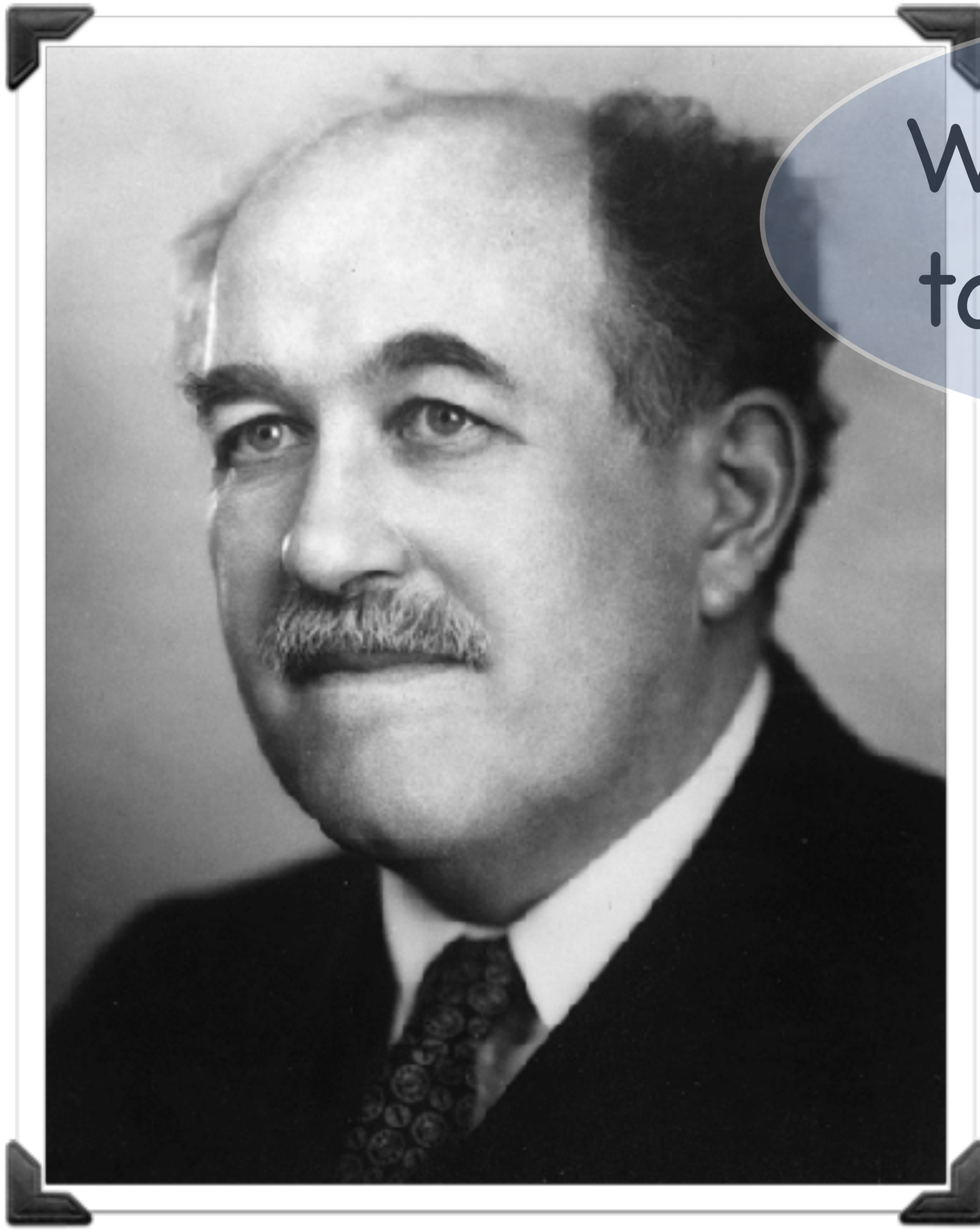
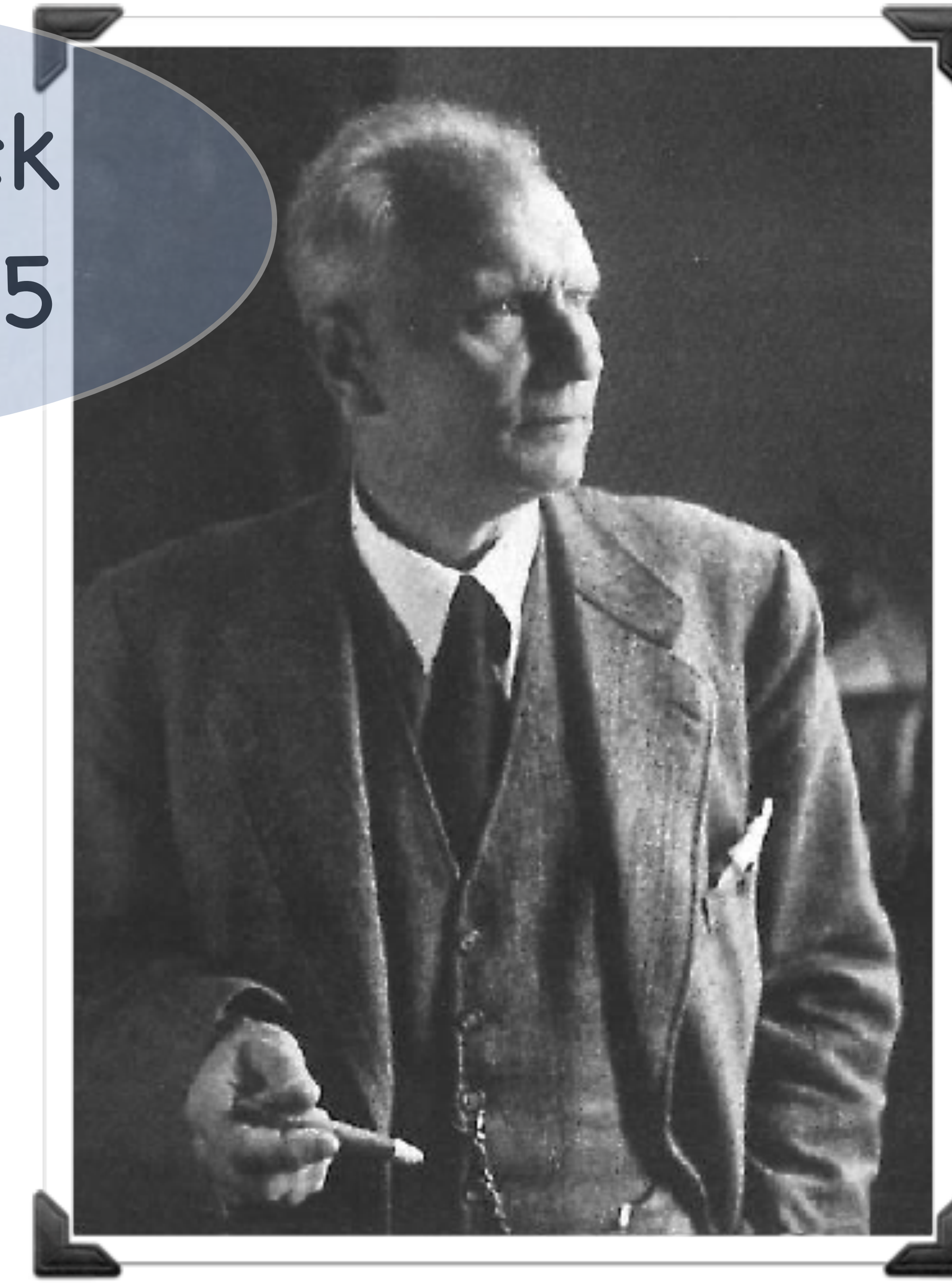
Welcome back to PHYS 3305



Otto Stern
1888 - 1969



Walther Gerlach
1889 - 1979

Today's Lecture:

Angular Momentum Quantization

Stern-Gerlach Experiment

ANNOUNCEMENTS

- Reading Assignment for Nov 14th: Harris 8.2 - 8.5.
- Problem set 13 is due Tuesday, Nov 21st at 12:30 pm.
- Regrade for problem set 12 is due Tuesday, Nov 21st at 12:30 pm.
- Problem sets 14 and 15 will have **no regrade** attempts.
- **Final presentations are due MONDAY, NOVEMBER 203th at 3 pm.** Email your pdf to cooley@physics.smu.edu before due date and time. If your talk is late less than one hour, your maximum grade will drop one increment (i.e. A —> A-). If your talk is late 1 - 2 hours, your maximum grade drops two increments (A —> B+) and so on.

Review Question

What are the quantum numbers n and l for a hydrogen atom with

$$E = -\frac{13.6}{9} eV$$

$$L = \sqrt{2}\hbar$$

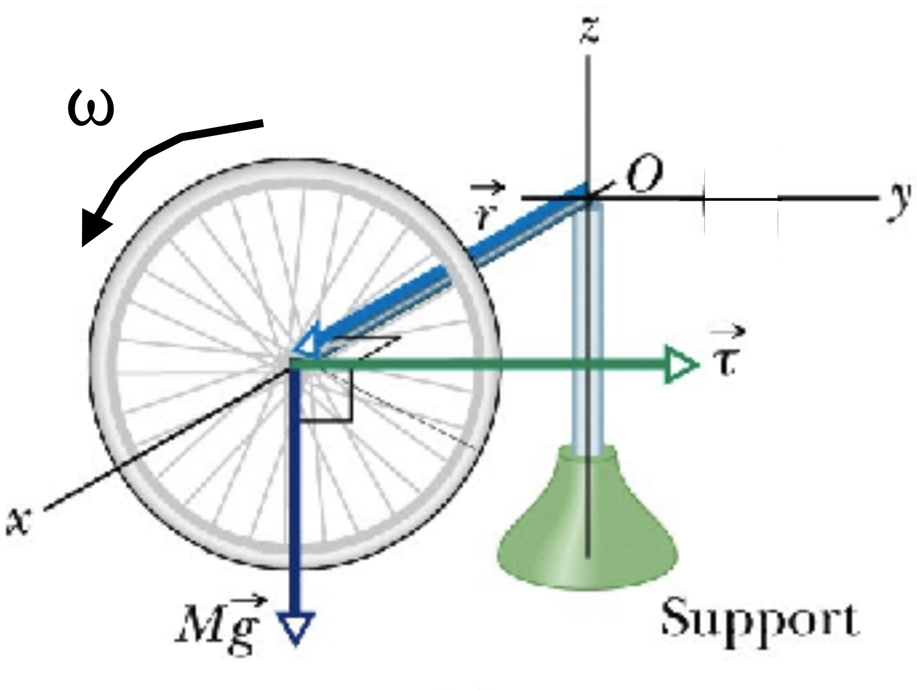
Answer: n = 3, l = 1, m_l = 0, ±1

$$E_n = \frac{13.6}{n^2}$$

$$|\vec{L}| = \sqrt{\ell(\ell+1)}\hbar \qquad (\ell = 0, 1, 2, ...(n-1))$$

$$L_z = m_\ell \hbar \qquad (m_\ell = 0, \pm 1, \pm 2, ... \pm \ell)$$

Demo: Gyroscope



(a)

Initially L is directed to the left (positive x-direction.

The weight of the wheel produces a torque about the origin.

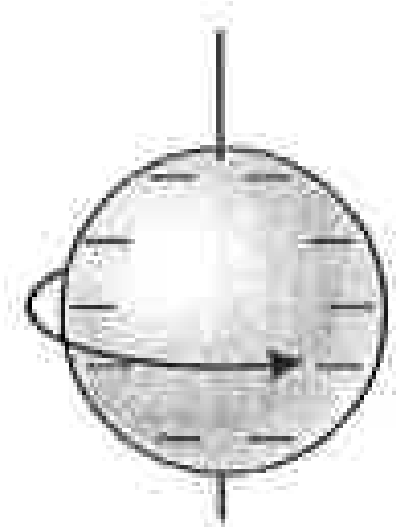
$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times m\vec{g}$$

$$|\vec{\tau}| = mgr \text{ (positive y-direction)}$$

Torque is related to angular momentum: $\vec{\tau} = \frac{d\vec{L}}{dt}$

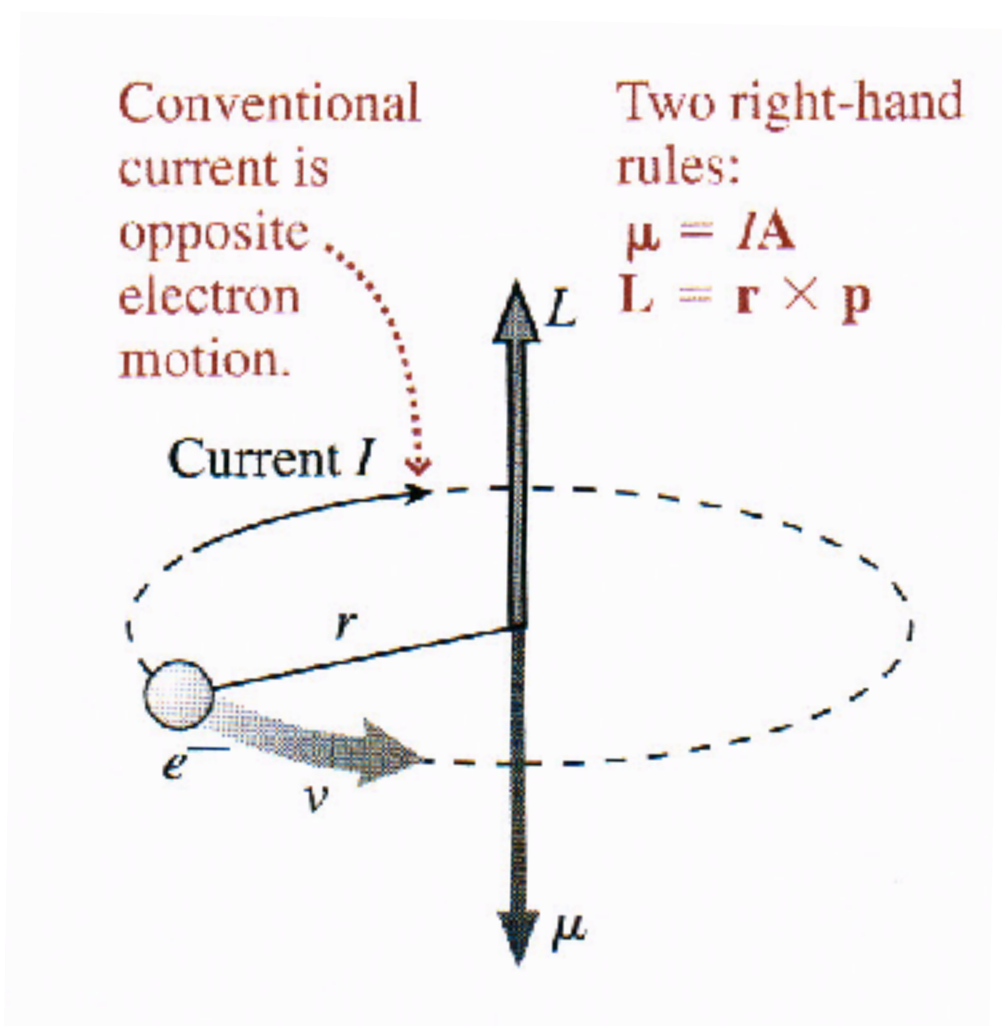
Change is in direction only –> L rotates around z-axis

The figure shows a spinning ball of negative charge. Does the magnetic moment of this spinning charge point up, point down, to the right, or to the left?



From Lecture Video:

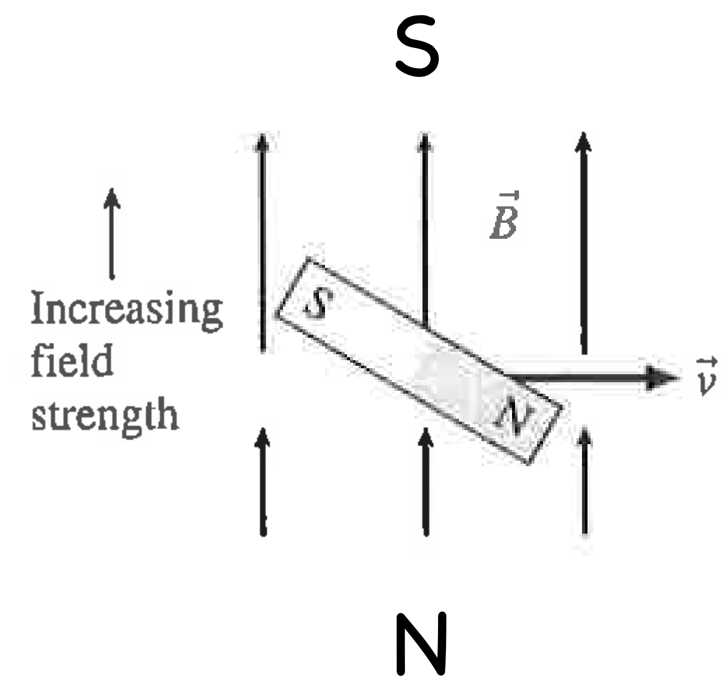
Ans: Points Down



The **magnetic moment** for an electron orbiting CCW is given by

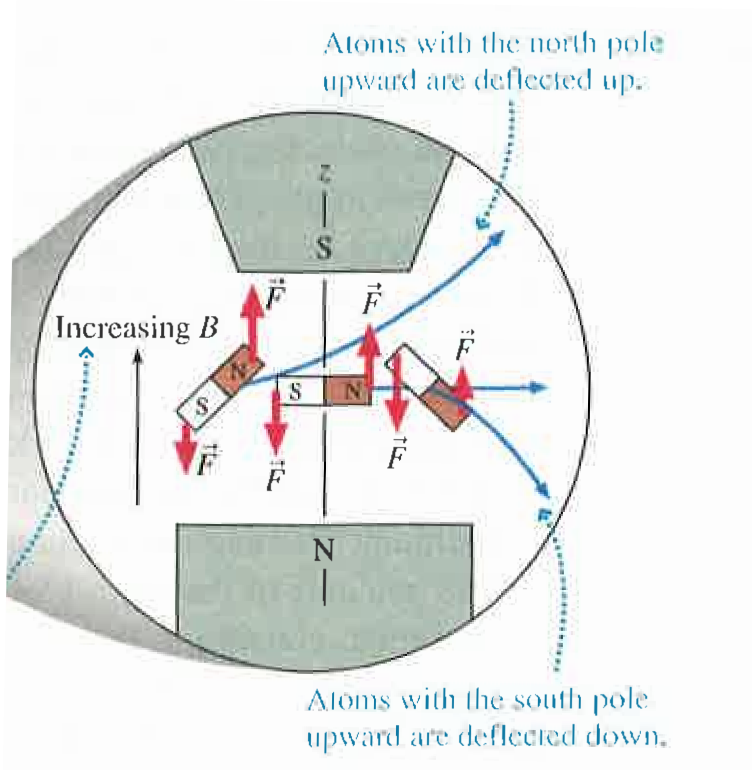
$$\boxed{\vec{\mu_L} = -\frac{e}{2m_e}\vec{L}}$$

A bar magnet is moving to the right though a nonuniform magnetic field. The field is weaker toward the bottom of the page and stronger toward the top.



a) Is there a force on the magnet? If so, in which direction?

Ans: Yes, the magnet will feel a downward force. The South pole of the bar magnet will feel a downward force. This force is greater than the upward force felt by the North pole of the bar magnet.



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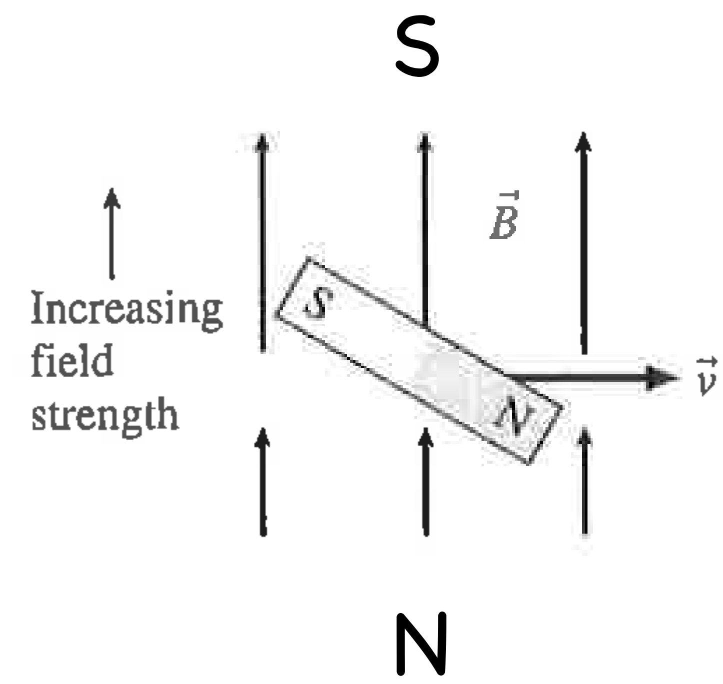
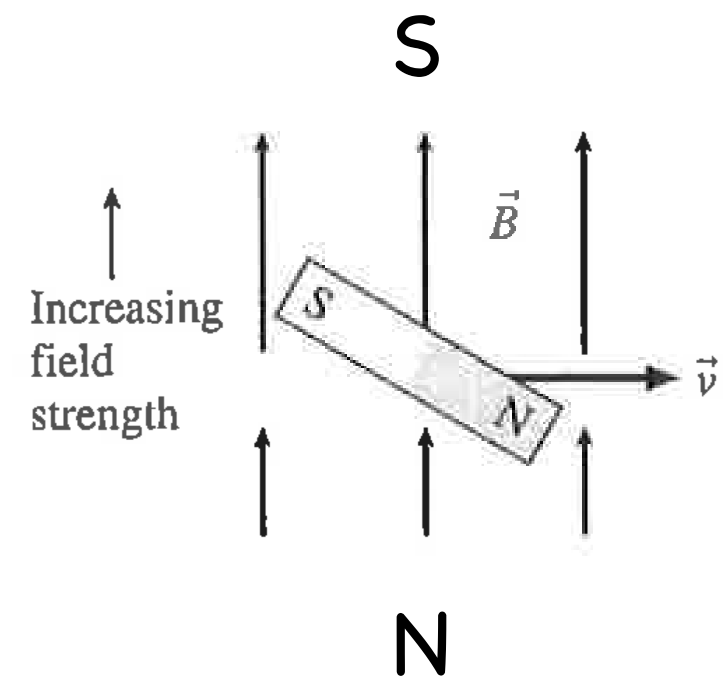


b) Will the magnet be deflected by the field? If so, in which direction?

Ans: Yes, the bar magnet will be deflected downward because the downward force will be greater than the upward force.

A bar magnet is moving to the right though a nonuniform magnetic field. The field is weaker toward the bottom of the page and stronger toward the top.



c) Would the magnet be deflected by a UNIFORM magnetic field? If so, in which direction?

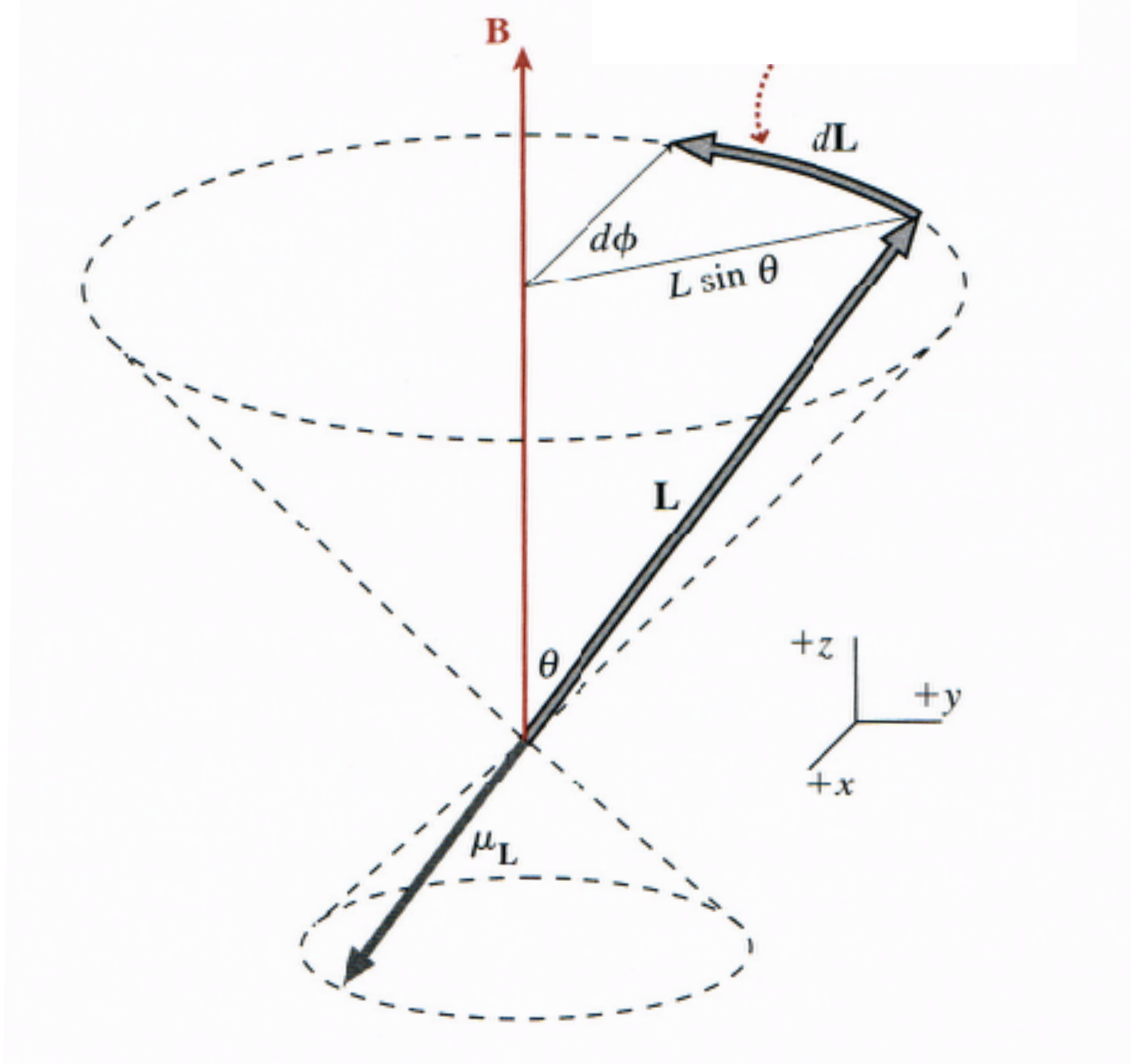
Ans: No, the net force would be 0.

From Lecture Video:

A dipole in a magnetic field also feels a torque: $|\vec{\tau}| = |\vec{\mu_L} \times \vec{B}|$

What direction is the torque?

Counterclockwise



We also know that torque is related to angular frequency by the rotational second law:

$$|\vec{\tau}| = \left|\frac{d\vec{L}}{dt}\right| = \left| -\frac{e}{2m_e}\vec{L} \times \vec{B}\right|$$

$$\boxed{\frac{d\phi}{dt} = \frac{eB}{2m_e}}$$

Larmor frequency

From Lecture Video:

$$\mathbf{F} = -\frac{e}{2m_e} m_\ell \hbar \frac{\partial B_z}{\partial z} \hat{\mathbf{z}} \qquad m_\ell = -\ell, \ldots, +\ell$$

The **magnetic quantum number, m_l,** is an important factor for governing the effect of the magnetic field.

Question: Consider hydrogen in the ground state. What force do we expect and why?

Answer: F = 0. For the ground state, $\ell = 0$, thus, m_ℓ would equal zero.

From Lecture Video:

Stern Gerlach:

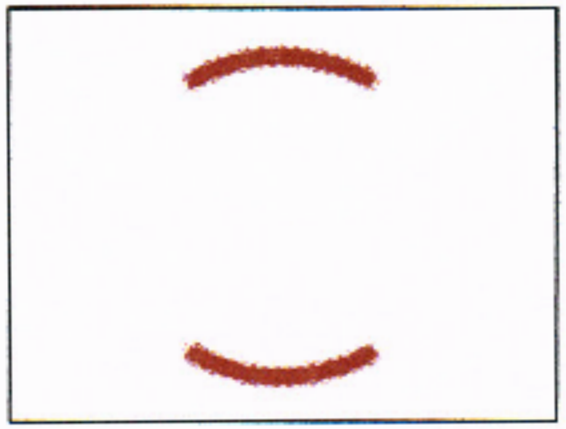
- Take hydrogen atoms in the l = 0 state and pass them through the magnetic field.
- Expect to see 1 line on the screen corresponding to the only possible m_l state (m_l = 0) (no magnetic moment).

What was seen -- 2 lines!

- Take hydrogen atoms in the l = 1 state and pass them through the magnetic field.
- Expect to see 3 lines on the screen corresponding to the 3 possible m_l states.

What was seen -- Only 2 lines!

$\ell = 0$ expectation



$\ell = 0$ observation

From Lecture Video:

Intrinsic Angular Momentum is given the name SPIN, $(\vec{S})$.

$$S = \sqrt{s(s+1)}\hbar$$ (length of spin vector)

Remember: an intrinsic property is one that is fundamental to the particles nature.

The magnitude of a particle's spin vector depends on a dimensionless value s.

TABLE 8.1 Spins of some selected particles

Fermions (Half-integral spin)		**Bosons** (Integral spin)	
Particle	***s***	**Particle**	***s***
Electron, e^-	$\frac{1}{2}$	Pion, π^0	0
Proton, p	$\frac{1}{2}$	Alpha particle, α (helium nucleus)	0
Neutron, n	$\frac{1}{2}$	Photon, γ	1
Neutrino, ν	$\frac{1}{2}$	Deuteron, d (bound n-p)	1
Omega, Ω^-	$\frac{3}{2}$	Graviton	2

s is not a quantum number and can not take on different values!

From Lecture Video:

Similar to orbital angular momentum, the z-component of **intrinsic angular momentum** is quantized.

$$\boxed{\begin{gathered} S_z = m_s \hbar \\ m_s = -s, (-s+1), ...(s-1), s \end{gathered}}$$

Spin is a new Quantum Number bringing the total number to 4. The **spin quantum number** is given by **m_s**.

$$(n, \ell, m_\ell, m_s)$$

Can the spin angular momentum vector lie in the xy-plane? Justify your answer.

Answer: No, m_s = +1/2 or -1/2. Thus, the z-component of the intrinsic angular momentum, S_z, cannot be zero.

The subatomic particle Ω has spin s = 3/2. What angles might its intrinsic angular momentum vector make with the x-axis?

We know

$$S = \sqrt{s(s+1)}\hbar = \sqrt{\frac{3}{2}(\frac{3}{2}+1)\hbar} = \frac{\sqrt{15}}{2}\hbar$$

The S_z components are $\pm\frac{3}{2}\hbar,\ \pm\frac{1}{2}\hbar$

Thus,

$$\theta = \cos^{-1}(\frac{S_z}{S})$$

$$\boxed{\theta = 140.8^o, 105^o, 75^o, 39.2^o}$$

Example: Stern-Gerlach

In a Stern-Gerlach type of experiment, the magnetic field varies with distance in the z direction according to $dB_z/dz = 1.4$ T/mm. Silver atoms travel a distance $x = 3.5$ cm through the magnet. The most probable speed of the atoms emerging from the oven is $v=750$ m/s. Find the separation of the two beams as they leave the magnet. The mass of a silver atom is 1.8×10^{-25} kg, and its magnetic moment is about 1 Bohr magneton.

Hint: Use a combination of what we just learned about Forces and B-fields and what we know of kinematics to solve this problem.

Step 1: Find the force felt by the atom.

The magnetic field is acting only in the z-direction. So, we can use the expression we just derived to relate the magnetic moment and the magnetic field to the force.

$$\mathbf{F} = \mu_z \frac{\partial B_z}{\partial z} \hat{\mathbf{z}}$$

Step 2: Relate force to the acceleration of the atom.

The acceleration of a silver atom of mass m as it passes through the magnet is

$$a = \frac{F_z}{m} = \frac{\mu_z (dB_z/dz)}{m}$$

Step 3: The vertical deflection of the beam can be found from kinematics.

$$a = \frac{F_z}{m} = \frac{\mu_z(dB_z/dz)}{m}$$

$$\Delta z = \frac{1}{2}at^2$$

The time, t, to transverse the magnet is given by

$$t = \frac{x}{v} \longrightarrow \Delta z = \frac{1}{2}a\left(\frac{x}{v}\right)^2$$

The spread in the beam is two times the distance Δz.

$$d = \frac{\mu_z(dB_z/dz)x^2}{mv^2}$$

$$= \frac{(9.27 \times 10^{-24}\ \mathrm{J/T})(1.4 \times 10^{3}\ \mathrm{T/m})(3.5 \times 10^{-2}\ \mathrm{m})^2}{(1.8 \times 10^{-25}\ \mathrm{kg})(750\ \mathrm{m/s})^2}$$

$$\boxed{d = 0.16mm}$$

A particle's intrinsic magnetic dipole moment is related to its intrinsic angular momentum.

$$\boxed{\vec{\mu_s} = g\frac{q}{2m}\vec{S}}$$

q = charge
m = mass
g = gyromagnetic ratio

NOTE: μ_S is related to the intrinsic angular momentum, S
μ_L is related to the orbital angular momentum, L

Values of g depend upon the particle. For an electron the value is close to 2. For a proton it is 5.6.

The predominant effect in the Stern-Gerlach experiment is due to the hydrogen atom's electron. Why?

Although the gryromagnetic ratio is greater for the proton than for the electron, the mass of the proton is much, much larger than the mass of the electron. Thus, it's magnetic moment is quite small compared to that of the electron.

From Lecture Video:

Revisit Wave Functions

Now that we have a 4th quantum number, we need to adjust our wave functions.

$$\psi_{n,\ell,m_\ell,m_s} = \psi_{n,\ell,m_\ell} m_s$$

spacial state

spin state

For an electron (spin 1/2), m_s can take on only two values:

$$\psi_{n,\,\ell,\,m_\ell,\,+\frac{1}{2}} = \psi_{n,\,\ell,\,m_\ell}(r,\theta,\phi)\uparrow \qquad \text{Spin up}$$

$$\psi_{n,\,\ell,\,m_\ell,\,-\frac{1}{2}} = \psi_{n,\,\ell,\,m_\ell}(r,\theta,\phi)\downarrow \qquad \text{Spin down}$$

By including electron spin, what is the degeneracy of the n = 2 energy level of hydrogen?

Answer: 8

Each n, ℓ and m_ℓ value has 2 possible spin states. Without m_s, the degeneracy was given by $(2\ell+1)$ at each level. Thus, our new degeneracy would be $2(2\ell+1)$ at each level.

n	ℓ	m_ℓ	$R(r)$	$\Theta(\theta)$	$\Phi(\phi)$
1	0	0	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	0	0	$\frac{1}{(2a_0)^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
2	1	0	$\frac{1}{\sqrt{3}(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\sqrt{\frac{3}{2}}\cos\theta$	$\frac{1}{\sqrt{2\pi}}$
2	1	±1	$\frac{1}{\sqrt{3}(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$	$\mp\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	0	0	$\frac{2}{(3a_0)^{3/2}}\left(1-\frac{2r}{3a_0}+\frac{2r^2}{27a_0^2}\right)e^{-r/3a_0}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2\pi}}$
3	1	0	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}}\left(\frac{r}{a_0}-\frac{r^2}{6a_0^2}\right)e^{-r/3a_0}$	$\sqrt{\frac{3}{2}}\cos\theta$	$\frac{1}{\sqrt{2\pi}}$
3	1	±1	$\frac{8}{9\sqrt{2}(3a_0)^{3/2}}\left(\frac{r}{a_0}-\frac{r^2}{6a_0^2}\right)e^{-r/3a_0}$	$\mp\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	2	0	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\sqrt{\frac{5}{8}}(3\cos^2\theta-1)$	$\frac{1}{\sqrt{2\pi}}$
3	2	±1	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\mp\sqrt{\frac{15}{4}}\sin\theta\cos\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$
3	2	±2	$\frac{4}{27\sqrt{10}(3a_0)^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{1}{\sqrt{2\pi}}e^{\pm 2i\phi}$

$\ell=0$: $2(0+1) = 2$

$\ell=1$: $2(2+1) = 6$

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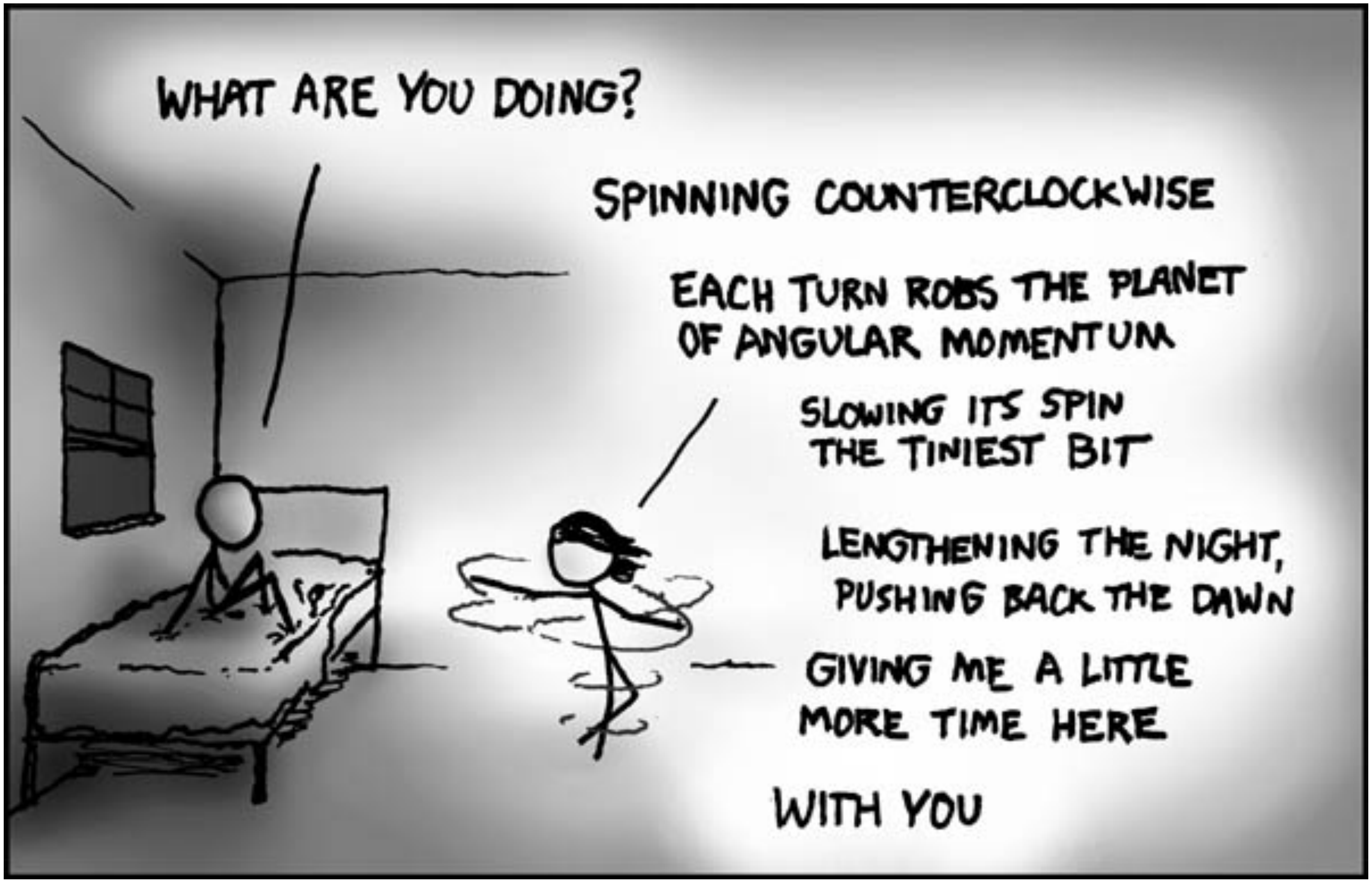
degeneracy = 8

The end
(for today)



xkcd.com