

Welcome back to PHY 3305

<u>Today's Lecture:</u> Identical Particles Periodic Table

Wolfgang Ernst Pauli 1900 - 1958

From Last Time:

Intrinsic Angular Momentum is given the name SPIN, $(ec{S})$.

 $S=\sqrt{s(s+1)\hbar}$ (length of spin vector)

Remember: an intrinsic property is one that is fundamental to the particles nature.

The magnitude of a particle's spin vector depends on a dimensionless value s.

Fermions		Bosons		
(Half-integral spin)		(Integral spin)		
Particle	\$	Particle	\$	
Electron, e ⁻	$\frac{1}{2}$	Pion, π^0	0	
Proton, p	$\frac{1}{2}$	Alpha particle, α (helium nucleus)	0	
Neutron, n	$\frac{1}{2}$	Photon, γ	1	
Neutrino, v	$\frac{1}{2}$	Deuteron, d (bound n-p)	1	
Omega, Ω ⁻	$\frac{3}{2}$	Graviton	2	

s is not a quantum number and can not take on different values!

From Last Time:

Similar to <u>orbital angular momentum</u>, the z-component of **intrinsic angular momentum** is quantized.

$$S_z = m_s \hbar$$

 $m_s = -s, (-s+1), ...(s-1), s$

<u>Spin</u> is a new <u>Quantum Number</u> bringing the total number to 4. The spin quantum number is given by m_s .

$$(n, \ell, m_\ell, m_s)$$

A particle's intrinsic magnetic dipole moment is related to its intrinsic angular momentum.

$$\vec{\mu_s} = g \frac{q}{2m} \vec{S}$$

q = charge m = mass g = gyromagnetic ratio

NOTE: μ_{s} is related to the intrinsic angular momentum, S μ_{L} is related to the orbital angular momentum, L

Values of g depend upon the particle. For an electron the value is close to 2. For a proton it is 5.6.

The predominant effect in the Stern-Gerlach experiment is due to the hydrogen atom's electron. Why?

Although the gryromagnetic ratio is greater for the proton than for the electron, the mass of the proton is much, much larger than the mass of the electron. Thus, it's magnetic moment is quite small compared to that of the electron.

Revisit Wave Functions

Now that we have a 4th quantum number, we need to adjust our wave functions.



For an electron (spin 1/2), m_s can take on only two values:

$$\psi_{n,\,\ell,\,m_{\ell},\,+\frac{1}{2}} = \psi_{n,\,\ell,\,m_{\ell}}\left(r,\,\theta,\,\phi\right) \uparrow \qquad \text{Spin up}$$

$$\psi_{n,\,\ell,\,m_{\ell},\,-\frac{1}{2}} = \psi_{n,\,\ell,\,m_{\ell}}\left(r,\,\theta,\,\phi\right) \downarrow \qquad \text{Spin down}$$

Identical Particles

An identical particle is one for whom all quantum numbers and there mass are the same. The only thing that might distinguish them is their location in space.

Could you distinguish them if they occupied the same space? For example – if they were in a box?

Answer: No.

Consider the situation where two identical particles are involved.

The SWE for this situation is

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}\right)\psi(x_1, x_2) + U(x_1, x_2)\psi(x_1, x_2) = E\psi(x_1, x_2)$$

We need a **TWO-PARTICLE WAVE FUNCTION** to describe the system.

$$\psi(x) = \psi(x_1, x_2)$$

With identical particles, the probability density and outcomes can not change if the particles are swapped.

$$|\psi(x_1, x_2)|^2 = |\psi(x_2, x_1)|^2$$

Question: Is the following choice for the two-particle wave function unchanged under the swap of particles 1 and 2?

$$\psi(x_1, x_2) = \psi_n(x_1)\psi_{n'}(x_2)$$

where

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$$

Answer: Not necessarily. Consider what happens if you have an electron in state 1 and another in state 2. If you swap them in space, the probability densities are not necessarily the same anymore.

Language: individual-particle state $\psi_n(x_1)$ multiparticle state $\psi(x_1, x_2)$ Consider the case n = 4 and n' = 3. What will the wave function look like?

$$\psi(x_1, x_2) = \psi_4(x_1)\psi_3(x_2) = \frac{2}{L}\sin\frac{4\pi x_1}{L}\sin\frac{3\pi x_2}{L}$$

The probability density, $P(x_1, x_2)$ is then

$$P(x_1, x_2) = \psi^2(x_1, x_2)$$
$$= \frac{4}{L^2} \sin^2 \frac{4\pi x_1}{L} \sin^2 \frac{3\pi x_2}{L}$$

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 $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$



 $P(\frac{1}{2}L, x_2) = \frac{4}{L^2} \sin^2 \frac{4\pi(\frac{1}{2}L)}{L} \sin^2 \frac{3\pi x_2}{L} = 0$

Their are 4 bumps along x_1 and only 3 bumps along x_2 .

Particles 1 & 2 are identical. We can not claim that particle 2 can be found and particle 1 not found.

Case:
$$x_2 = 1/2$$
 L
 $P(x_1, \frac{1}{2}L) = \frac{4}{L^2} \sin^2 \frac{4\pi x_1}{L} \sin^2 \frac{3\pi (\frac{1}{2}L)}{L} = \frac{4}{L^2} \sin^2 \frac{4\pi x_1}{L} \neq 0$

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Professor Jodi Cooley

Requirement: Probability density must be unchanged if the labels of indistinguishable particles are switched.

What will meet our requirement of the particles being indistinguishable?

Adding or subtracting the same function with the labels swapped.

$$\psi_{S}(x_{1}, x_{2}) \equiv \psi_{n}(x_{1})\psi_{n'}(x_{2}) + \psi_{n'}(x_{1})\psi_{n}(x_{2})$$
Symmetric
$$\psi_{A}(x_{1}, x_{2}) \equiv \psi_{n}(x_{1})\psi_{n'}(x_{2}) - \psi_{n'}(x_{1})\psi_{n}(x_{2})$$
Antisymmetric

Spin and the total wave function

It is not sufficient to consider only space. We must include SPIN as well.

Why is this the case?

Wether or not the total wave function of a system of particles is symmetric or asymmetric depends <u>ENTIRELY</u> on the spin of the particles.

Bosons: particles for which s = 0, 1, 2, ... manifest a symmetric multiparticle state.

Fermions: particles for which s = 1/2, 3/2, ... manifest an antisymmetric multiparticle state.

<u>Building blocks</u> – electrons, protons, neutrons are fermions <u>Particles that transmit forces</u> are bosons. What happens if we have two fermions that occupy the exact same state?

$$\psi_n(1)\psi_n(2) - \psi_n(1)\psi_n(2) = 0$$

Hence, their probability density is zero. Fermions may not occupy the same state.

Pauli Exclusion Principle:

No two indistinguishable fermions may occupy the same individual particle state.

Articulated by Wolfgang Pauli in 1924 and earned him the Nobel Prize in 1945. Note: The Pauli Exclusion Principle does <u>NOT</u> apply to bosons. Bosons can all occupy the same state.

Multielectron Atoms

Brief Review of Chemistry:

A neutral multielectron atom consists for Z electrons surrounding a nucleus with Z protons.

Z = atomic number

The <u>atomic number</u> is the order in which elements are listed in the periodic table.

Z = 1	Hydrogen
Z = 2	Helium
Z = 3	Lithium

Connection to Pauli Exclusion Principle



- No two electrons can fill the same state.
- Thus, the lowest energy state of an atom with Z protons would fill up the Z lowest-energy states.

The spacial states are distinguished by n, l, m_l .

The set of orbits with a certain value of n are known as an atomic shell.



The levels with a certain value of n and l are called subshells (i.e. 2s or 3d).

The Pauli Exclusion Principle tells us that the maximum number of electrons that can be placed in each subshell is



n	1	Subshell	Capacity 2(2 <i>l</i> +1)
1	0	1s	2
2	0	2 <i>s</i>	2
2	1	2 <i>p</i>	6
3	0	35	2
3	1	3 <i>p</i>	6
4	0	45	2
3	2	3 <i>d</i>	10
4	1	4 <i>p</i>	6
5	0	55	2
4	2	4 <i>d</i>	10
5	1	5 <i>p</i>	6
6	0	6 <i>s</i>	2
4	3	4 <i>f</i>	14
5	2	5 <i>d</i>	10
6	1	6 <i>p</i>	6
7	0	7 <i>s</i>	2
5	3	5 <i>f</i>	14
6	2	6 <i>d</i>	10

From this you can find the capacity of each of the subshells.

2(2l +1) = max electrons in subshell number of different values of m_l for each l two different values of m_s

The levels are filled using a rule of thumb:

- the lowest value of n+l,
- the state of n being lower

in the case of equal n+l.

Periodic Table



Terminology: valence electrons are weakly bound, dangling at the periphery of the electron cloud. For Lithium (Z=3), there is one valence electron (2s).

Valence electrons are directly related to the chemical behavior of elements. The ordering of the filling of the shells produces the periodic behavior of the chemical behavior.



Sodium has valence +1, chemically active, gives electron.

Fluorine has valence -1, chemically active, captures electron.

Neon has valence 0, chemically inactive.

What is first ionization energy?

energy needed to remove the first electron from the atom

We can also observe periodicity in the first ionization energies.



Notice that maxima occur for the noble gases in which the electrons are tightly bound. Similarly, minima occur for Li, Na, ... where the energy required to remove a valence electron drops.

Caveat: The Periodic Table

The periodic table was published by Dmetri Mendeleev in 1869.

Quantum Mechanics was first formulated in the early 1900s.

Quantum Mechanics is not the motivation for the periodic table.



Characteristic X-rays

In ground state atoms, electrons completely fill the lowest-energy states up to a maximum energy. Excited valence electrons can jump around among higher unfilled energy levels. What is the typical energy range in the EM spectrum of the photons produced?



In chapter 3 we learned that smashing atoms into a target produced a continuous spectrum of X-rays via bremsstrahlung.

Something else also happens:



Electrons jump around within the atom when a hole is made in the innershell! Any speculation of what the spectrum of x-rays might look like? (Is it continuous, discontinuous, something in-between?)

Atomic energy levels are quantized --- So, only certain X-rays can be emitted by a given element.

Each element is different, so these x-rays can act as a fingerprint by which we can distinguish different elements.



Notation: X-ray photons produced in the n=1 shell are referred to as K-shell, n=2 are referred to as L-shell, and so on.

Characteristic X-rays are used in many ways.

- Those of a known target material are used in crystallography (the study of the arrangement of atoms in crystals).
- The intensity and value of X-rays coaxed from samples of unknown composition can be matched to known values to determine the composition of the sample.
- Protons and helium nuclei are also commonly used in cases where a lower bremsstrahlung background is needed. This technique is known as PIXE (particleinduced X-ray emission).

The end (for today)