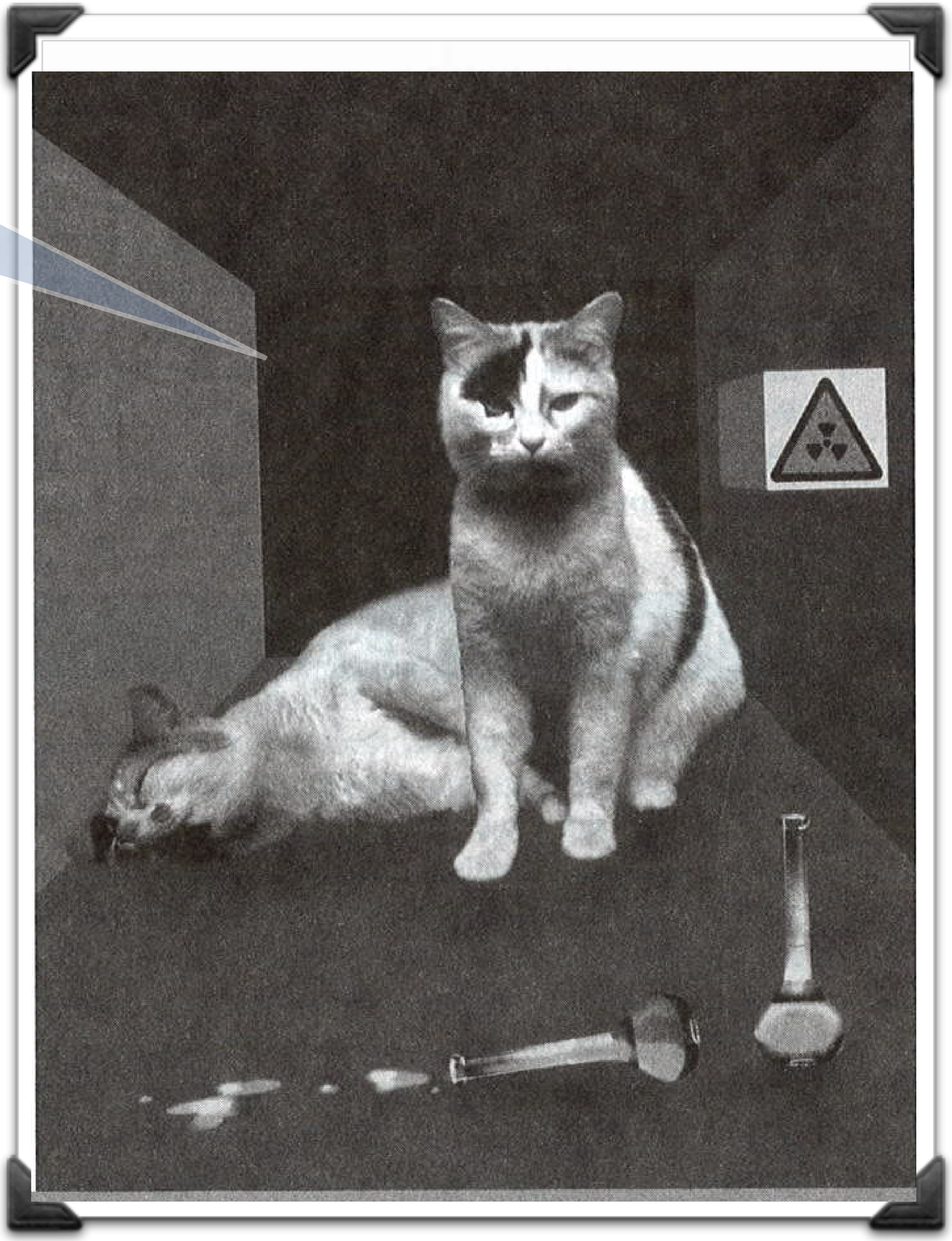
Welcome back to PHY 3305



Today's Lecture:

More Schoedinger Equation!

Schrödinger's Cat

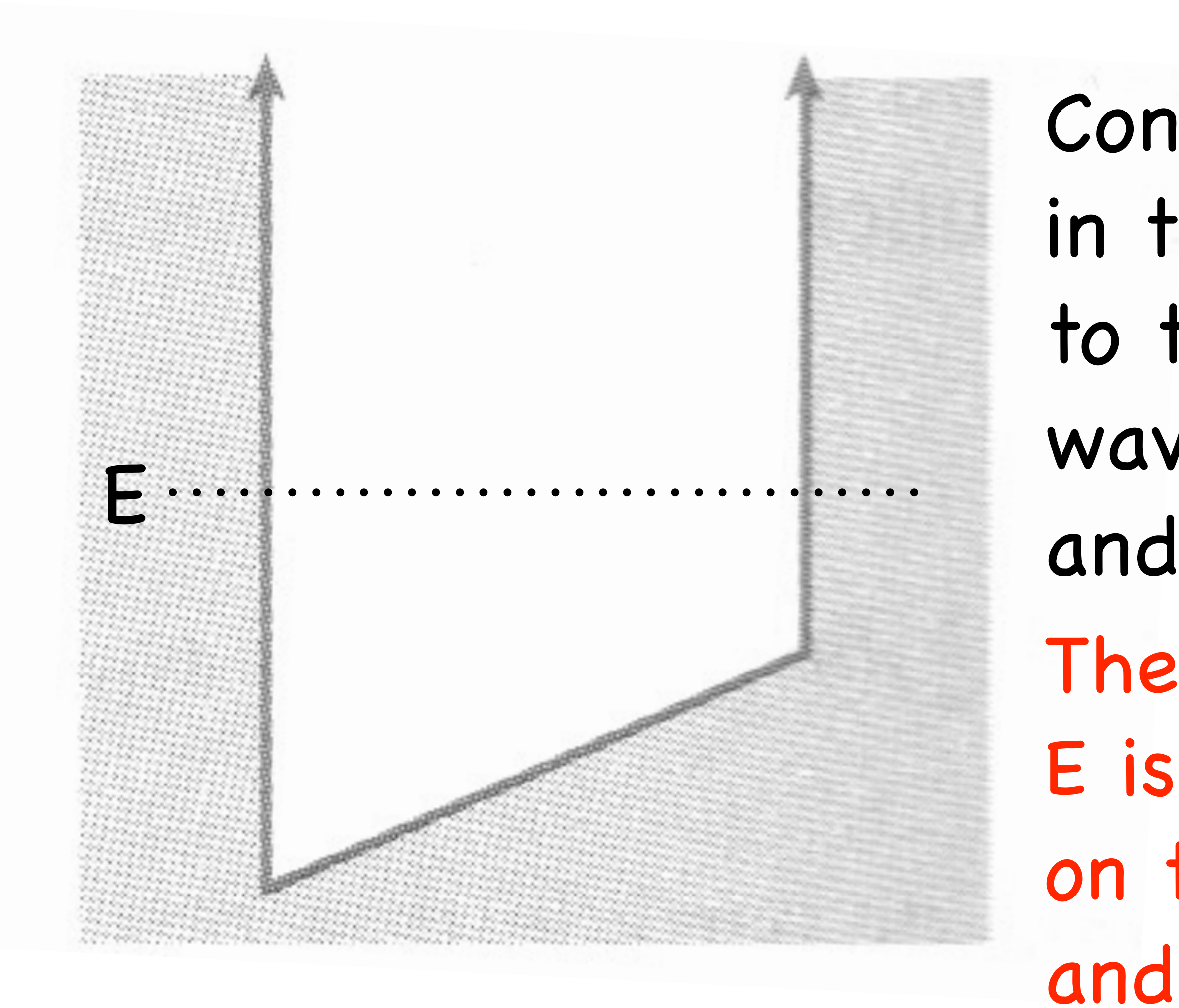
Alive or Dead?

Professor Jodi Cooley

ANNOUNCEMENTS

- Reading Assignment for Tuesday, October 24th: Chapter 5.7.
- Problem set 9 is due Tuesday, October 24th at 12:30 pm.
- Regrade for problem set 8 is due Tuesday, October 24th at 12:30 pm.
- Next week, detailed outlines for your presentations will be due.
- Exam 2 is in class, Thursday, October 19th at 12:30 pm. It will directly cover chapters 3 and 4. This does not mean you can forget everything from the beginning of the course as the material builds.
- Dr. Cooley will be out of town October 15 - 18th and will be available by appointment via zoom for office hours.

Review Question



Consider a particle with energy, E, bound in the potential well in the figure shown to the left. Decide how, if at all, its wavelength and amplitude should vary and sketch a plausible wave function.

The wavelength cannot be constant. Since E is constant, the kinetic energy decreases on the right, resulting in lower momentum and a larger wavelength.

We most often find the particle in a region where the particle moves more slowly. Hence the probability density (and thus the amplitude) will be largest when KE is smallest.

$$KE = \frac{p^2}{2m} \quad \& \quad p = \frac{h}{\lambda}$$

From Video Lecture:

Bound States II: Particle in a Finite Well

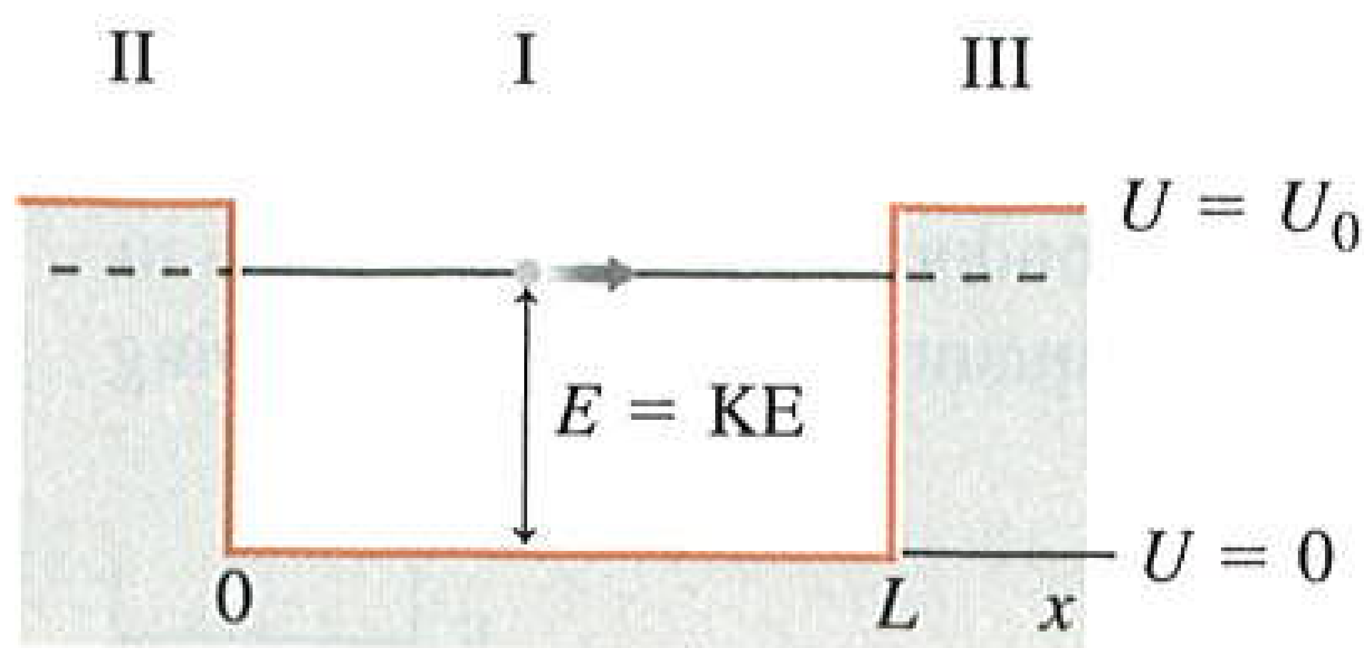
To solve we need to write the SWE in 3 different regions.

Regions II and III:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U_0\psi(x) = E\psi(x)$$

Regions I:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$



From Video Lecture:

$$2\cot(kL) = \frac{k}{\alpha} - \frac{\alpha}{k} \qquad k \equiv \sqrt{\frac{2mE}{\hbar^2}} \qquad \alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}}$$

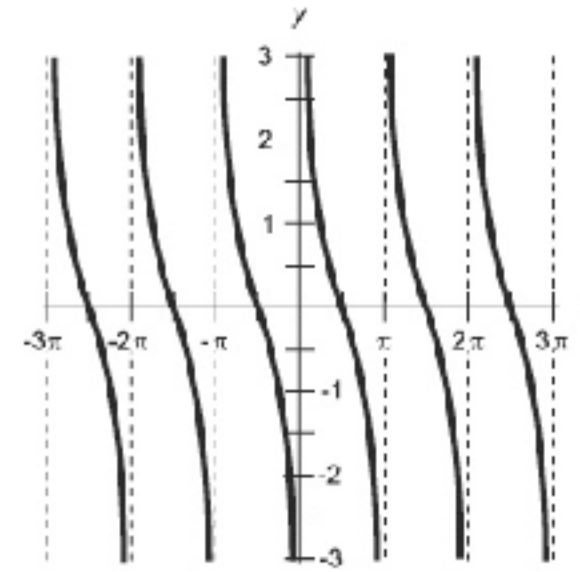
Limiting Case: U_0 -> infinity:

α -> infinity.

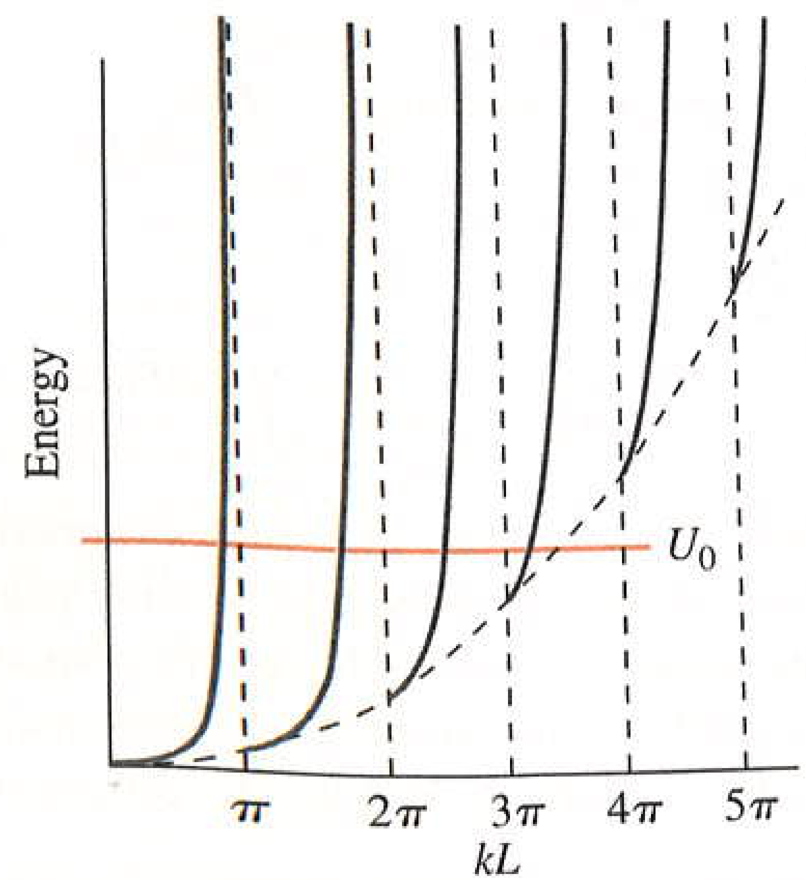
$$\cot(kL) = -\infty$$

$$kL = n\pi$$

This is the same as the infinite square well!



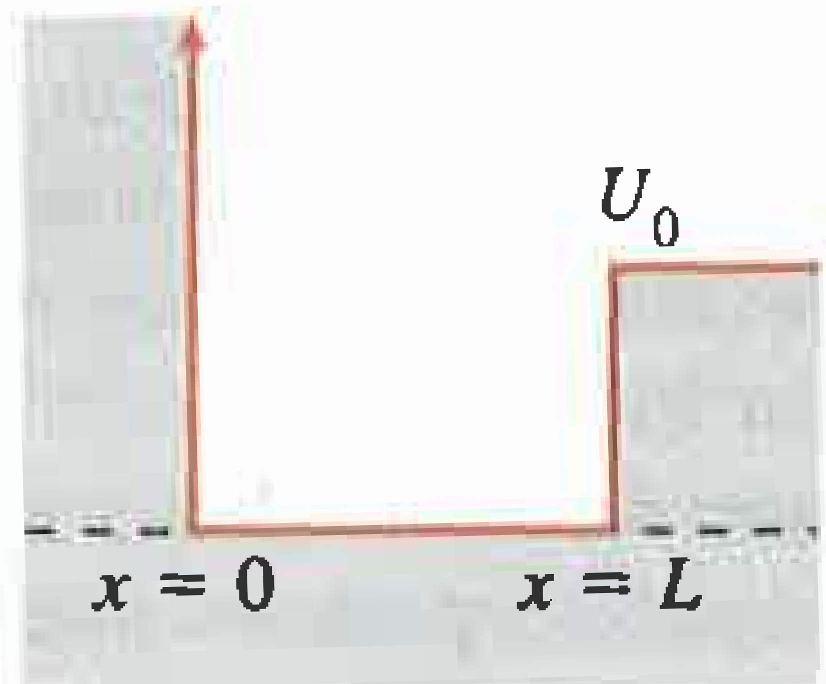
Graph this quantization -
A little math
(substitute k and α)
puts the transcendental
eqn ito energy (U_0).



Consider the well below. What is the maximum wavelength possible in the well?

The LHS is an infinite well. As such, it must have a node at zero at the LHS boundary. The RHS is a finite well. Inside the wall, the solution is an exponential. Thus, the wave function must be coming downward (or at least be flat) at the RHS boundary. Thus, the largest wavelength will be 4L.

$$U(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ U_0 & x \geq L \end{cases}$$



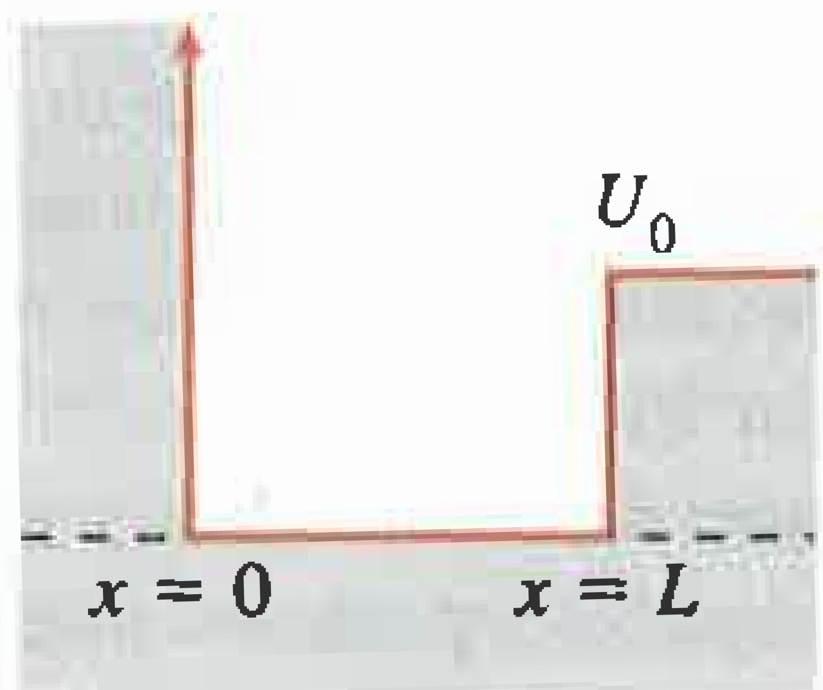
If the wavelength is at most 4L, what is the value of the kinetic energy in the well?

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$= \frac{h^2}{2m(4L)^2}$$

$$\boxed{KE = \frac{h^2}{32mL^2}}$$

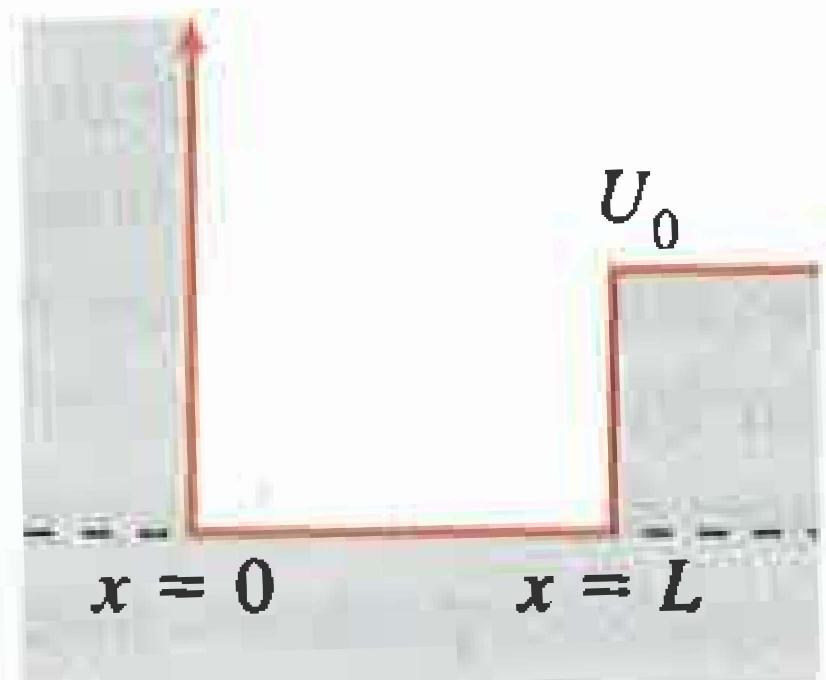
$$U(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ U_0 & x \geq L \end{cases}$$



What must the minimum value of U_0 be if there is to be a bound state in this well?

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ U_0 & x \ge L \end{cases}$$

We found that in the well when $U_0 = 0$, $KE = h^2/32mL^2$. By conservation of energy, $E = KE = h^2/32mL^2$.



To be a bound state $E <= U_0$. Thus, $U_0 >= h^2/32mL^2$.

Write solutions to the Schrodinger equation appropriate to the two regions in this example.

Region I:

$$\psi(x) = A\sin kx + B\cos kx$$

where $k = \dfrac{\sqrt{2mE}}{\hbar}$

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ U_0 & x \ge L \end{cases}$$

Region II:

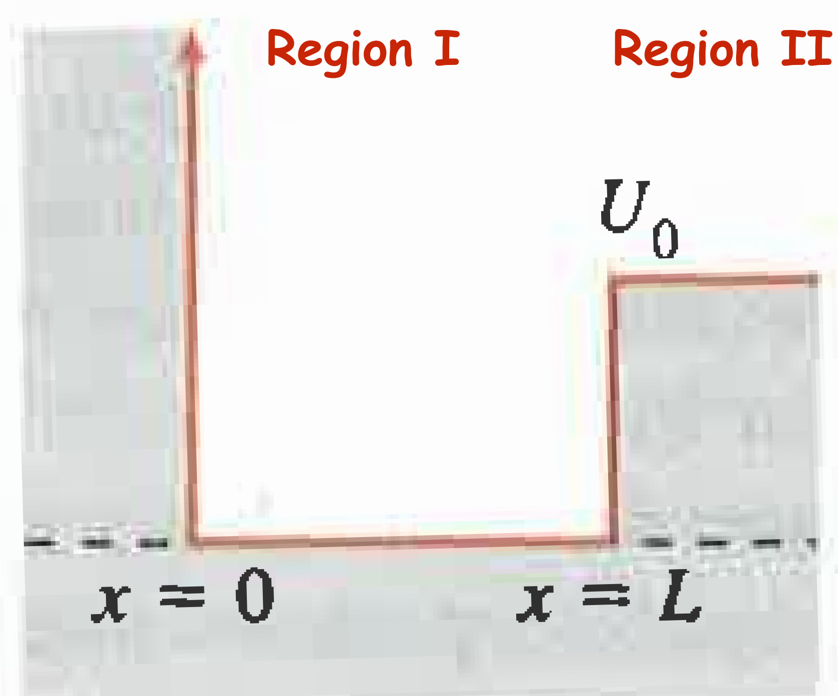
D=0

$$\psi(x) = Ce^{-\alpha x} + \cancel{De^{+\alpha x}}$$

Note: $e^{\alpha x}$ diverges as x approaches positive infinity. Thus, it is physically unacceptable.

$$\psi(x) = Ce^{-\alpha x}$$

where $\alpha = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$

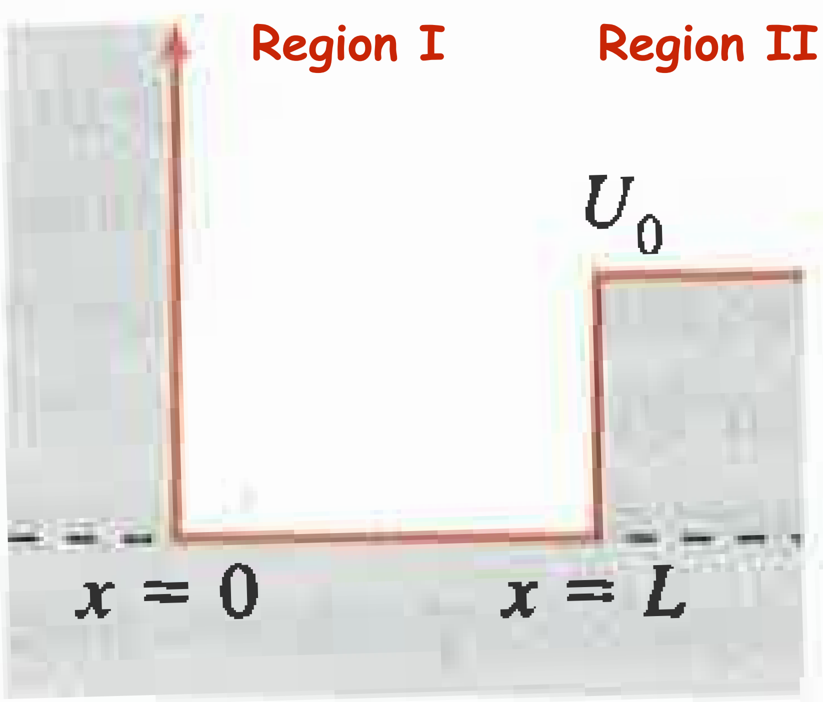


Impose required continuity conditions and obtain the energy quantization

$$\sqrt{E}cot(\frac{\sqrt{2mE}}{\hbar}L) = \sqrt{U_0 - E}$$

$$U(x) = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ U_0 & x \geq L \end{cases}$$

In order to solve, we need to impose the smoothness conditions at the boundaries.



$\psi(x) = A \sin kx + B \cos kx$ and $\psi(x) = Ce^{-\alpha x}$

$\psi(x < 0) = 0$

Continuity of Wave Function:

apply boundary conditions:

x = 0: $A \sin(k0) + B \cos(k0) = 0$

$$B = 0$$

Thus, $\boxed{\psi(x) = A \sin kx}$

x = L: $A \sin kL + B \cos kL = Ce^{-\alpha L}$

$$\boxed{A \sin kL = Ce^{-\alpha L}}$$

$$\psi(x) = A\sin kx \quad \text{and} \quad \psi(x) = Ce^{-\alpha x}$$

$$\boxed{A\sin kL = Ce^{-\alpha L}}$$

Continuity of First Derivative of Wave Function:

$$\frac{d\psi(x)}{dx} = kA\cos kx \quad \text{and} \quad \frac{d\psi(x)}{dx} = -\alpha Ce^{-\alpha x}$$

apply boundary conditions at x = L:

$$\boxed{kA\cos kL = -\alpha Ce^{-\alpha L}}$$

Divide the boxed equations:

$$\frac{kA\cos kL = -\alpha Ce^{-\alpha L}}{A\sin kL = Ce^{-\alpha L}} \longrightarrow k\cot kL = -\alpha$$

$$\boxed{\sqrt{E}\cot\left(\frac{\sqrt{2mE}}{\hbar}L\right) = -\sqrt{U_0 - E}}$$

From Video Lecture:

There exists some probability of finding the particle in the classically forbidden region in a finite square well.

In this region, E_{tot} is less than U_0. This implies that there is negative KE. Classically this is not allowed.

The measure of how far $\psi(x)$ extends into the forbidden region is given by α.

$$\boxed{\delta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(U_0 - E)}}}$$

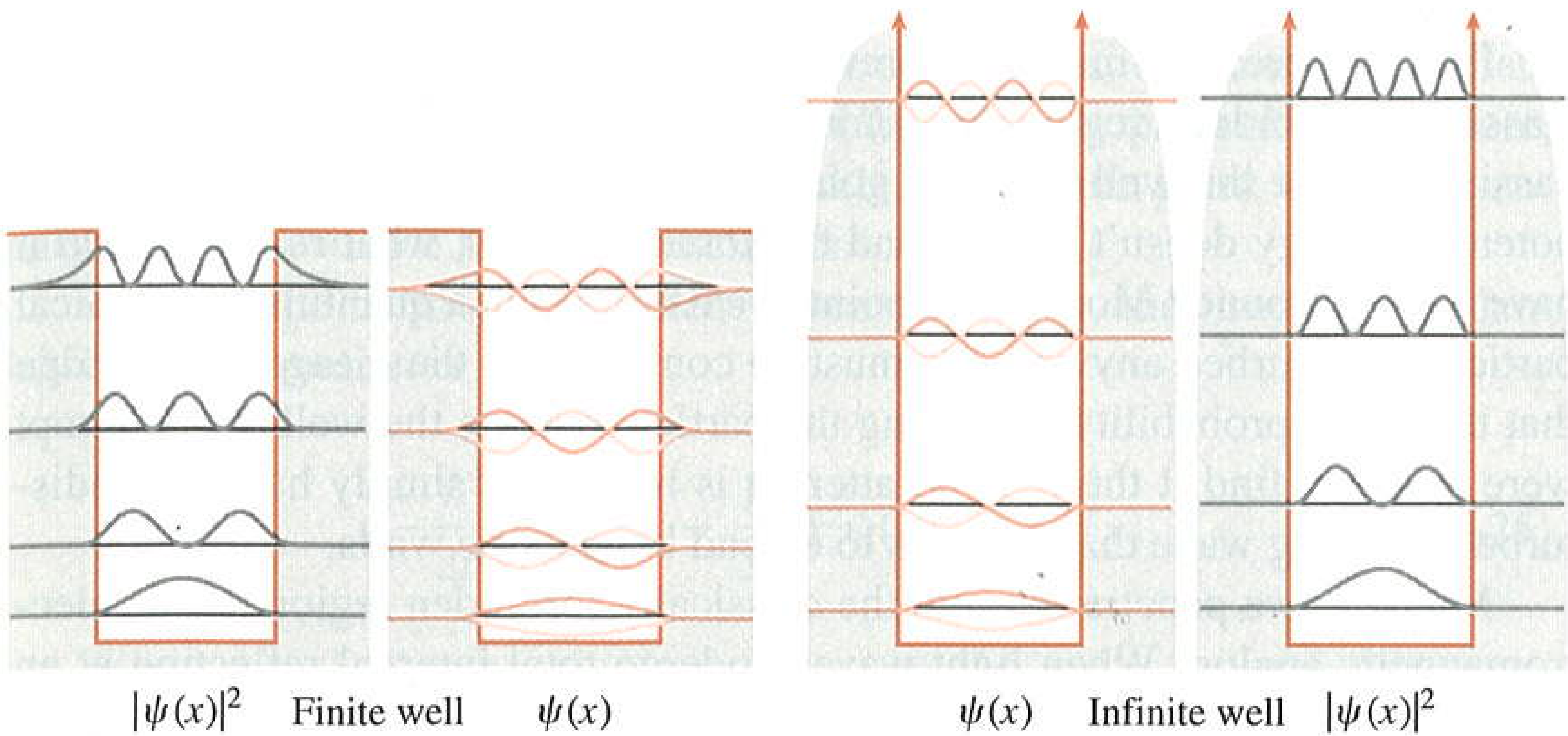
A 50 eV electron is trapped between electrostatic walls 200 eV high. How far does its wave function extend beyond the walls?

$$\delta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

$$= \frac{1.055 \times 10^{-34}\ J \cdot s}{\sqrt{2(9.11 \times 10^{-31}\ kg)(200\ eV - 50\ eV)(1.6 \times 10^{19}\ J/eV)}}$$

$$\boxed{\delta = 1.6 \times 10^{-11}\ m}$$

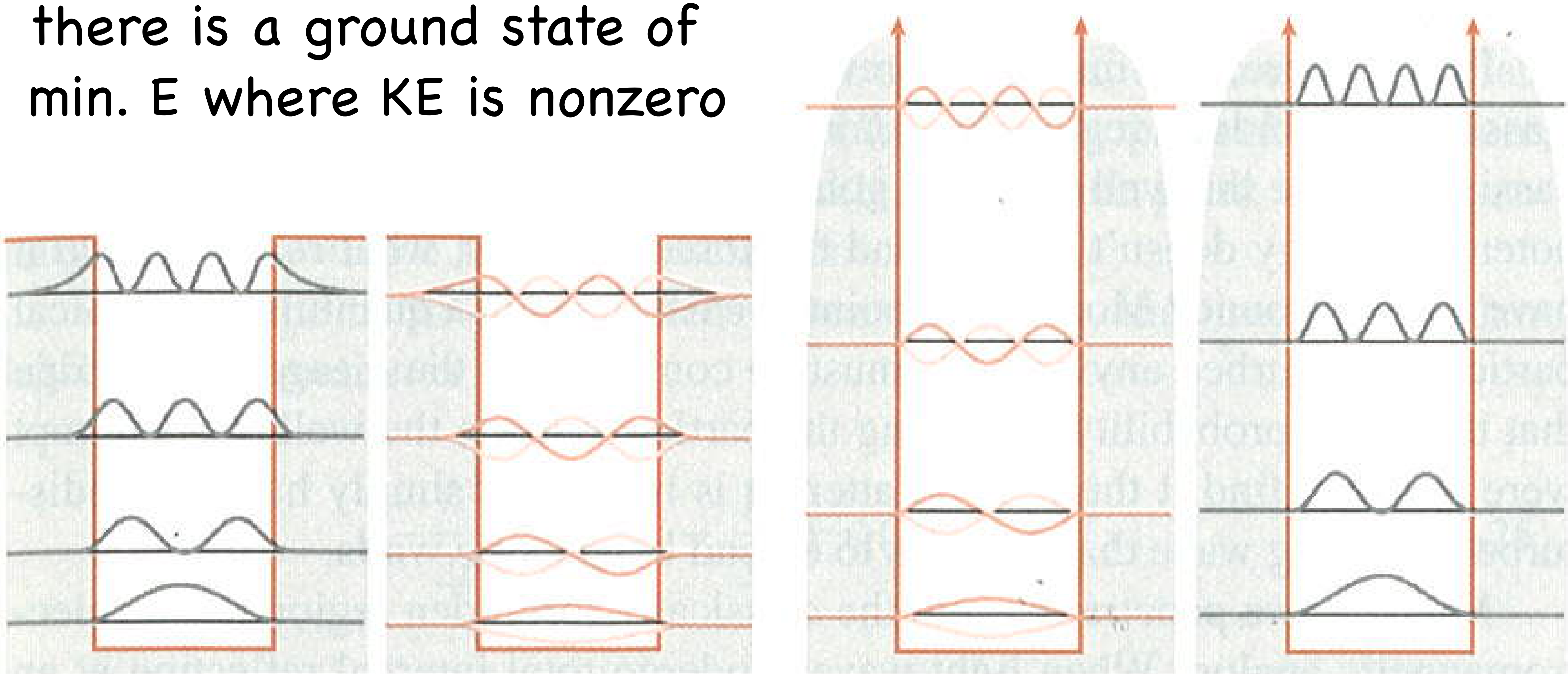
Sketch the wave function and the probability distribution for the n = 4 state for the finite square well potential and for the infinite square well.



Comment on the similarities and differences.

same

energy is quantized
there is a ground state of
min. E where KE is nonzero

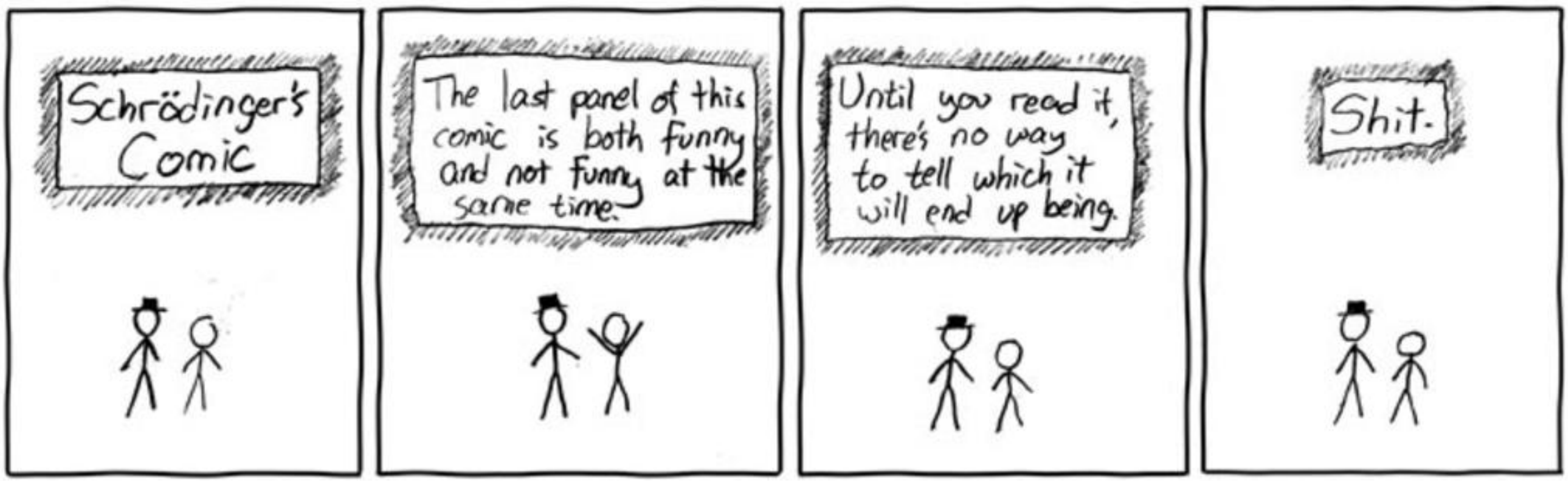


$|\psi(x)|^2$ Finite well $\psi(x)$ $\psi(x)$ Infinite well $|\psi(x)|^2$

different

Finite well - waves penetrate into forbidden region
energy levels are closer together

THE END
(FOR TODAY)



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