Welcome back to PHY 3305

Today's Lecture:

More Hydrogen Atom

Spin!

Wolfgang Ernst Pauli
1900 - 1958

Professor Jodi Cooley

ANNOUNCEMENTS

- Reading Assignment for Nov 9th: Harris 8.1 .
- Problem set 12 is due Tuesday, Nov 14th at 12:30 pm.
- Regrade for problem set 11 is due Tuesday, Nov 14th at 12:30 pm.
- You will be required to practice you talk in front of another person this week. Make sure your slides are in near final form.

Review Question 1

Recall our experiments with the electroscope. A large excess charge was placed on a piece of metal, then separately we shine light sources of two pure but different colors at it. The first source is extremely bright, but shows no change in the net charge. The second is much dimmer, but the charge disappears. Explain what evidence this provides for the particle nature of light.

The amount of light clearly is not the deciding factor. The dimmer light has less intensity, but what intensity it has is composed of little particles each of which has enough energy to knock an electron out of the metal. The bright light has many particles, but each has insufficient energy to eject an electron.

Review Question 2

What is a quantum number and how does it arise?

It is a quantity that takes on different discrete values, each of which corresponds to some physical quantity (which is accordingly "quantized"). They arise from imposing physical conditions, such as continuity or normalization on mathematically solutions of governing differential equations.

Review Question 3

Generally speaking, why is the wave nature of matter so counterintuitive?

Because we live in a world in which common dimensions are much larger than the electron's wavelength. We never experience its particle nature, so a wave nature is unexpected.

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Review Question 4

Will a particle with a longer wavelength tunnel more easily?

Longer wavelength suggests more wave-like behavior and less classical particle-like behavior. Matter waves have shorter wavelengths and act more like classical particles. Hence, longer wavelengths will tunnel more easily.

Can anyone think of an example we talked about earlier in the class that demonstrates this?

Matter waves have shorter wavelengths and higher frequencies. Hence, the wave-like nature of matter is harder to detect. Think of electrons. They have wavelengths that are on the order of pm whereas EM waves have wavelengths on the order of nm.

An electron is in an n = 4 state of the hydrogen atom.

a) What is its energy?

We know that

$$E_n = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}\frac{1}{n^2} = -\frac{13.6\ eV}{n^2}$$

$$E_4 = -\frac{13.6\ eV}{4^2}$$

$$\boxed{E_4 = -0.85\ eV}$$

An electron is in an n = 4 state of the hydrogen atom.

b) What properties besides energy are quantized and what values might be found if these properties are measured?

The magnitude of the angular momentum and the z-component of the angular momentum may be quantized.

$$L = \sqrt{\ell(\ell+1)}\hbar$$ where ℓ = 0, 1, 2, or 3

$$\boxed{L = 0, \sqrt{2}\hbar, \sqrt{6}\hbar, 2\sqrt{3}\hbar}$$

$$L_z = m_\ell \hbar$$ where m_ℓ = 0, ±1, ±2, or ±3

$$\boxed{L_z = 0, \pm\hbar, \pm 2\hbar, \pm 3\hbar}$$

Video Lecture:

Radial Probability

The radial probability (per unit distance) is

$$\boxed{P(r) = \frac{dP}{dr} = r^2 R^2(r)}$$

Group 1:

Find the most likely distance(s) from the origin of an electron in the n = 1 state.

Group 2:

Find the most likely distance(s) from the origin of an electron in the n = 2, ℓ = 0 state.

n	l	m_l	$R(r)$	$\Theta(\theta)$	$\Phi(\phi)$
1	0	0	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	0	0	$\frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	1	0	$\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\sqrt{\frac{3}{2}} \cos\theta$	$\frac{1}{\sqrt{2\pi}}$
2	1	±1	$\frac{1}{\sqrt{3}(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\mp\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	0	0	$\frac{2}{(3a_0)^{3/2}} \left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
3	1	0	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right) e^{-r/3a_0}$	$\sqrt{\frac{3}{2}} \cos\theta$	$\frac{1}{\sqrt{2\pi}}$
3	1	±1	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}} \left(\frac{r}{a_0} - \frac{r^2}{6a_0^2}\right) e^{-r/3a_0}$	$\mp\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	2	0	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\sqrt{\frac{5}{8}} (3\cos^2\theta - 1)$	$\frac{1}{\sqrt{2\pi}}$
3	2	±1	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\mp\sqrt{\frac{15}{4}} \sin\theta\cos\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$
3	2	±2	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{\sqrt{15}}{4} \sin^2\theta$	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$

Group 1:

An electron in the n=1 state can only have values of ℓ=0 and $m_\ell = 0$. Thus,

$$R(r)_{1,0,0} = \frac{2}{a_0^{3/2}} e^{-\frac{r}{a_0}}$$

The radial probability is then

$$P(r) = r^2 |R(r)_{1,0}|^2 = \frac{4}{a_0^3} r^2 e^{-\frac{2r}{a_0}}$$

To find the maxima, we set the derivative of probability equal to zero and solve for r.

$$P(r) = r^2|R(r)_{1,0}|^2 = \frac{4}{a_0^3} r^2 e^{-\frac{2r}{a_0}}$$

Crunch:

$$\frac{dP}{dr} = \frac{4}{a_0^3}\left[2re^{-2r/a_0} - r^2\left(\frac{2}{a_0}\right)e^{-2r/a_0}\right]$$

$$= \frac{8r}{a_0^3} e^{-2r/a_0}\left(1 - \frac{r}{a_0}\right) = 0$$

The possible values of r are 0, infinity and a_0. In this case, 0 and infinity are minima.

The most likely value is a_0.

(Remember, to determine if min or max, determine if the second derivative is positive or negative). Or you can plot the function.

Group 2:

An electron in the n=2, ℓ=0 state can only have m_ℓ=0. Thus,

$$R(r)_{2,0,0} = \frac{1}{(2a_0)^{3/2}}(2 - \frac{r}{a_0})e^{-\frac{r}{2a_0}}$$

The radial probability is then

$$P(r) = r^2|R(r)_{2,0,0}|^2 = \frac{r^2}{(4a_0)^3}(2 - \frac{r}{a_0})^2 e^{-\frac{r}{a_0}}$$

$$P(r) = \frac{1}{8a_0^3}(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2})e^{-\frac{r}{a_0}}$$

To find the maxima, we set the derivative of probability equal to zero and solve for r.

$$P(r) = \frac{1}{8a_0^3}(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2})e^{-\frac{r}{a_0}}$$

Crunch:

$$\frac{dP}{dr} = \frac{1}{8a_0^3} re^{-r/a_0}\left(8 - \frac{16r}{a_0} + \frac{8r^2}{a_0^2} - \frac{r^3}{a_0^3}\right)$$

$$= \frac{1}{8a_0^3} re^{-r/a_0}\left(2 - \frac{r}{a_0}\right)\left(4 - \frac{6r}{a_0} + \frac{r^2}{a_0^2}\right) = 0$$

The possible values of r are 0, infinity, $2a_0$ and $(3 \pm \sqrt{5})a_0$.

In this case, the maxima are at

$$r = (3 \pm \sqrt{5})a_0$$

What is the probability of finding a n=2, ℓ=1 electron between a_0 and $2a_0$?

The radial wave function for a n=2, ℓ=1 electron is

$$R_{2,1}(r) = \frac{1}{\sqrt{3}(2a_0)^{\frac{3}{2}}} \frac{r}{a_0} e^{-r/2a_0}$$

We then have

$$P(r) = r^2 \left| R_{2,1}(r) \right|^2$$

To find probability, we need to integrate P(r) between a_0 and $2a_0$.

$$P(r) = r^2 \left|R_{2,1}(r)\right|^2 = r^2 \frac{1}{24a_0^3} \frac{r^2}{a_0^2} e^{-r/a_0}$$

Integrate:

$$P(a_0 : 2a_0) = \int_{a_0}^{2a_0} P(r)\, dr = \frac{1}{24a_0^5} \int_{a_0}^{2a_0} r^4 e^{-r/a_0} dr$$

The solution to this integral is equation 7.4 in your reading.

$$\int x^n e^{-cx} dx = -\frac{e^{cx}}{c} \times (x^n + \frac{nx^{n-1}}{c} + \frac{n(n-1)x^{n-2}}{c^2} + \ldots + \frac{n!}{c^n})$$

$$P(a_0 : 2a_0) = \frac{1}{24a_0^5} [-a_0 e^{-r/a_0} (r^4 + 4a_0 r^3 + 12a_0^2 r^2 + 24a_0^3 r = 24a_0^4)]_{a_0}^{2a_0}$$

$$= -\frac{168e^{-2}}{24} + \frac{65e^{-1}}{24}$$

$$\boxed{P(a_0 : 2a_0) = 0.049}$$

Video Lecture:

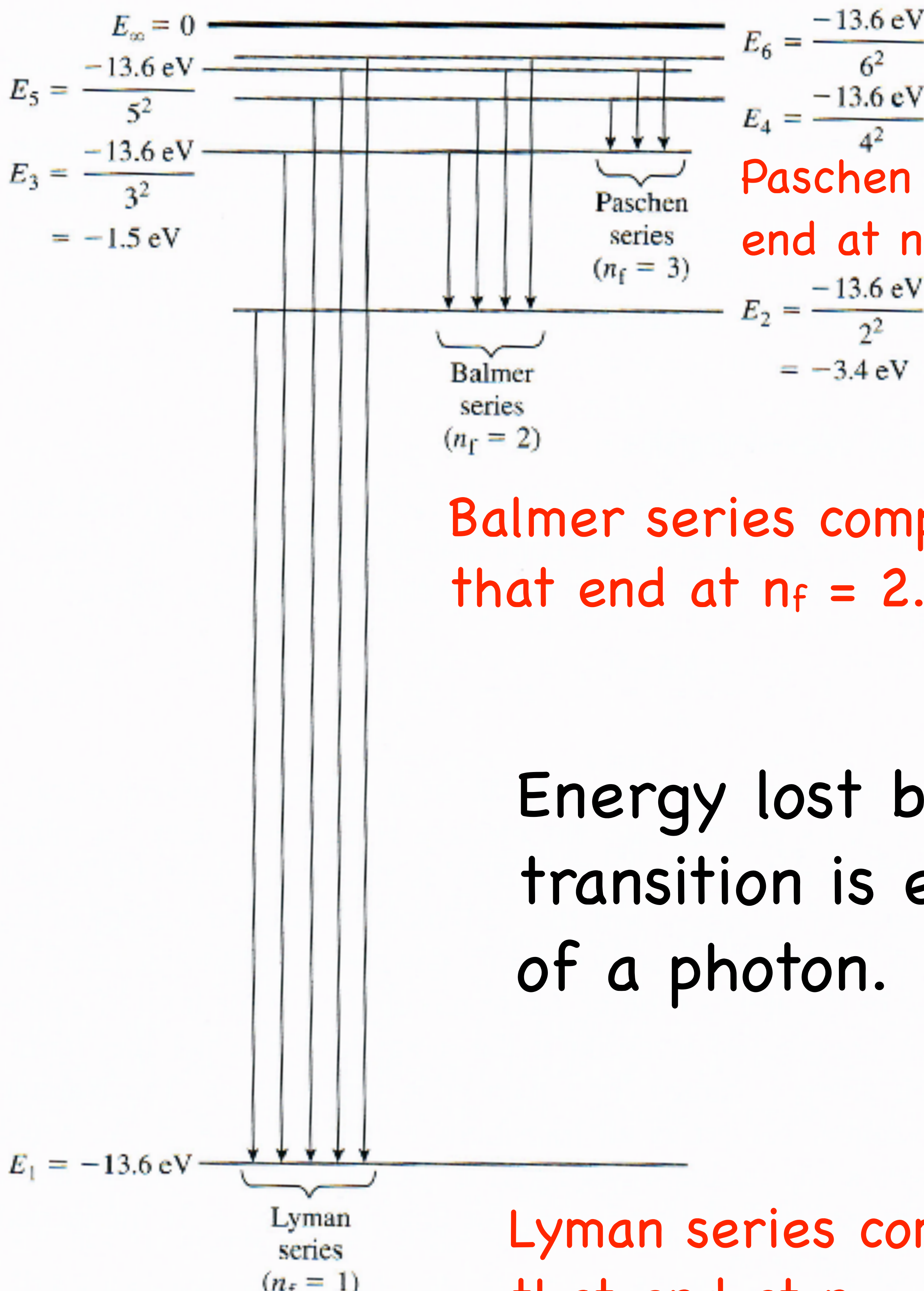
Angular Probability Density

$$P(\theta, \phi) = |\Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)|^2$$

n	l	m_l	$R(r)$	$\Theta(\theta)$	$\Phi(\phi)$
1	0	0	$\frac{2}{a_0^{3/2}}e^{-r/a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	0	0	$\frac{1}{(2a_0)^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	1	0	$\frac{1}{\sqrt{3}(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\sqrt{\frac{3}{2}}\cos\theta$	$\frac{1}{\sqrt{2\pi}}$
2	1	±1	$\frac{1}{\sqrt{3}(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\mp\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	0	0	$\frac{2}{(3a_0)^{3/2}}\left(1-\frac{2r}{3a_0}+\frac{2r^2}{27a_0^2}\right)e^{-r/3a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
3	1	0	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}}\left(\frac{r}{a_0}-\frac{r^2}{6a_0^2}\right)e^{-r/3a_0}$	$\sqrt{\frac{3}{2}}\cos\theta$	$\frac{1}{\sqrt{2\pi}}$
3	1	±1	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}}\left(\frac{r}{a_0}-\frac{r^2}{6a_0^2}\right)e^{-r/3a_0}$	$\mp\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	2	0	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\sqrt{\frac{5}{8}}(3\cos^2\theta-1)$	$\frac{1}{\sqrt{2\pi}}$
3	2	±1	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\mp\sqrt{\frac{15}{4}}\sin\theta\cos\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	2	±2	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm 2i\phi}$

Note: The l = 0 wave functions are spherically symmetric, no dependence on angle.

Video Lecture:



Paschen series comprises all transitions that end at $n_f = 3$.

Balmer series comprises all transitions that end at $n_f = 2$.

Energy lost by the downward transition is emitted in the form of a photon.

Lyman series comprises all transitions that end at $n_f = 1$.

Calculate the series limit of the Lyman series of spectral lines. As a reminder, this is defined as the shortest wavelength possible of a photon emitted in a transition from a higher energy level to the final energy level.

The shortest wavelength corresponds to the largest energy. The Lyman series has an endpoint of n = 1.

$$n = \infty \to 1$$

Thus,

$$E_\infty - E_1 = \frac{-13.6\text{eV}}{\infty^2} - \frac{-13.6\text{eV}}{1^2} = 13.6\text{eV}$$

We know

$$E = \frac{hc}{\lambda} \longrightarrow \lambda = \frac{hc}{E} = \frac{1240 eV \cdot nm}{13.6\ eV} \longrightarrow \boxed{\lambda = 91.2\ nm}$$

An electron is in the n = 3 state of hydrogen. To what states can the electron make transitions on its way to the ground state, and what are the energies of the emitted radiations?

The initial energy state is

$$E_3 = -\frac{13.6}{3^2} = -1.51\ eV$$

An electron can make transitions to any lower energy state with energies corresponding to n = 2 or 1. Eventually it will transition to the ground state.

$$E_2 = -\frac{13.6}{2^2} = -3.40\ eV$$

$$E_1 = -\frac{13.6}{1^2} = -13.6\ eV$$

The transitions and corresponding energies are

Transition	Emitted Energy
3 —> 1	$E_3 - E_1 = -1.51\ eV - (-13.6)\ eV = 12.0\ eV$
3 —> 2	$E_3 - E_2 = -1.51\ eV - (-3.40)\ eV = 1.89\ eV$
2 —> 1	$E_2 - E_1 = -3.40\ eV - (-13.6\ eV) = 10.2\ eV$

The end (for today)