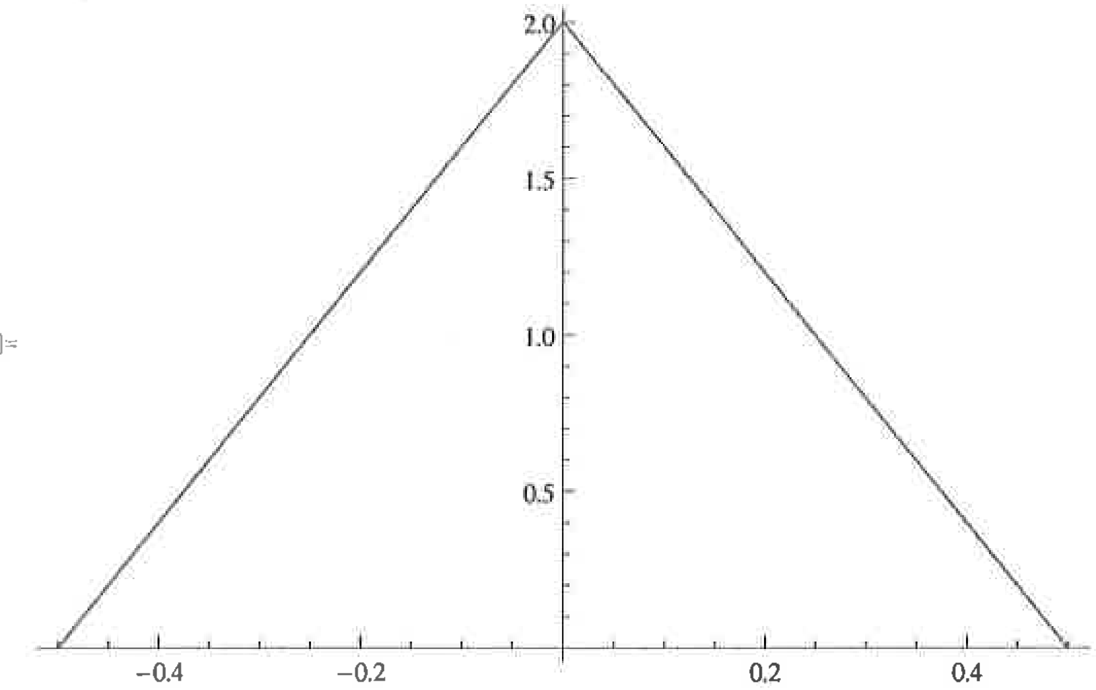
Mathematica Solution:

```
In[1]:= f[x_] = If[x < 0, 4 x + 2, -4 x + 2]

Out[1]= If[ x < 0, 4 x + 2, -4 x + 2]

In[4]:= Plot[f[x], {x, -.5, .5}]
```

Out[4]=

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In[5]:= Plot[Sqrt[f[x]], {x, -.5, .5}]
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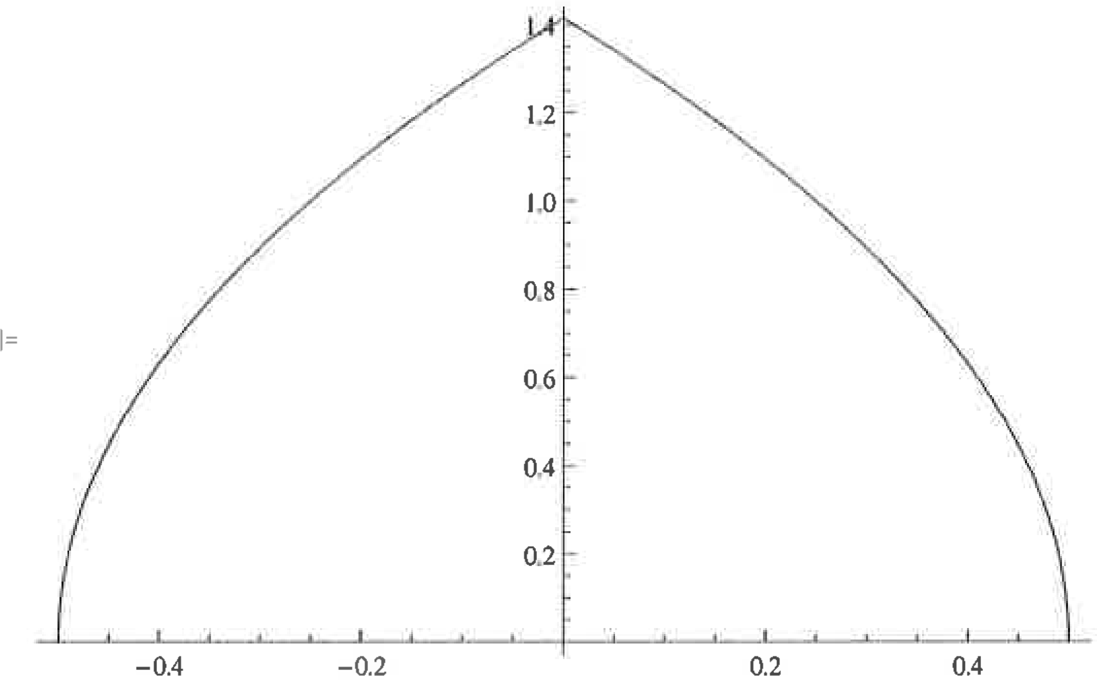
Out[5]=



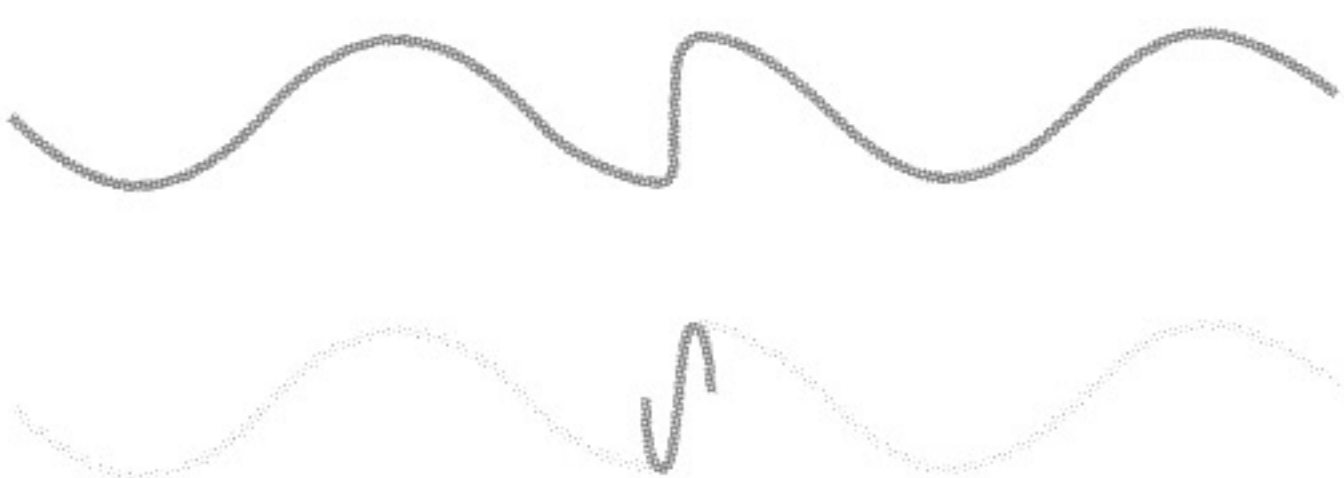
From video lecture:

To be physically acceptable, a wave function must be smooth.

The wave function and it's first derivative must be continuous.

Why must the wave function be continuous?

Say that the wave function has a point where it is infinite. This would signal a wave with a place of infinite kinetic energy - not realistic.



Abrupt jumps act like short wavelengths (high frequencies) which mean huge energies.

$$E = hf = \hbar\omega$$

From video lecture:

Why must the first derivative be continuous?

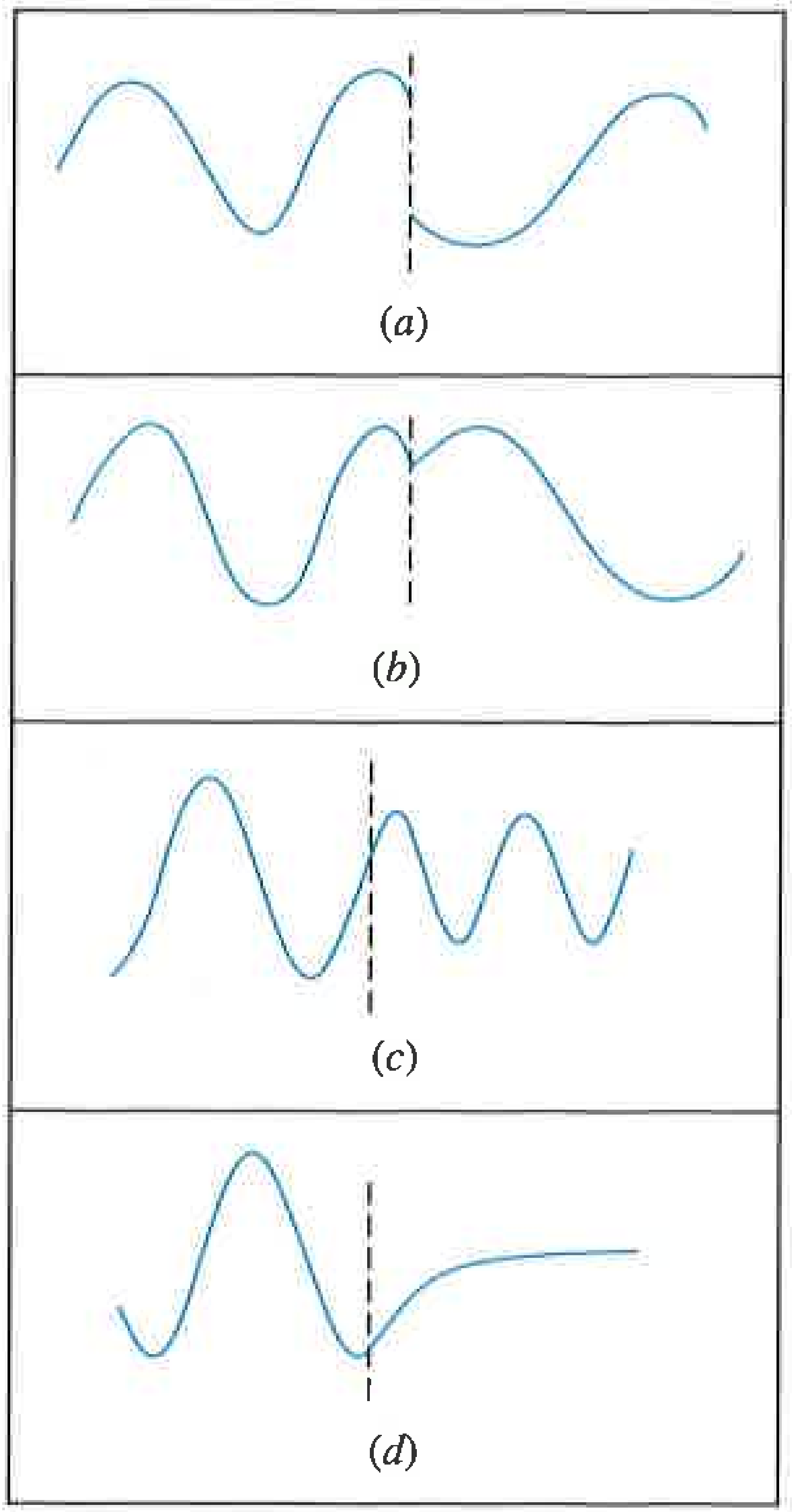
- The first derivative is the slope of the wave function.
- If the first derivative is continuous, the second derivative (slope of the first derivative) is finite. Thus the kinetic energy is finite. This must be true for the Schrodinger equation to hold (since U(x) and E_{total} are finite.

$$(KE + U(x))\Psi(x,t) = (E)\Psi(x,t)$$

Note: There is an exception. The first derivative can be discontinuous if we require the potential energy to be infinite at some point in space. infinite = so large as to completely oppose any motion of the particle

Which of these wave functions is/are acceptable?

C & D are acceptable.
A is a discontinuous wave.
B is a continuous wave with a discontinuous slope.



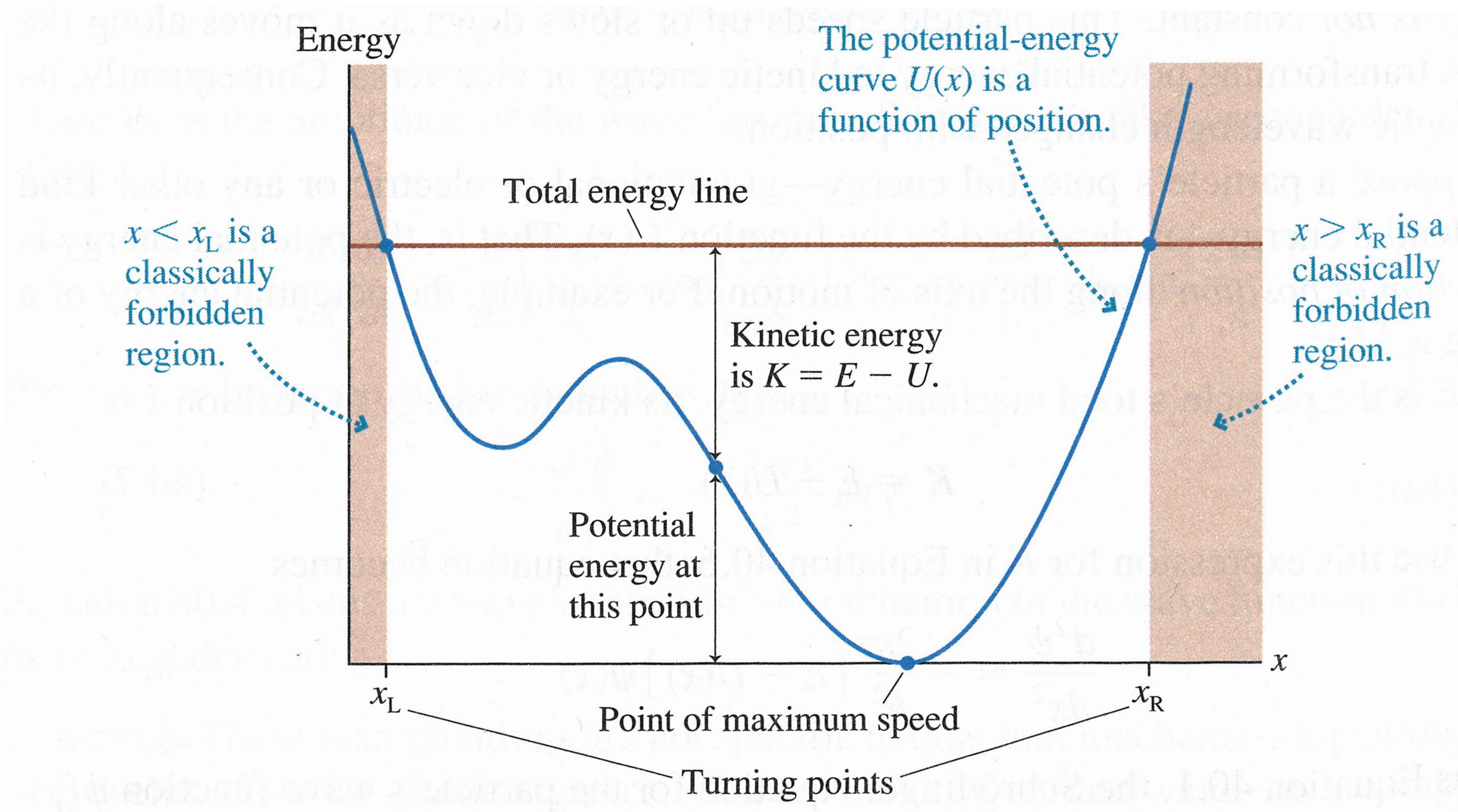
From video lecture:

What is a bound state?

Cases where a particle's motion is restricted by a force. The motion is restricted to a finite region.

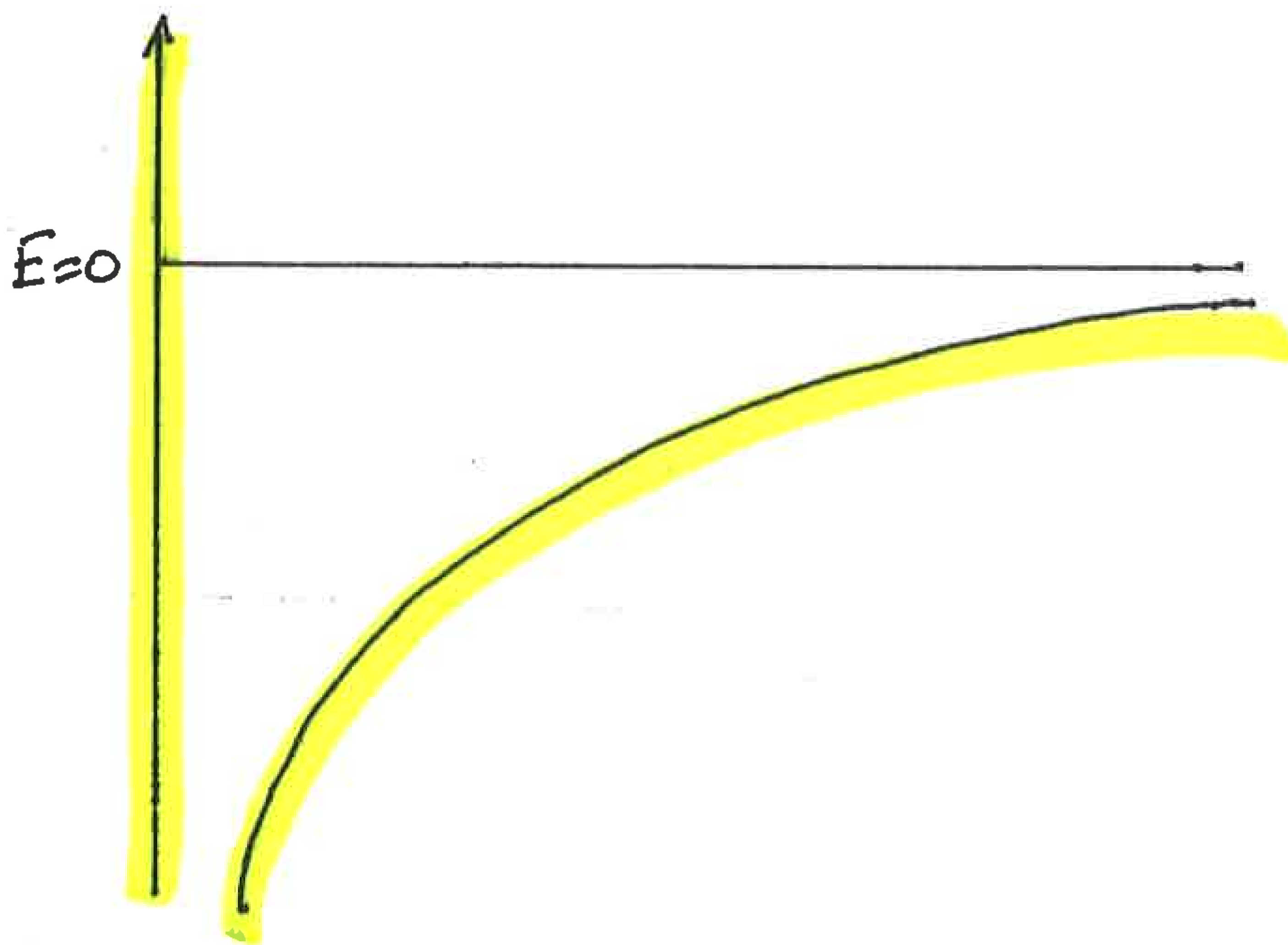
- States that are NOT free of forces. They are states that act under the influence of forces.
- These forces have only a spacial component. They can be described by adding a space-dependent potential, U(x), to the SWE.

INTERPRETING AN ENERGY DIAGRAM



A particle is subject to potential energy that has an infinitely high wall at the origin, but for positive values of x is of the from U(x) = -b/x.

a) Sketch this potential energy.



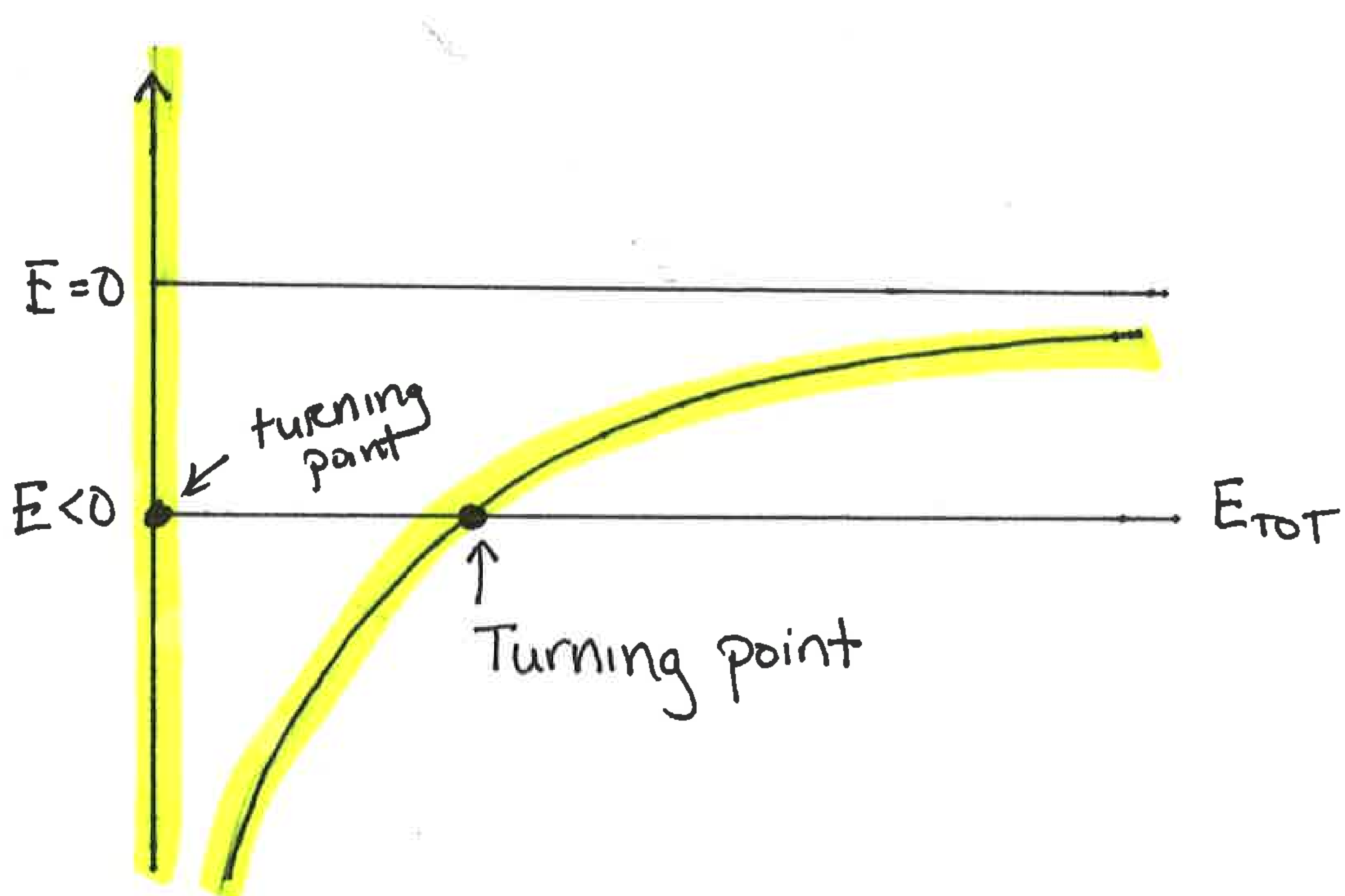
A particle is subject to potential energy that has an infinitely high wall at the origin, but for positive values of x is of the from $U(x) = -b/x$.

b) How much energy could this particle have and still be bound by such a potential energy?

The particle will be bound so long as the total energy is negative.

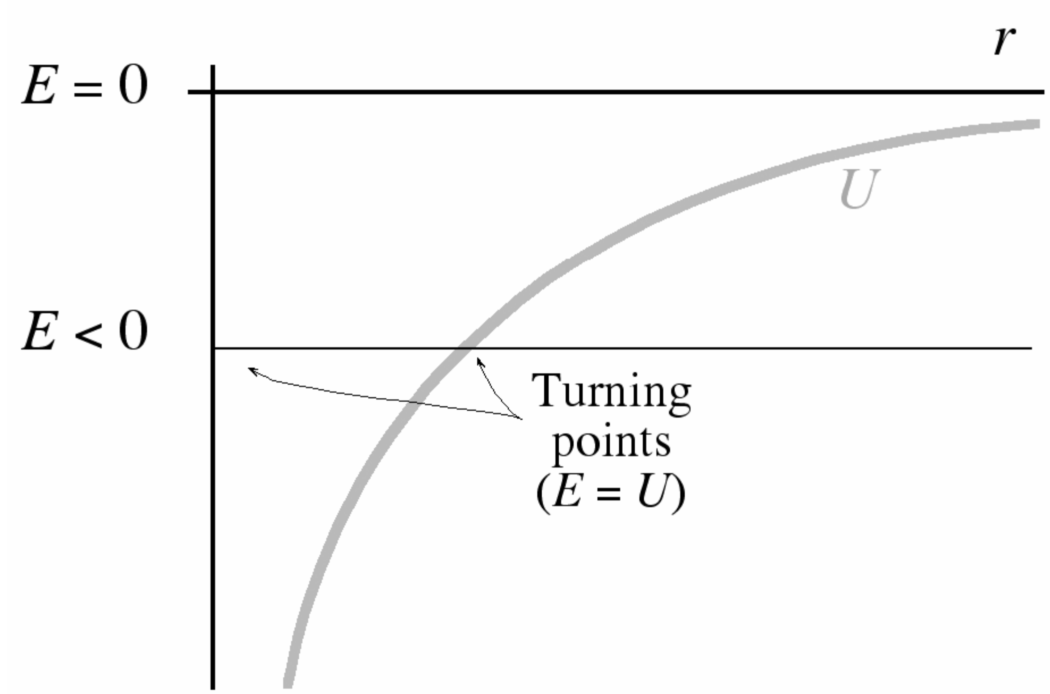
A particle is subject to potential energy that has an infinitely high wall at the origin, but for positive values of x is of the from U(x) = -b/x.

c) Add to the sketch a plot of E for a bound particle and indicate the classical outer turning points.



Professor Jodi Cooley

A comet in an extremely elliptical orbit about a star has a maximum orbit radius. By comparison, its minimum orbit radius may be nearly 0. Make plots of the potential energy and a plausible total energy E versus radius on the same set of axes. Identify the classical turning points on your plot.



How does this relate to Quantum Mechanically Bound States?

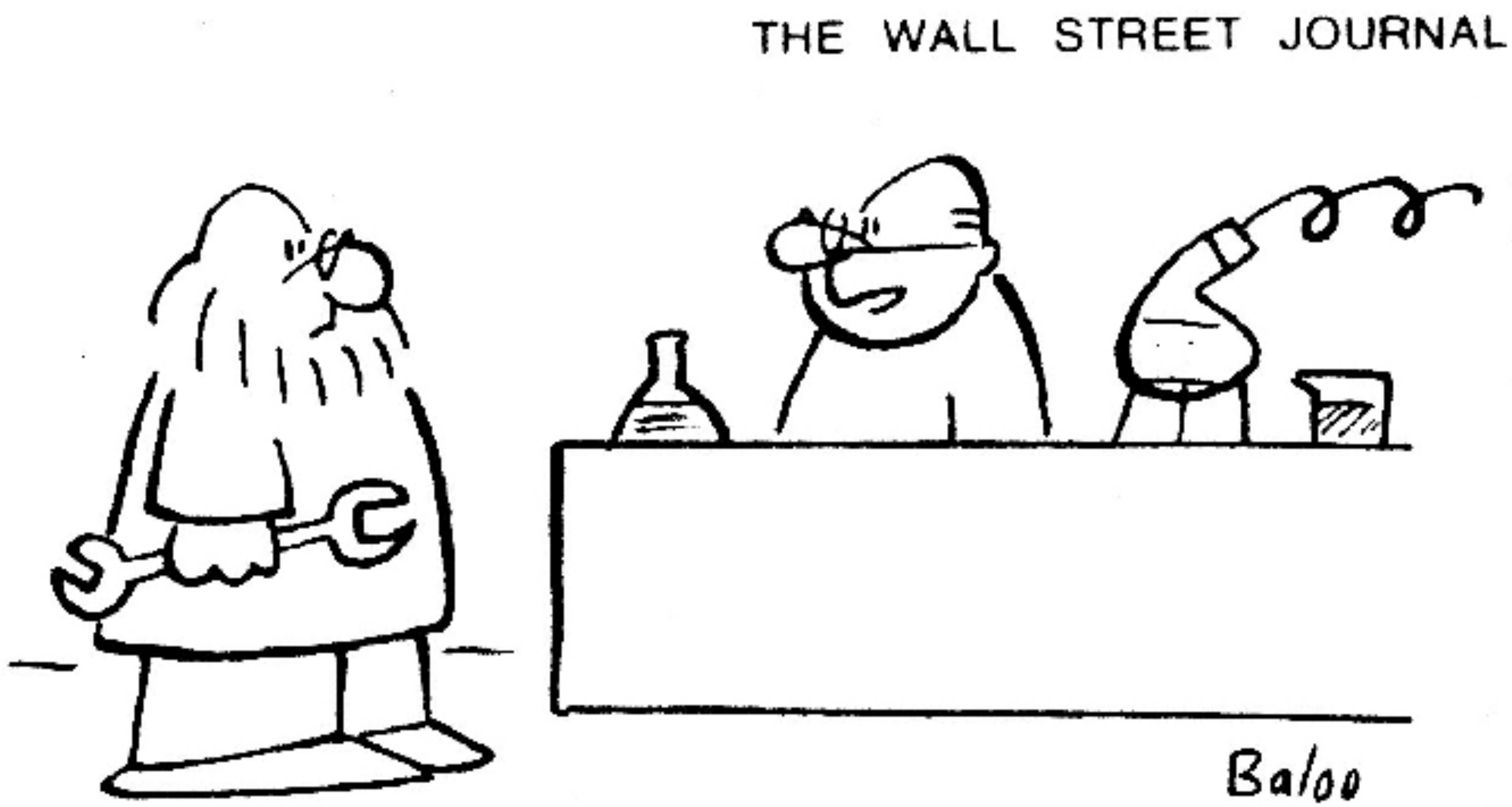
Recall our requirements:

- The player(s) are described as quantum waves of probability density.
- The waves must be continuous.
- The first derivatives of the waves (in space) will nearly always be continuous.

The quantum bound states of a particle under the influence of a potential will be STANDING WAVES. There are only discrete states allowed for particles in such bound states. (think wave on a string, bound at two ends)

Quirk: Wave functions can exist in the "classically forbidden" area. i.e. You can find the particle where, classically, it can not be.

THE END (FOR TODAY)



"Blumenkraft, I'm afraid you have the wrong idea about quantum mechanics."