LIFE OF A MUON

Assume that a muon is created in the atmosphere 3 km above Earth's surface, traveling downward at 0.98c. It survives 2.2 μs in its own reference frame before decaying.

Define the S reference frame as the surface of Earth with the x-direction along the path of the muon. The S′ reference frame is that of the muon.

This means that the muon is moving at +v as seen from Earth and Earth is moving at -v as seen from the muon.

Classical Physics:

1. How far will the muon travel before it decays?

$$d = vt \; = 0.98(3.00 \times 10^8 m/s)(2.2 \times 10^{-6} s)$$

$$d = 647m$$

The muon would decay before reaching Earth's surface.

2. How much longer would it have to live in order to reach Earth?

$$t = \frac{d}{v} \; = \frac{3000m - 647m}{0.98(3.00 \times 10^8)m/s}$$

$$t = 8.0 \times 10^{-6} s = 8.0 \mu s$$

Special Relativity:

$$t = \gamma_\nu(\frac{v}{c^2}x' + t')$$

1. According to an observer on Earth, how long will the muon survive before decaying?

We can approach the problem in 2 ways - Lorentz transformation equations or time dilation formula (which is derived from Lorentz transformations).

= 0

$$\Delta t = \gamma_\nu(\frac{v}{c^2}\Delta x' + \Delta t')$$

In the muon's frame, it is standing still. $\Delta x' = 0$.

$$\Delta t = \gamma_\nu \Delta t'$$

Note: Proper time is assigned to the S' reference frame where events occur in the same location. This agrees with our formula here. $\Delta t = \gamma_\nu \Delta t_0$

$$\Delta t = \gamma_\nu \Delta t'$$

Now just calculate. Start with the Lorentz factor.

$$\gamma_\nu = \frac{1}{\sqrt{1 - \frac{(0.98\cancel{c})^2}{\cancel{c^2}}}} = 5.03$$

Then, apply to our case.

$$\Delta t = \gamma_\nu \Delta t' = 5.03 \times (2.2 \times 10^{-6}\ \mathrm{s})$$

$$\Delta t = 11 \mu s$$

2. Will the muon reach the surface?

$$d = vt = 0.98(3.00 \times 10^8 \frac{m}{s}) \times (11 \times 10^{-6} s) = 3.2 km$$

Yes, the muon will reach the surface.

Special Relativity:

$$x = \gamma_\nu (x' + vt')$$

3. Now, let's ask ourselves how long it takes to reach Earth from the muon's perspective

Use the Lorentz transformation equations to find Δt'.

$$\Delta x = \gamma_\nu (\Delta x' + v\Delta t')$$

= 0

In the muon's frame, it sees itself standing still. Thus, Δx' = 0.

$$\Delta t' = \frac{\Delta x}{v\gamma_\nu} = \frac{3000m}{0.98 \times (3.00 \times 10^8 \frac{m}{s}) \times 5.03}$$

$$\Delta t' = 2.0 \times 10^{-6} s = 2.0 \mu s$$

Thus, from the perspective of the muon, it reaches Earth, penetrates Earth, and decays 0.2 μs later.

3. Suppose that we observe 10^4 muons at an altitude of 3000 m in some time interval. How many muons would we observe at sea level **from a classical perspective**?

Muons decay according to the statistical law of radioactivity.

$$N(t) = N_0 e^{(-t/\tau)}$$

N_0 = original number of muons
N(t) = number of muons at time t
τ = mean lifetime of the muon (a proper time interval)

The time it takes muon at an altitude of 3000 m which are traveling 0.98c to reach sea level is

$$t = \frac{L}{v} = \frac{3000}{0.98 \times 3 \times 10^8} \sim 10\mu s$$

The mean lifetime for muons is 2.2 μs. Thus, it take approximately 4.5 lifetimes to reach sea level and the number of particles to reach sea level is

$$N = N_0 e^{-t/\tau} = 10^4 e^{-4.5} = 111$$

$$\boxed{N = 111}$$

4. Suppose that we observe 10^4 muons at an altitude of 3000 m in some time interval. How many muons would we observe at sea level **taking into account special relativity**?

Due to length contraction, the distance is contracted to a length of

$$L = \frac{L_0}{\gamma_\nu} = \frac{3000}{5.03} = 600\ m$$

Thus, the actual time it takes muon to reach sea level is

$$t = \frac{L}{v} = \frac{600\ m}{0.98 \times 10^8\ m/s} \approx 2\ \mu s$$

Plugging into our formula for the number at sea level yields

$$N = N_0 e^{-t/\tau} = 10^4 e^{-2.0/2.2} \sim 3 \times 10^4$$

$N = 3 \times 10^4$ closer to real measurements