Principles of Astrophysics and Cosmology
Problem Set 4

JC-10) Chapter 3, problem 2 from your textbook.

JC-11) One idea proposed in the 19th century as the source of the sun's energy is gravitational contraction. If a massive body gets smaller, it releases gravitational potential energy, in this case in the form of heat.

a) Show that the gravitational potential energy of a star can be given by

$$U_g = -4\pi G \int_0^R M_r \rho r dr$$

where M_r is a spherically symmetric mass, ρ is the mass density of a shell.

b) An exact solution of the above equation requires knowledge of how ρ and consequently M_r depend on r. However, an approximate value can be obtained assuming that ρ is constant and equal to its average value. Using this information find an expression for ρ in terms of the total mass of the star, M, and the star radius, R.

c) Using the information from part b) find an expression for M_r in terms of r and $\bar{\rho}$

d) Apply the viral theorem to show that the total mechanical energy of the star is approximately

$$E \sim -\frac{3}{10}\frac{GM^2}{R}$$

e) Assuming the sun's original radius was much larger than it is today, $R_i << R_{sun}$, then how much energy was radiated away during the collapse to the radius we know it to be today?

f) Assume that the luminosity of the Sun has been roughly constant throughout its lifetime. Using this assumption, calculate how long the sun could radiate energy. (This is known as the Kelvin-Helmholz timescale).

g) Based on radioactive dating techniques, the estimated age of the rocks on the Moon's surface is over 4 x 10^9 years. Given this information, comment on the answer you found in part f).