Welcome back
to PHY 3305

Today's Lecture:

Particle in an Infinite Well

Erwin Rudolf Josef
Alexander Schrödinger

1887-1961

ANNOUNCEMENTS

- Reading Assignment for Tuesday, October 17th:
  Chapter 5.6.
- Problem set 8 is due Tuesday, October 17th at 12:30 pm.
- Regrade for problem set 7 is due Tuesday, October 17th at 12:30 pm. Watch your email for homework pickup instructions tomorrow.
- Exam 2 is in class, Thursday, October 19th at 12:30 pm. It will directly cover chapters 3 and 4. This does not mean you can forget everything from the beginning of the course as the material builds.
- Dr. Cooley will be out of town October 15 - 18th and will be available by appointment via zoom for office hours. Mr. Thomas will lead classroom discussions on Tuesday, October 17th.

Extra Credit Opportunity # 2:

1. Attend SMU Physics Dept. Student-Only meet and greet.
2. Ask a thoughtful question of Mr Frank during the event.
3. Write a reflective paragraph about the question and response using acceptable standard English/grammar.
4. RSVP - (ssekula@smu.edu). The deadline for RSVPs is Oct. 13.

Adam Frank

Astrophysicist ◊ Science Communicator
University of Rochester

Allman Family Lecture Series

October 17, 2017
10:45-11:45am
Heroy Hall 153

Light refreshments will be provided by the DCII

Adam Frank is a Professor of Physics and Astronomy at the University of Rochester, as well as a gifted science communicator and founder of National Public Radio's blog "13.7: Cosmos & Culture."

Professor Frank's research is in the general area of Theoretical Astrophysics, and in particular the hydrodynamic and magneto-hydrodynamic evolution of matter ejected from stars. He is also actively involved in science outreach as a popular science writer. He has contributed articles to Discover and Astronomy magazines. He received the science-writing prize from the Solar Physics Division of the American Astronomical Society in 1999.

Join the SMU Physics Department for a students-only "meet and greet" with Prof. Frank!

SMU. Dedman College of Humanities & Sciences

Modern Physics Presentations

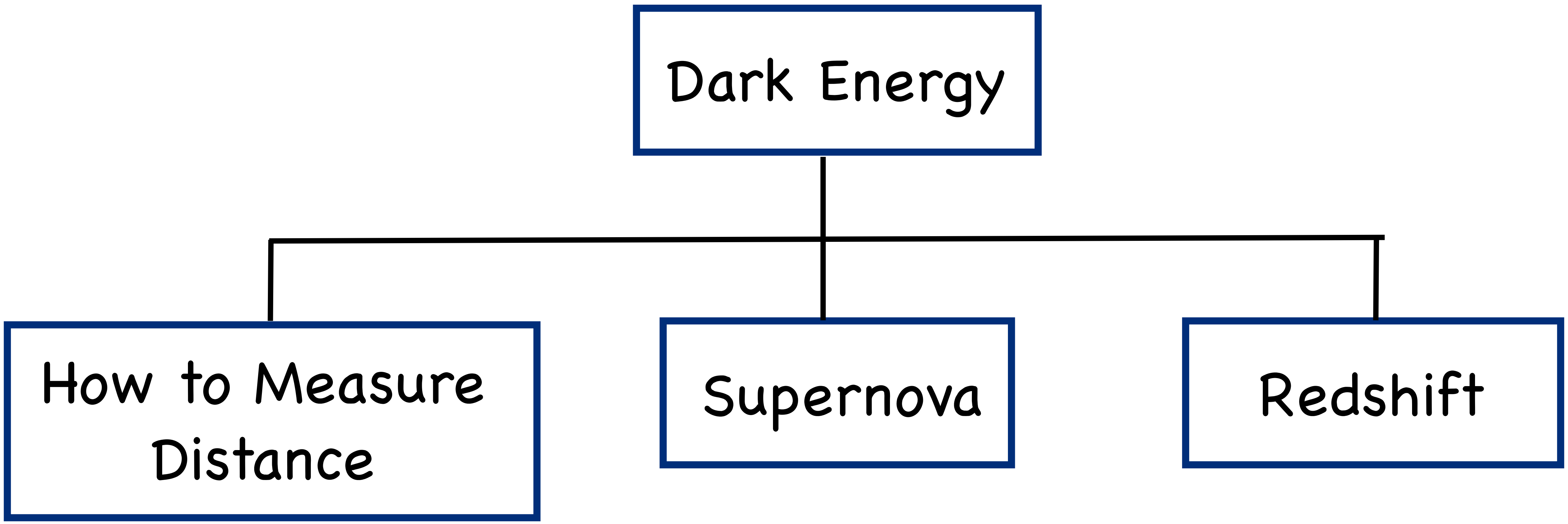
You will be expected to do the following:

1. Deliver a 15 minute presentation on the topic.
2. Adhere to the basic principles of good presentation design.
3. Answer questions from the audience on the subject.
4. Ask questions of your classmates on their subjects.

Outlines Due Next Week

- Outlines should include key ideas that will be explained during the presentation.
- Present you outline in outline form, using Roman Numerals for Key Ideas and letters for ideas supporting key ideas.
- Outlines must be typed.

Identify Key Ideas



Make sure to define concepts and parameters needed to explain your problem or idea.

Example Outline

I. Title Slide: Dark Energy

II. Dark Energy
   - A. Relevance
   - B. Definition

III. Distance Ladder - How to Measure Cosmic Distances
   - A. LIDAR
   - B. Geometry
   - C. Standard Candles

IV. Supernovae
   - A. Definition
   - B. How to Search for them.
   - C. Relevance

V. How to Measure Dark Energy
   - A. Redshift
   - B. Relationship Between Supernovae Distance and Redshift

From Video Lecture:

Case 1: Particle in a Box - The Infinite Well

In this case a particle is confined in x by a potential which represents infinitely steep "walls".

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L}) & (0 < x < L) \\ 0 & x < 0; \quad x > L \end{cases}$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

These are the normalized, continuous wave functions representing the allowed states of a particle in an infinite well and their corresponding energies.

Write out the total wave function $\Psi(x,t)$ for an electron in the n = 3 state of a 10.0 nm wide infinite well. Other than the symbols x and t, the function should only include numerical values.

Given:

$$L = 10.0\ nm = 10^{-8}\ m$$

$$n = 3$$

The total wave function for an infinite well can be written as

$$\Psi(x,t) = \psi(x)\phi(t) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} e^{-i(E/\hbar)t}$$

$$\Psi(x,t) = \psi(x)\phi(t) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}e^{-i(E/\hbar)t}$$

Need to find expression for E_3.

$$E_3 = \frac{3^2\pi^2\hbar^2}{2mL^2}$$

Put it all together.

$$\Psi(x,t) = \sqrt{\frac{2}{10^{-8}}}\sin\frac{3\pi x}{10^{-8}}e^{-i\left(\frac{3^2\pi^2(1.05\times10^{-34})}{2(9.11\times10^{-31})(10^{-8})^2}\right)t}$$

$$\Psi(x,t) = (1.41\times10^4\ m^{-1/2})\sin(9.42\times10^{16}\ m^{-1}\ x)e^{-i(5.12\times10^{13}\ s^{-1})t}$$

An electron in the n = 4 state of a 5.0 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

We know:

$$E_4 - E_1 = \frac{\pi^2 \hbar^2}{2mL}(4^2 - 1^2)$$

and

$$E = \frac{hc}{\lambda} \longrightarrow \lambda = \frac{hc}{E}$$

Put it together

$$\lambda = \frac{2mL^2hc}{\pi^2\hbar^2(4^2-1^2)} = \frac{2(9.11 \times 10^{-31}\ kg)(10^{-8}m)^2(6.62 \times 10^{-34}\ J \cdot s)(3 \times 10^2\ m/s)}{\pi^2(1.05 \times 10^{-34}\ J \cdot s)^2(4^2 - 1^2)}$$

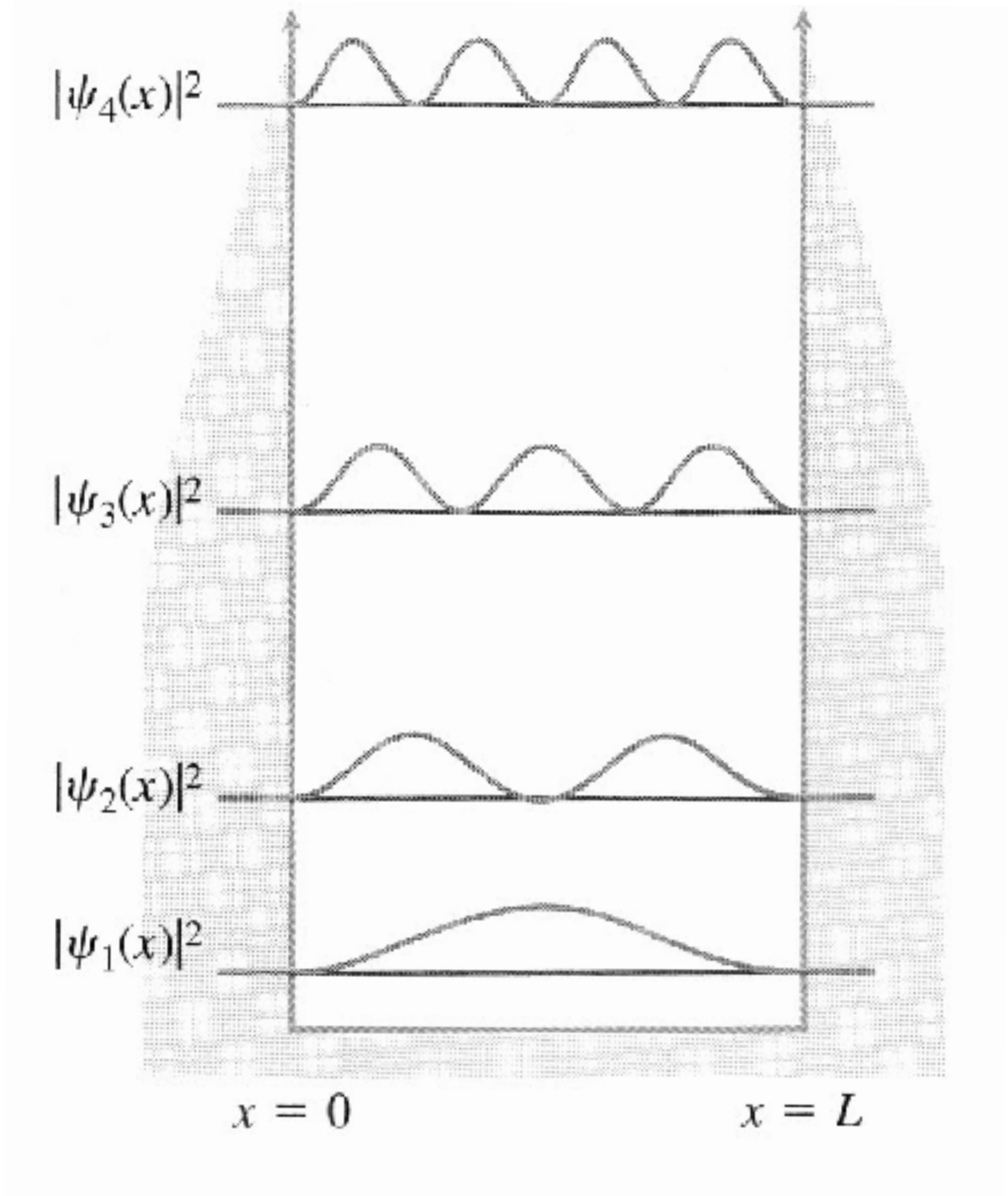
$$\boxed{\lambda = 5.5 \times 10^{-6}\ m}$$

From Video Lecture:

LESSONS LEARNED

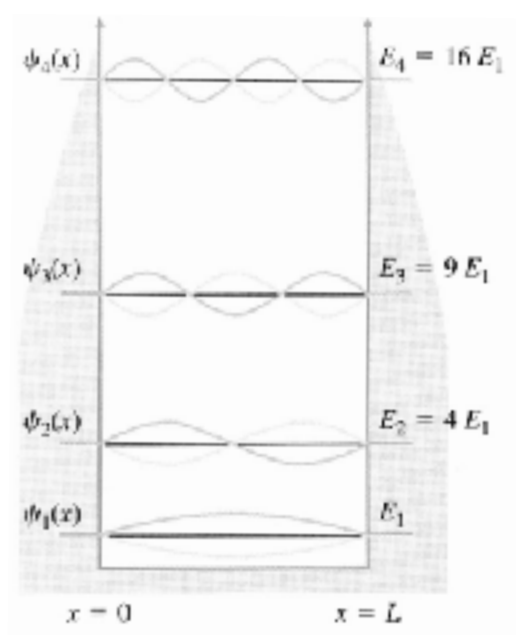
2. The probability densities have nodes. Thus, there are places where a particle is more likely to be found.

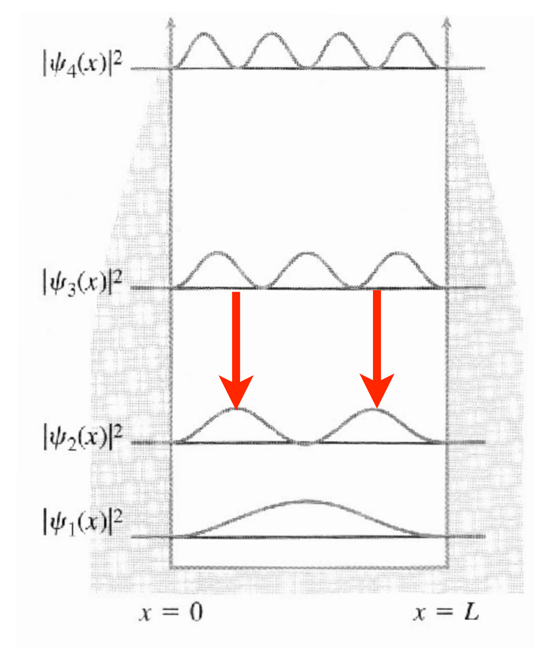
   Nodes are places where the probability density is zero.



Consider a particle in a box with infinite sides. If the particle is in the n=2 stationary state, where is the particle most likely to be found?

a) In the center of the box.
b) One-third of the way from either end.
c) One-quarter of the way from either end.
d) It is equally likely to be found at any point in the box.





$\psi(x)$ is the wave function for a particle moving along the x axis. The probability that the particle is in the interval from $x = a$ to $x = b$ is given by:

A) $\psi(b) - \psi(a)$

B) $|\psi(b)|^2 - |\psi(a)|^2$

C) $|\psi(b) - \psi(a)|^2$

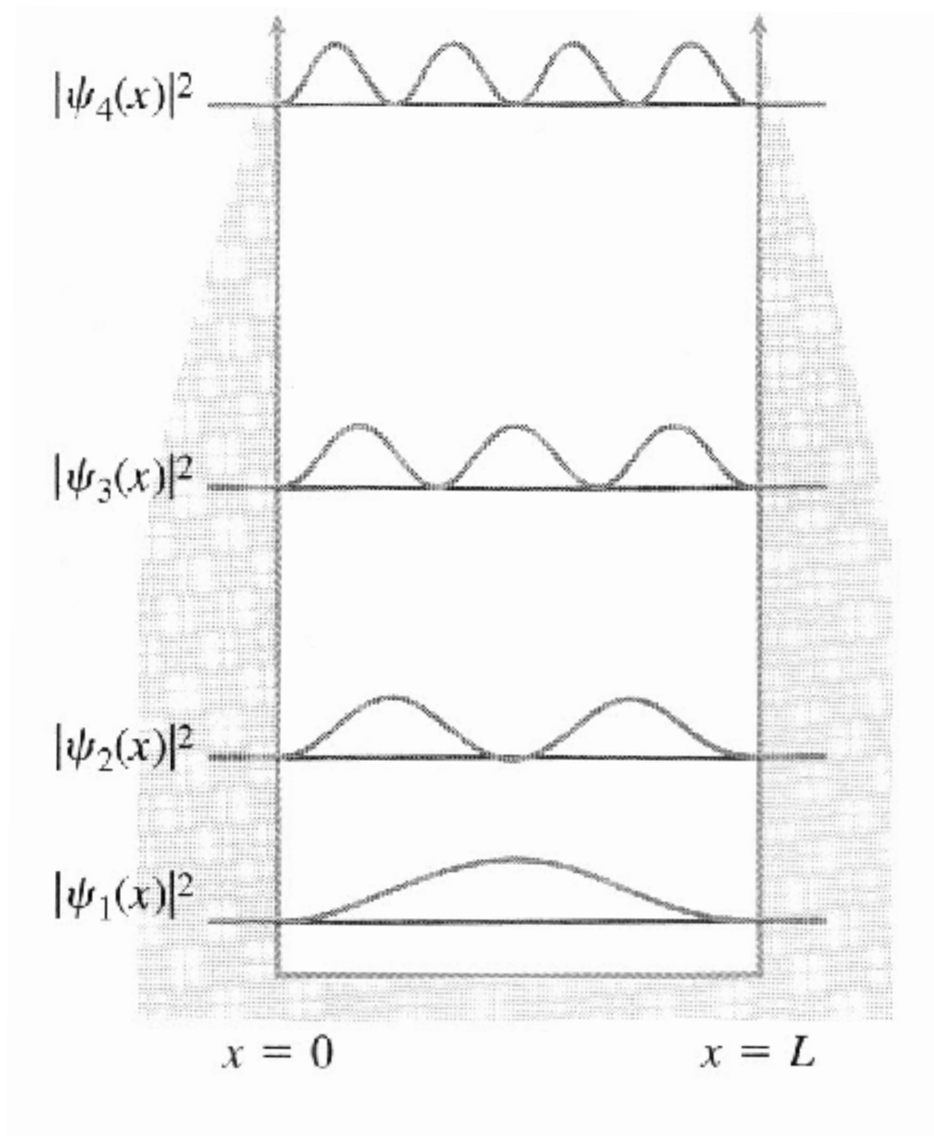
D) $\int_a^b \psi(x)\, dx$

E) $\int_a^b |\psi(x)|^2\, dx$

What is the probability that a particle in the first excited state of an infinite well would be found in the middle third of the well?

The wave function of the first excited state (n = 2) is given by:

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$



To find the probability of being in the middle third, we integrate.

$$Prob = \int_{\frac{1}{3}L}^{\frac{2}{3}L} |\psi(x)|^2 dx$$

$$Prob = \int_{\frac{1}{3}L}^{\frac{2}{3}L} |\psi(x)|^2 dx \quad = \int_{\frac{1}{3}L}^{\frac{2}{3}L} \psi^*(x)\psi(x)dx$$

$$= \frac{2}{L}\int_{\frac{1}{3}L}^{\frac{2}{3}L} \sin^2(\frac{\pi x}{mL})$$

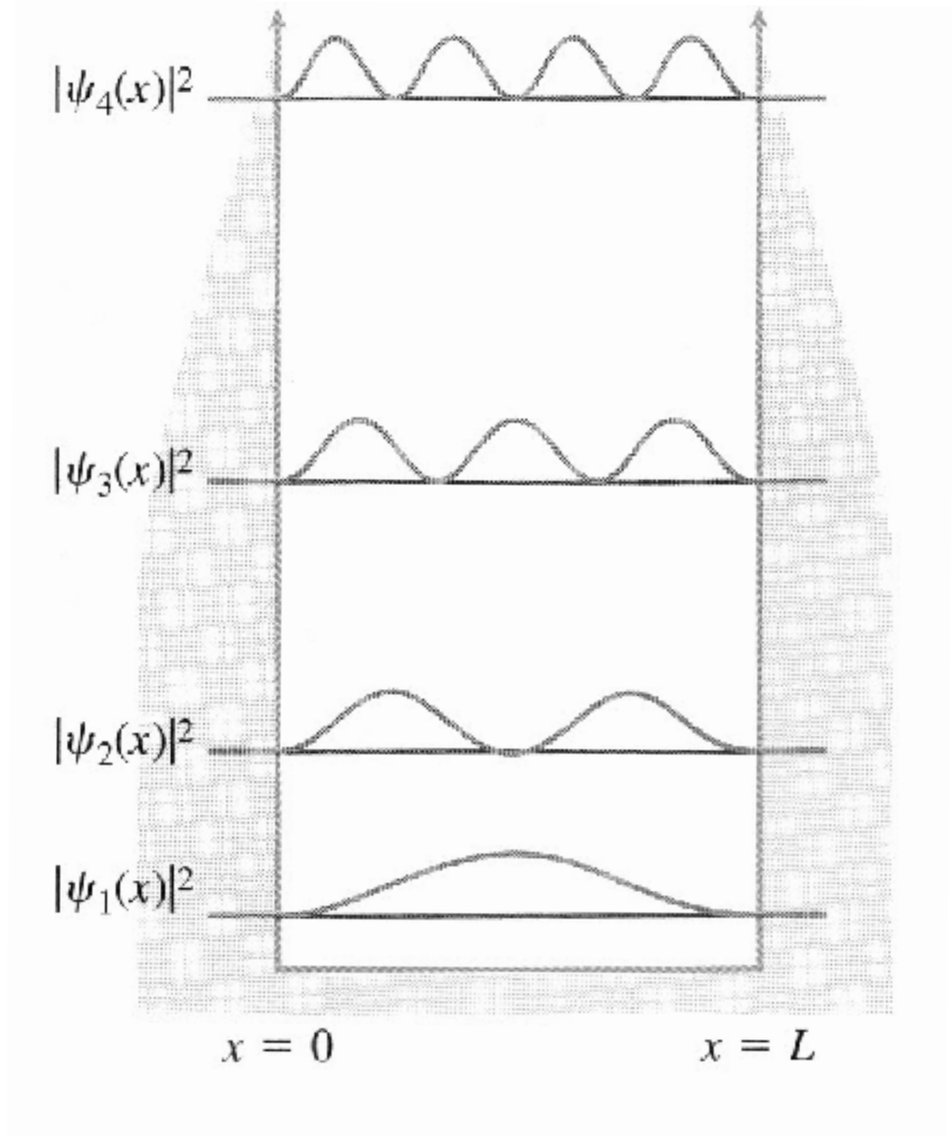
Either solve or "look up" the solution to this integral in Schaum's or your textbook.

$$P = \frac{2}{L}[\frac{x}{2} - L\frac{\sin\frac{4\pi x}{L}}{8\pi}]_{\frac{1}{3}L}^{\frac{2}{3}L} = \frac{2}{L}[\frac{L}{6} - L\frac{\sin\frac{8\pi}{3} - \sin\frac{4\pi}{3}}{8\pi}]$$

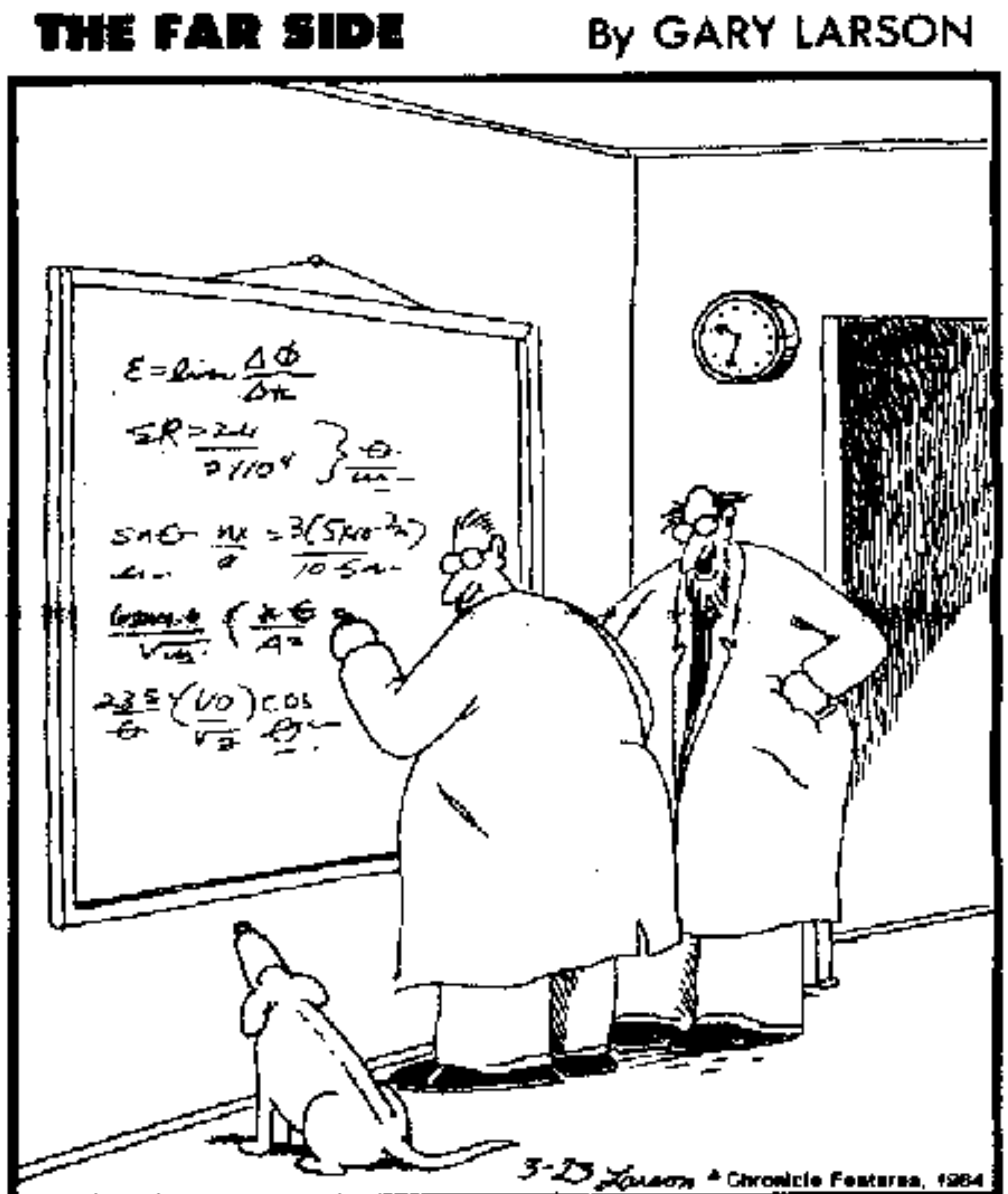
$$P = 0.196$$

How does your answer compare with the classical expectation?

Classically, the probability should be 1/3 = 0.333. Our value is lower due to the fact the region is centered on the node.



THE END (FOR TODAY)



"Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics."