Welcome back to PHY 3305

Today's Lecture:

Identical Particles

Periodic Table

Wolfgang Ernst Pauli
1900 - 1958

ANNOUNCEMENTS

- Problem set 13 is due Tuesday, Nov 21st at 12:30 pm.
- Regrade for problem set 12 is due Tuesday, Nov 21st at 12:30 pm.
- Problem sets 14 will have **no regrade** attempt.
- **Final presentations are due MONDAY, NOVEMBER 20th at 3 pm.** Email your pdf to cooley@physics.smu.edu before due date and time. If your talk is late less than one hour, your maximum grade will drop one increment (i.e. A —> A-). If your talk is late 1 - 2 hours, your maximum grade drops two increments (A —> B+) and so on.
- PhysBowl teams for Thursday, November 16th:
  - Luke, Chris, Andrew
  - Rebecca, Hope, Ali
  - Robert, Connor, Gabriel

Review Question

The magnetic quantum number m_s is most closely associated with what property of the electron in an atom?

a) Magnitude of the orbital angular momentum

b) Energy

c) z component of the spin angular momentum

d) z component of the orbital angular momentum

e) Radius of the orbit

From Lecture Video:

Identical Particles

An identical particle is one for whom all quantum numbers and there mass are the same. The only thing that might distinguish them is their location in space.
The SWE for this situation is

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}\right)\psi(x_1, x_2) + U(x_1, x_2)\psi(x_1, x_2) = E\psi(x_1, x_2)$$

We need a **TWO-PARTICLE WAVE FUNCTION** to describe the system.

$$\psi(x) = \psi(x_1, x_2)$$

With identical particles, the probability density and outcomes can not change if the particles are swapped.

$$|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$$

From Lecture Video:

Requirement: Probability density must be unchanged if the labels of indistinguishable particles are switched.

What will meet our requirement of the particles being indistinguishable?

Adding or subtracting the same function with the labels swapped.

$$\psi_S(x_1, x_2) \equiv \psi_n(x_1)\psi_{n'}(x_2) + \psi_{n'}(x_1)\psi_n(x_2) \qquad \text{Symmetric}$$

$$\psi_A(x_1, x_2) \equiv \psi_n(x_1)\psi_{n'}(x_2) - \psi_{n'}(x_1)\psi_n(x_2) \qquad \text{Antisymmetric}$$

From Lecture Video:

Spin and the total wave function

Wether or not the total wave function of a system of particles is symmetric or asymmetric depends ENTIRELY on the spin of the particles.

Bosons: particles for which s = 0, 1, 2, ... manifest a symmetric multiparticle state.

Fermions: particles for which s = 1/2, 3/2, ... manifest an antisymmetric multiparticle state.

Building blocks - electrons, protons, neutrons are fermions
Particles that transmit forces are bosons.

The Pauli exclusion Principle is obeyed by:

A) all particles

B) all charged particles

C) all Fermions

D) all Bosons

E) only electrons

From Lecture Video:

Pauli Exclusion Principle:

No two indistinguishable fermions may occupy the same individual particle state.

Articulated by Wolfgang Pauli in 1924 and earned him the Nobel Prize in 1945.

Note: The Pauli Exclusion Principle does NOT apply to bosons. Bosons can all occupy the same state.

What is the minimum possible energy for four (noninteracting) spin 1/2 particles of mass m in a one dimensional box of length L?

There may be 2 particles in the n=1 state, and 2 particles in the n=2 state.

$$E_{tot} = 2 \times \frac{(1)^2 \hbar^2 \pi^2}{2mL^2} + 2 \times \frac{(2)^2 \hbar^2 \pi^2}{2mL^2}$$

$$\boxed{E_{tot} = \frac{10 \hbar^2 \pi^2}{2mL^2}}$$

What is the minimum possible energy for four (noninteracting) spin 1/2 particles of mass m in a one dimensional box of length L?

What if the particles where instead spin-1?

In this case the particles are bosons and they would not obey the Pauli Exclusion Principle. Hence, they all would be in the ground state.

$$E_{tot} = 4 \times \frac{(1)^2 \hbar^2 \pi^2}{2mL^2}$$

$$\boxed{E_{tot} = \frac{4\hbar^2 \pi^2}{2mL^2}}$$

What is the minimum possible energy for four (noninteracting) spin 1/2 particles of mass m in a one dimensional box of length L?

What if the particles where instead spin 3/2?

In this case the particles are fermions and they would obey the Pauli Exclusion Principle. However, in this case, the possible spins are -3/2, -1/2, 1/2 and 3/2 (4 in total). Thus, 4 particles could occupy the ground state.

$$E_{tot} = 4 \times \frac{(1)^2 \hbar^2 \pi^2}{2mL^2}$$

$$\boxed{E_{tot} = \frac{4\hbar^2 \pi^2}{2mL^2}}$$

Whether a neutral whole atom behaves as a boson or a fermion is independent of Z, instead depends entirely on the number of neutrons in its nucleus. Why? What is it about this number that determines whether the atom is a boson or fermion?

Ans: There are as many electrons as protons in a neutral atom. Both are fermions, so together they will always add to an even number of fermions. Neutrons are also fermions. Thus, an odd number of neutrons means an odd total number of fermions, and thus fermonic behavior for the unit (known as composite fermion). An even neutron number gives an even total, and boson behavior (or composite boson).

Lithium exists as two isotopes in nature: lithium-6 (3 neutrons) and lithium-7 (4 neutrons).

a) As independent atoms, would these behave as bosons or fermions?

Lithium-6 has an odd number of neutrons in its nucleus and hence an odd number of electrons. It behaves as a fermion. Lithium-7 has an even number of neutrons in its nucleus and hence an even number of electrons. It behaves as a boson.

Lithium exists as two isotopes in nature: lithium-6 (3 neutrons) and lithium-7 (4 neutrons).

a) Might a gas of lithium-6 behave as a gas of bosons?

Yes, a gas of lithium-6 could behave as a gas of boson, provided that the atoms "pair up". Each pair would then have an even number of electrons and hence, behave as a boson.

When a lithium atom is made from a helium atom by adding a proton (and neutron) to the nucleus and an electron outside, the electron goes into an $n = 2$, $l = 0$ state rather than an $n = 1$, $l = 0$ state. This is an indication that electrons:

A) obey the Pauli exclusion principle

B) obey the minimum energy principle

C) obey the Bohr correspondence principle

D) are bosons

E) and protons are interchangeable

From Lecture Video:

The spacial states are distinguished by n, ℓ, m_ℓ.

The set of orbits with a certain value of n are known as an atomic shell.

n	1	2	3	4	5
Shell	K	L	M	N	O

The levels with a certain value of n and ℓ are called subshells (i.e. 2s or 3d).

The Pauli Exclusion Principle tells us that the maximum number of electrons that can be placed in each subshell is

$$2(2\ell + 1) = \text{max electrons in subshell}$$

two different values of m_s

number of different values of m_ℓ for each ℓ

From Lecture Video:

n	l	Subshell	Capacity $2(2l+1)$
1	0	1s	2
2	0	2s	2
2	1	2p	6
3	0	3s	2
3	1	3p	6
4	0	4s	2
3	2	3d	10
4	1	4p	6
5	0	5s	2
4	2	4d	10
5	1	5p	6
6	0	6s	2
4	3	4f	14
5	2	5d	10
6	1	6p	6
7	0	7s	2
5	3	5f	14
6	2	6d	10

From this you can find the capacity of each of the subshells.

$2(2\ell+1)$ = max electrons in subshell

number of different values of m_ℓ for each ℓ

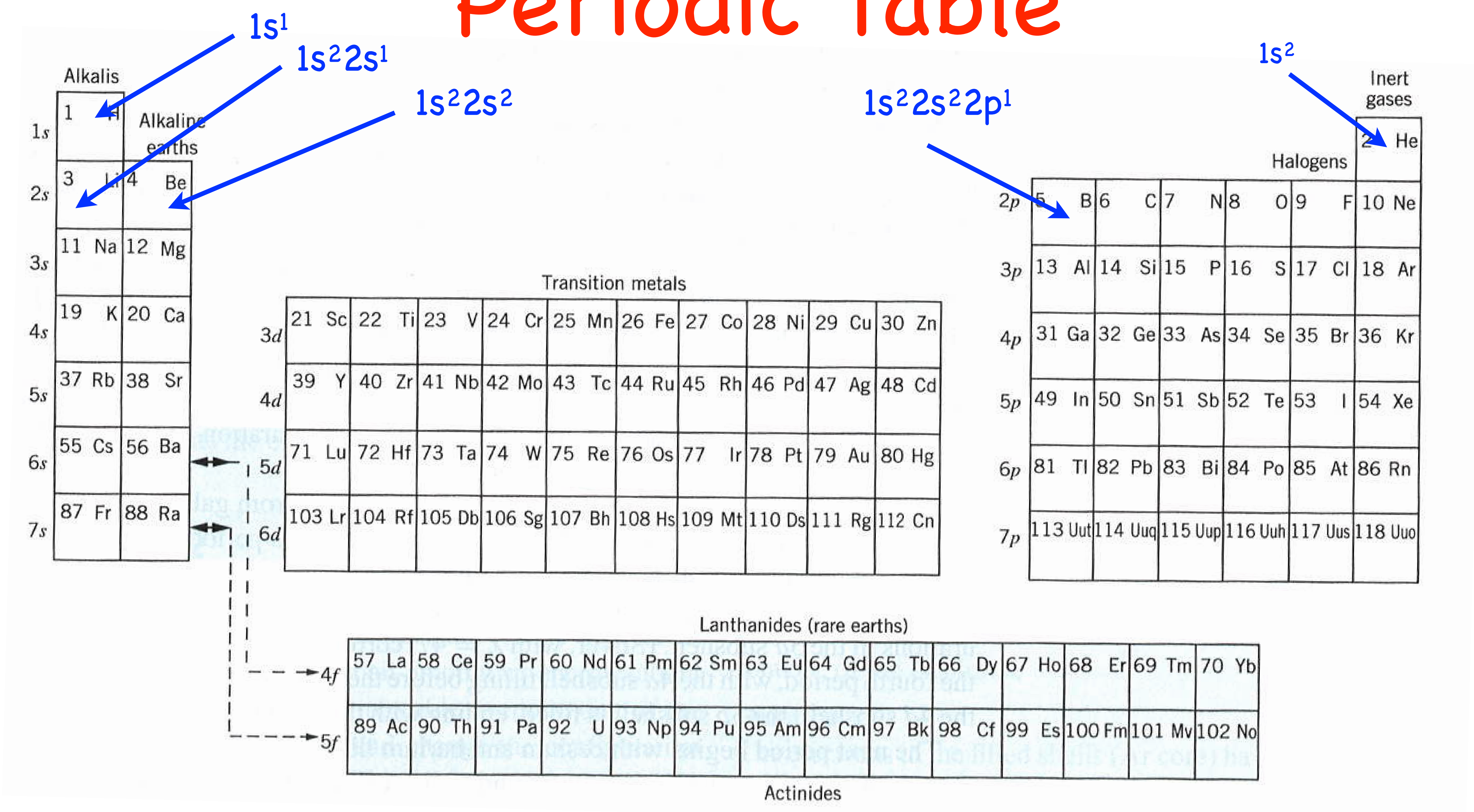
two different values of m_s

The levels are filled using a rule of thumb:

- the lowest value of $n+\ell$,
- the state of n being lower in the case of equal $n+\ell$.

From Lecture Video:

Periodic Table



Terminology: valence electrons are weakly bound, dangling at the periphery of the electron cloud. For Lithium (Z=3), there is one valence electron (2s).

Write the electronic configuration of phosphorus.

$$\boxed{\text{Phosphorus } (Z = 15)\text{: } 1s^2 2s^2 2p^6 3s^2 3p^3}$$

Write the electronic configuration of germanium.

$$\boxed{\text{Germanium } (Z = 32)\text{: } 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^2}$$

Write the electronic configuration of cesium.

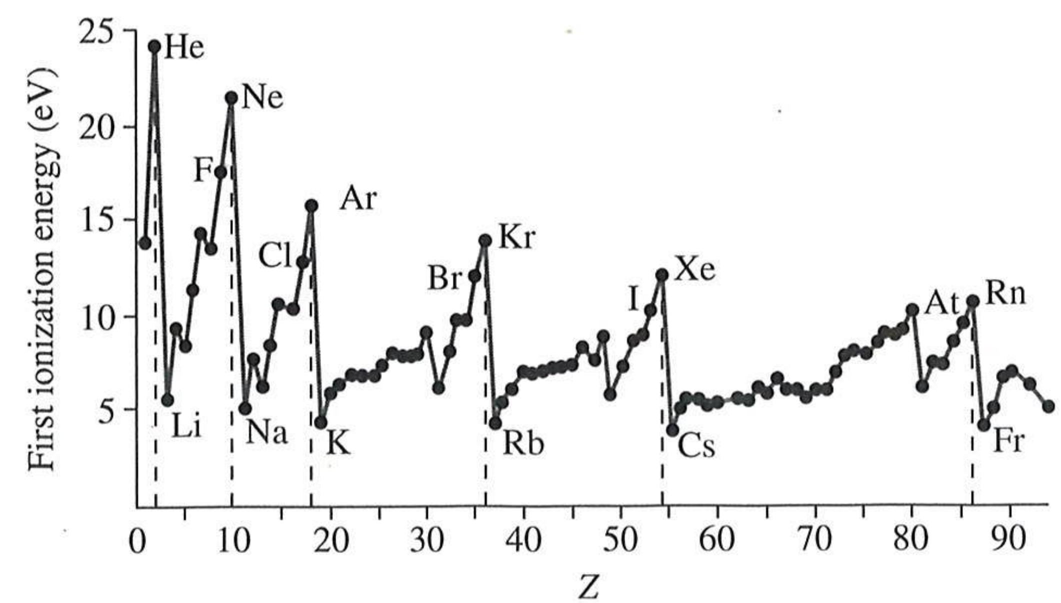
$$\boxed{\text{Cesium } (Z = 55)\text{: } 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6 6s^1}$$

A good electron thief needs a trap at low energy to entice its prey. A poor electron shepherd will have at least some of its flock dangling out at high energy.

Consider rows 2 and 5 in the periodic table. Why should fluorine in row 2 be more reactive than iodine in row 5, while lithium in row 2 is less reactive than rubidium in row 5?

Outer electrons are less tightly bound. I is less tightly bound than F and Rb is less tightly bound than Li.

If the element's role is to "give up" an electron (Li, Rb), it is more reactive if the electron is less tightly bound. If the role is to "seize" an electron (F, I), it is more reactive if it has a deeper hole (more tightly bound) to entice them.



Which of the following statements concerning electromagnetic waves emitted from atoms is true?

A) A collection of atoms emits electromagnetic radiation only at specific wavelengths.

B) Atoms only emit radiation in the visible part of the electromagnetic spectrum.

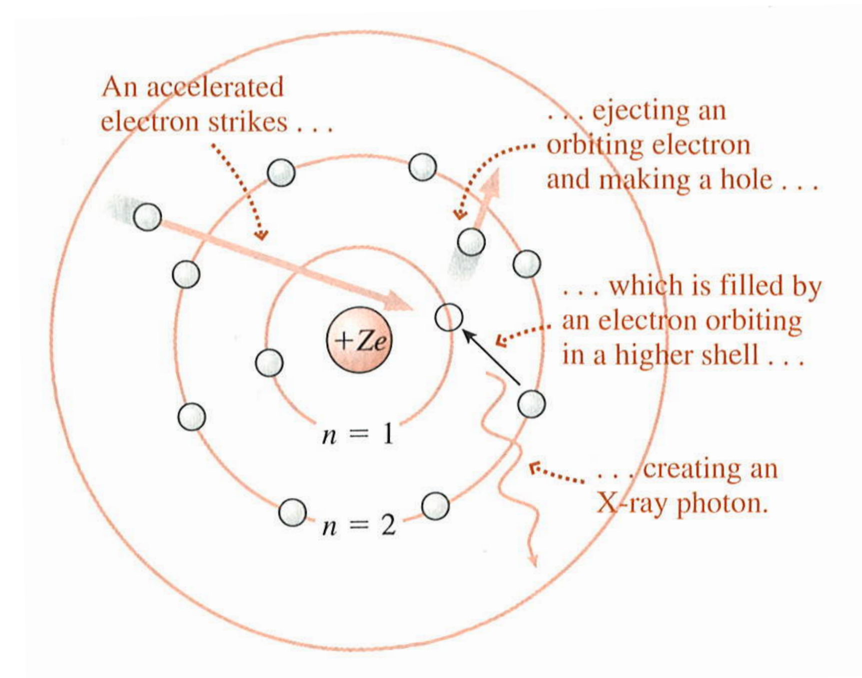
C) Free atoms have 3n unique lines in their atomic spectra, where n is the number of electrons.

D) The wavelengths of electromagnetic radiation emitted by free atoms is specifically characteristic of the particular element.

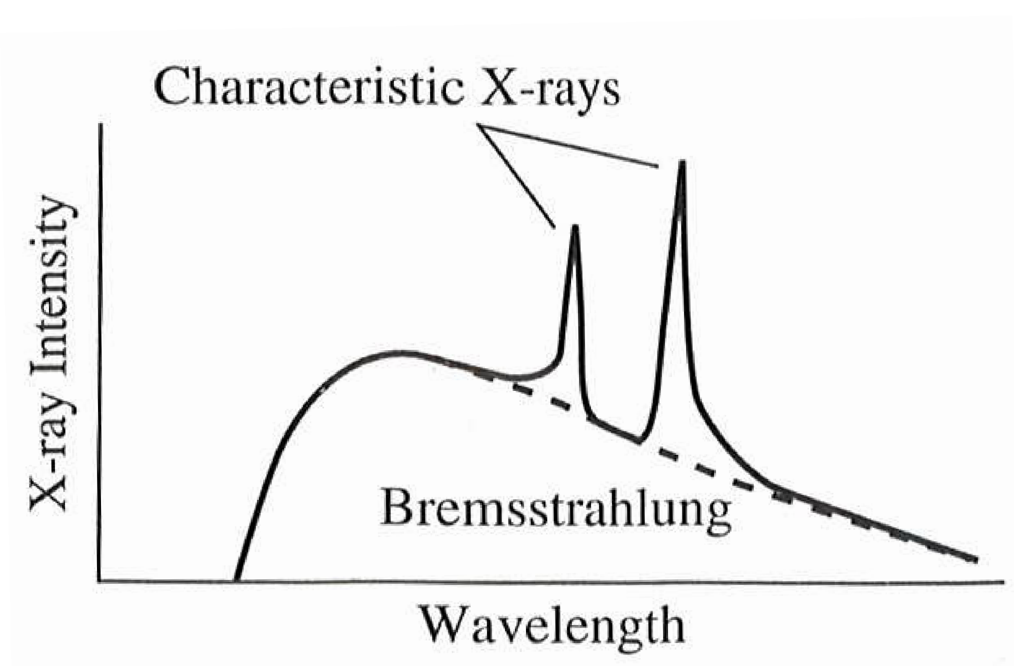
From Lecture Video:

Electrons jump around within the atom when a hole is made in the inner-shell!

Atomic energy levels are quantized --- So, only certain X-rays can be emitted by a given element.



Each element is different, so these x-rays can act as a fingerprint by which we can distinguish different elements.



Notation: X-ray photons produced in the n=1 shell are referred to as K-shell, n=2 are referred to as L-shell, and so on.

Notation:

n=1 —> K-shell
n=2 —> L-shell
n=3 —> M=shell

This notation is also used to indicate the shell at which a transition terminates.

A subscript advancing from α (and going on through the Greek alphabet) designates how many shells higher the transition began.

L_β is from the N-shell (n=4) down to the L-shell (n = 2)

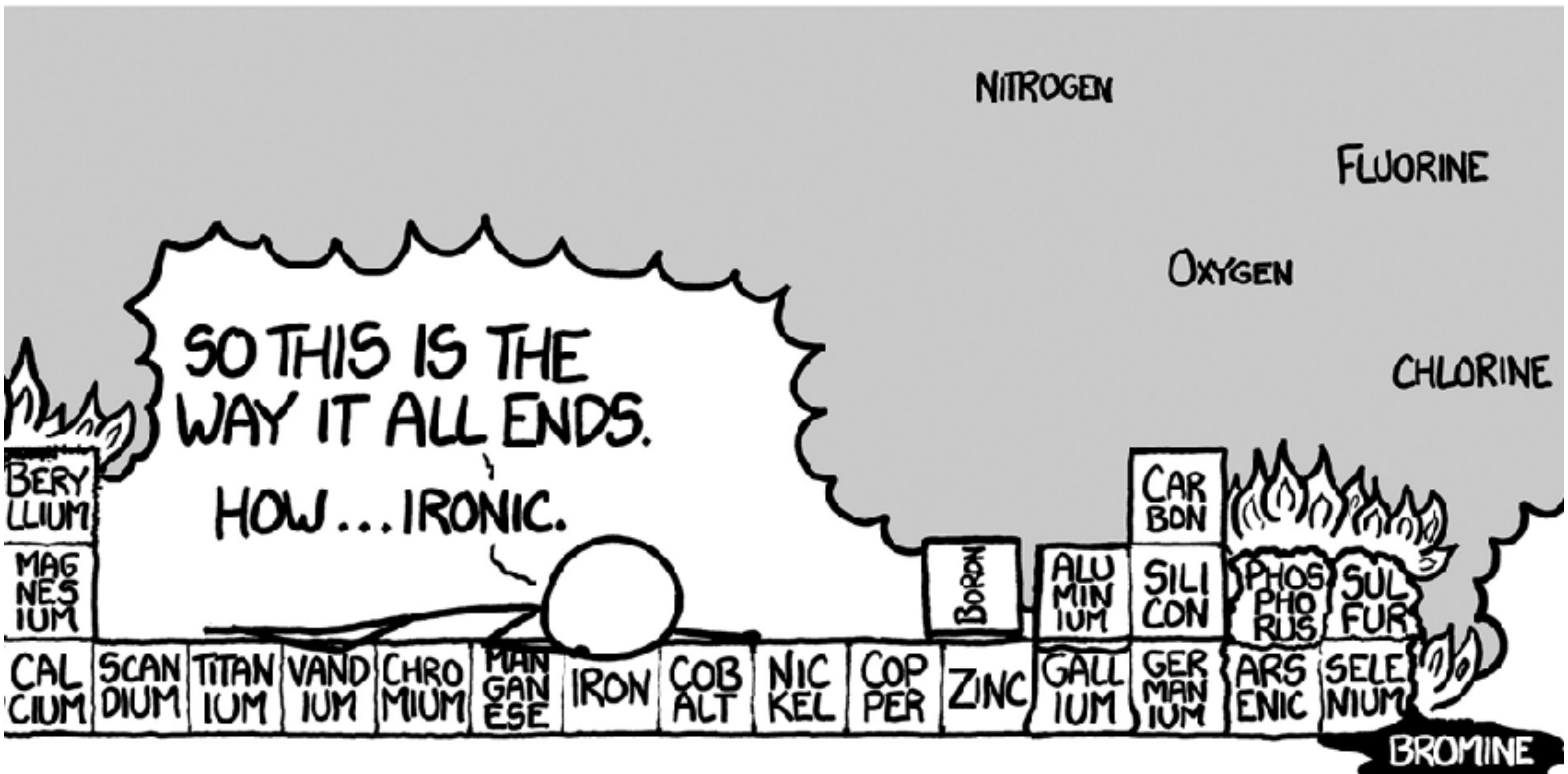
Rank the following lines according to increasing wavelength: K_α, K_β, and L_α.

Ans: K_β, K_α and L_α.

The $K\alpha$ comes from a transition from n=2 to n=1,the K_β from n=3to n=1, and the $L\alpha$ from n=3 to n=2. Energy levels tend to get closer together as n increases, so the 2 to 1 jump is bigger than the 3 to 2. Therefore, the highest energy/shortest wavelength photon is the K_β, next is the $K\alpha$, then the $L\alpha$ is the lowest energy/longest wavelength.

The end (for today)

Ever wonder what happens if you build a periodic table out of bricks, where each brick is made of the corresponding element?



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