Principles of Astrophysics and Cosmology

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April 23, 1858 - October 4, 1947

Announcements

- Reading Assignments: Chapter 2 all.
- Problem Set 2 is due Wednesday, February 4th, 2015.
- Next lab is Monday, February 9th. Be sure to report to FOSC 032 that day.
- Note on Lab reports.

Last Time:

- We discussed different ways we could get information from astronomical observations.
  - Human eye, CCDs, telescopes
- Discussed why angular resolution is important.
- Started discussing imaging methods that take advantage of the EM wave.

Last Time:

Consider an EM wave that is plane-parallel and monochromatic.

$$\mathbf{E} = \hat{\mathbf{e}} E(t) \cos(2\pi\nu t - \mathbf{k}\cdot\mathbf{r} + \phi)$$

$\hat{\mathbf{e}}$ = direction of polarization of the e-field

$E(t)$ = time-dependent amplitude of field

ν = frequency

$\mathbf{k}$ = wave vector (direction of wave propagation)

ϕ = phase shift

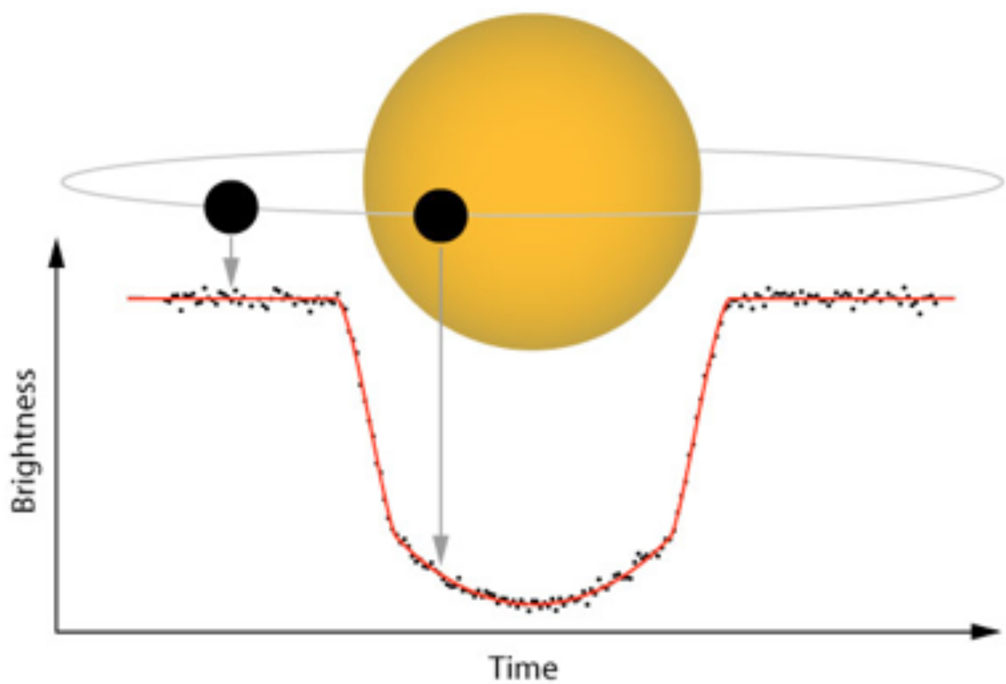
Recall Relations:

$$|\vec{k}| = \frac{2\pi}{\lambda} \qquad \text{and} \qquad \nu = \frac{\lambda}{c}$$

Last Time:

Photometry = measuring the photon flux from a source.

Time-Resolved Photometry = repeated photometric measurements as a function of time. This gives long-term time dependence of $<E^2>$



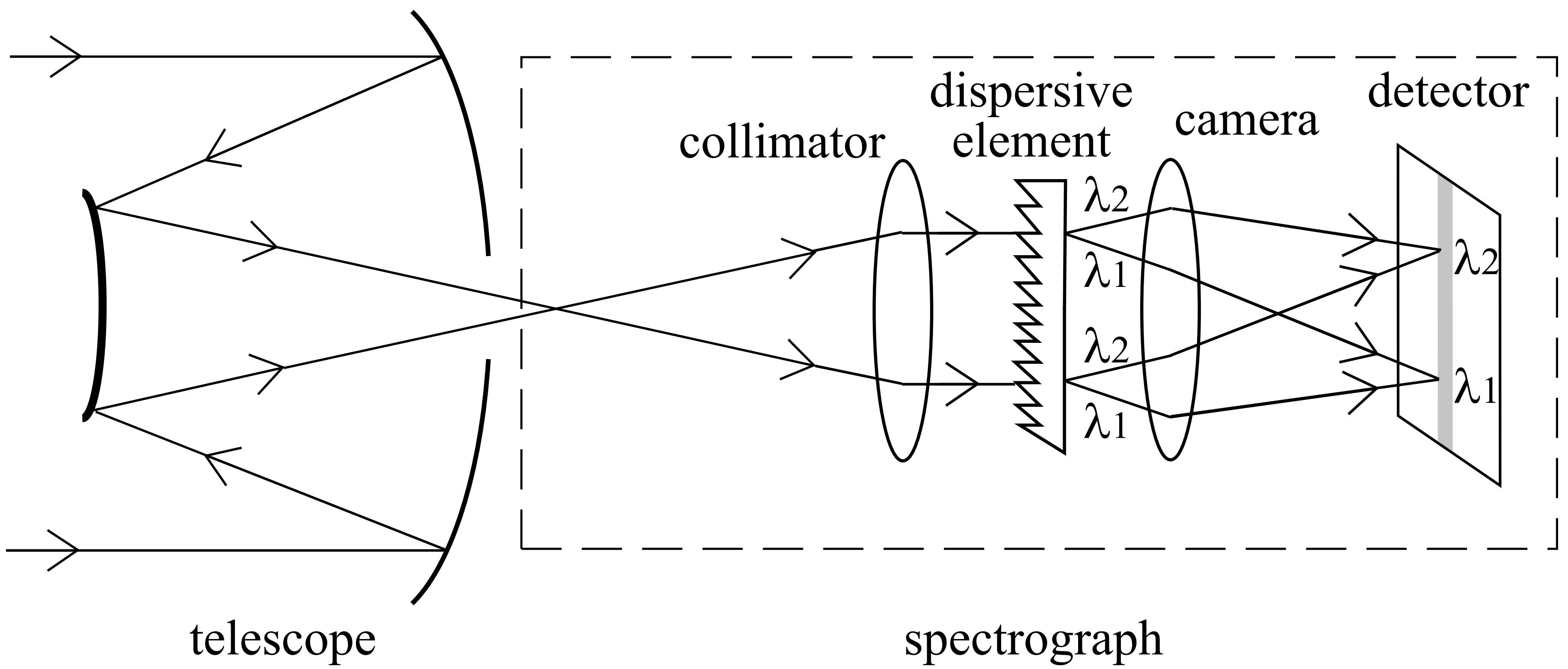
combined with inverse square law, determine luminosity if distance known (or vis versa)
study of light variation in variable stars, minor planets, AGN, supernova and transient exoplanets.

Last Time:

Wavelength and Frequency —

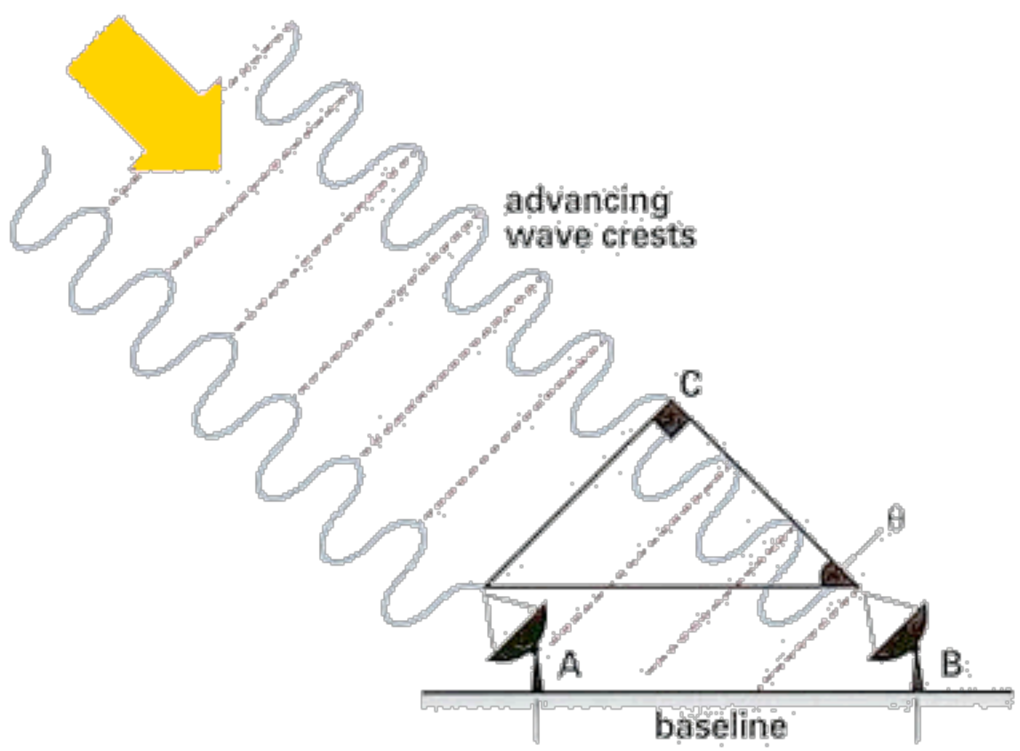
- Use a band-pass filter before detector allows radiation of only a certain frequency to pass.
- Reflection off or transmission through a dispersing element (think diffraction grating or prism)

Spectroscopy -



Phase Shift of light —

- Can reveal information on the precise direction of the source and on effects the wave underwent during its path from source to detector.
- Phase is measured by combining EM waves received from same source using several different telescopes and forming an interference pattern. (Interferometry)
- Baseline distance "B" between the two most widely spread telescopes determines angular resolution (λ/B).

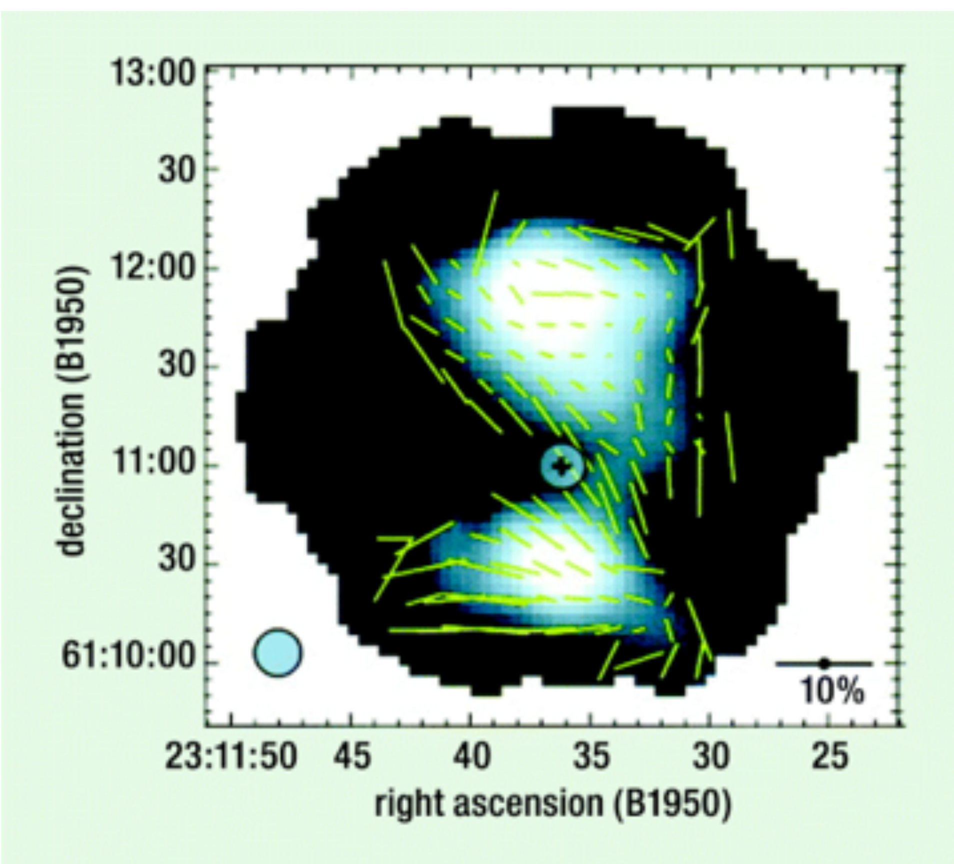


Very useful in radio astronomy where signals from radio telescopes around the globe and in space can provide baselines of 10^4 km.

http://www.hardhack.org.au/book/export/html/18

Polarization —

- Fraction of light that is polarized
- Type of polarization (linear, circular)
- Orientation of the polarization on the sky.
- **Polarimetry** is the measurement of polarization properties of a source.
- Can be done using filters that let only a particular polarization component to reach the detector.



Polarization vectors overlaid on an emission map of NGC 7538 (a star forming region). The length of each line is proportional to the polarization degree.

Interstellar dust: Short axis of dust grains align with the local magnetic field (unlike a compass).

CGS Units

<u>Gaussian Units</u> (also called cgs: centimeter-gram-second) are quite commonly used in astrophysics.

Name	Gaussian units	SI units
Lorentz force	$\mathbf{F} = q\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right)$	$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$
Coulomb's law	$\mathbf{F} = \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$	$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$
Electric field of stationary point charge	$\mathbf{E} = \frac{q}{r^2}\hat{\mathbf{r}}$	$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$
Biot–Savart law	$\mathbf{B} = \frac{1}{c}\oint \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$ [6]	$\mathbf{B} = \frac{\mu_0}{4\pi}\oint \frac{I d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$

Blackbody Radiation

To a rough approximation, stars shine with the spectrum of a blackbody. So, what is a **blackbody**?

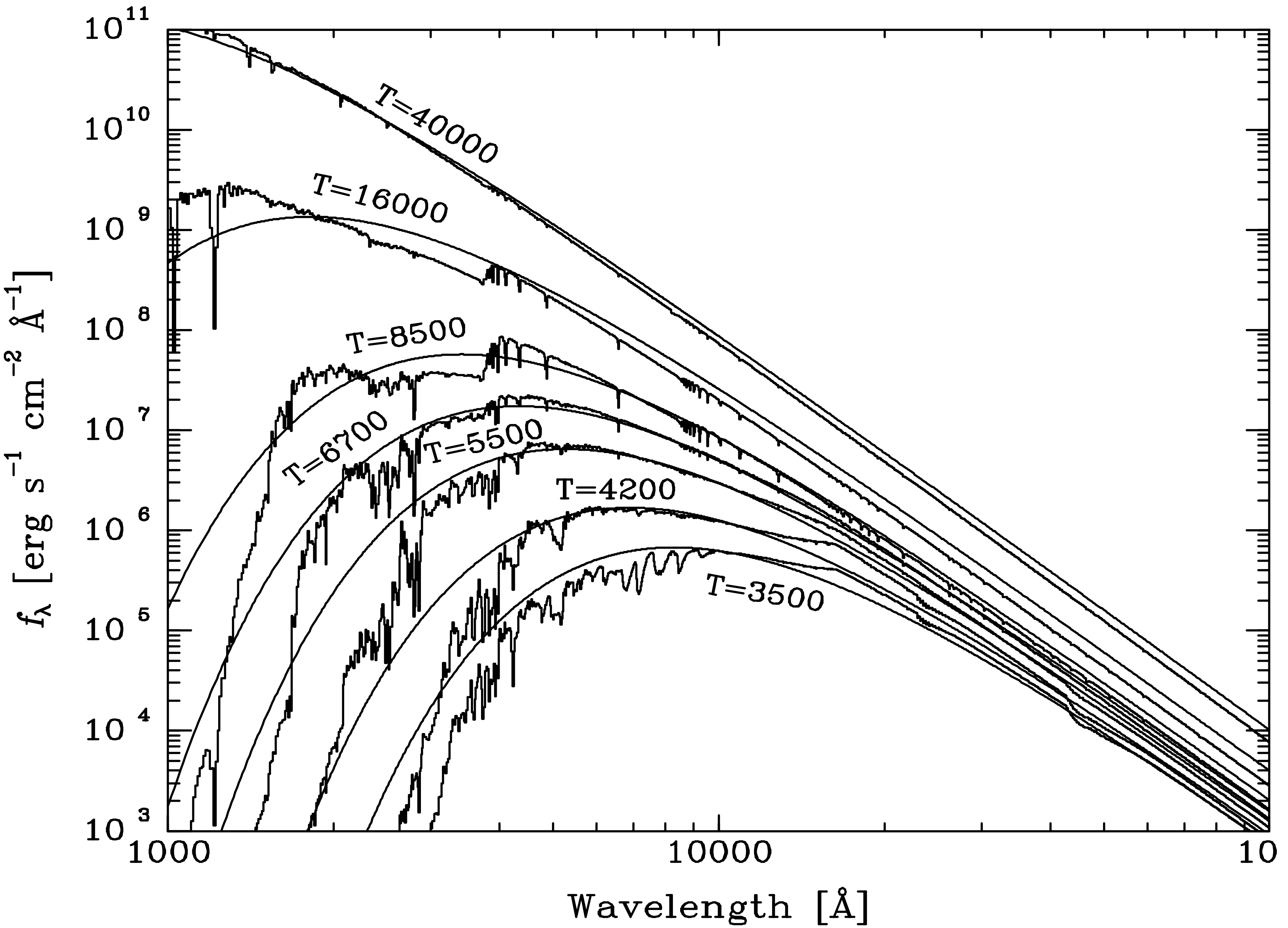
A blackbody is an object that absorbs all incident electromagnetic radiation regardless of frequency or angle of incidence. It is a body that is opaque and non-reflective.

What is **blackbody radiation**?

A blackbody in thermal equilibrium emits electromagnetic radiation that is called blackbody radiation.

Let's review properties of blackbody radiation!

Why Do We Care?



What is the unit [Å]? answer: angstrom = 10^{-8}

Energy Density

The **energy density** of blackbody radiation, per unit frequency interval

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}$$

ν = frequency
h = Planck's constant = 6.6×10^{-27} erg · s
k = Boltzmann's constant = 1.4×10^{-16} erg · K^{-1}
T = temperature (K)

What are the cgs units for energy density?

erg · cm^{-3} · Hz^{-1}

Intensity

Intensity is the net flow of blackbody energy radiation in a particular direction inside the radiator. It is given by

$$I_\nu = c\frac{du_\nu}{d\Omega} \quad = c\frac{u_\nu}{4\pi}$$

What is $d\Omega$?

$$d\Omega = \sin\theta d\theta d\phi$$

in spherical coordinates

Blackbody radiation is isotropic. So what is $d\Omega$?

$$d\Omega = 4\pi$$

Thus, putting it all together:

$$I_\nu = c\frac{du_\nu}{d\Omega} = c\frac{u_\nu}{4\pi}$$

and

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}$$

Gives the intensity of blackbody radiation:

$$\boxed{I_\nu = \frac{c}{4\pi}u_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \equiv B_\nu.}$$

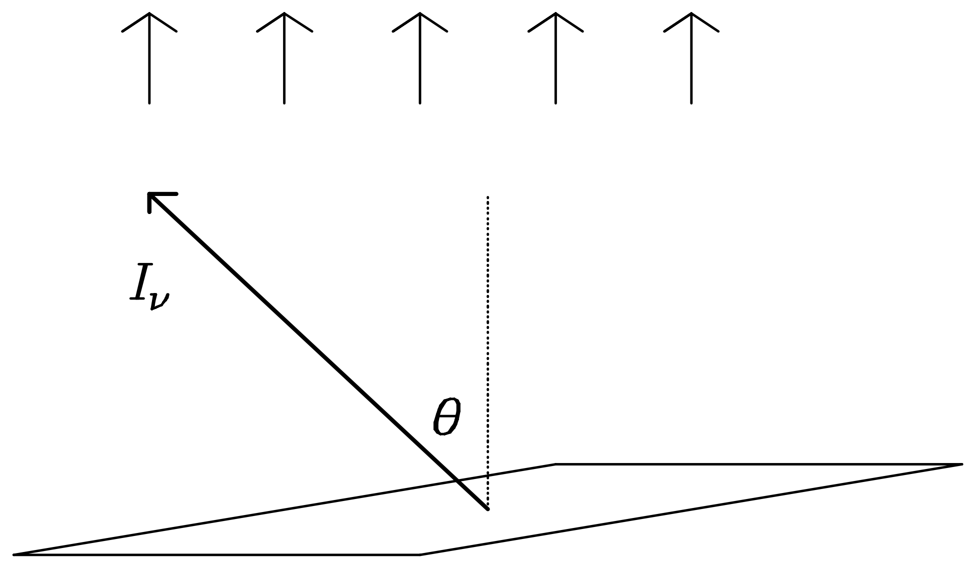
What are the cgs units for this quantity?

$$erg\,\boxed{s^{-1}}\,cm^{-2}\,\boxed{Hz^{-1}}\,steradian^{-1}$$

Does this make sense?

Emission from an Object

Consider the flow from a small unit area. How do we obtain the **flux**?



$$f_\nu = \int_{\theta=0}^{\pi/2} I_\nu \cos\theta d\Omega = I_\nu 2\pi \frac{1}{2} = \pi I_\nu = \frac{c}{4} u_\nu = \frac{2\pi h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Luminosity is the total power (energy per unit time) radiated by the star (spherical and isotropically emitting)

$$L_\nu = f_\nu(r_*) 4\pi r_*^2$$

r_*= radius of star

$$L_\nu = f_\nu(r_*) 4\pi r_*^2$$

The flux seen by an observer at distance d from the source is

$$f_\nu(d) = \frac{L_\nu}{4\pi d^2} = f_\nu(r_*)\frac{r_*^2}{d^2}$$

Note on language:

Flux density is the flux at a particular frequency.

Flux is the flux density integrated over some frequency.

Bolometric flux is the flux density integrated over all frequencies.

(similar for luminosity)

The Stefan-Boltzman law relates total energy density or flux of a blackbody to its temperature.

$$u = aT^4$$

$$f = \frac{c}{4} aT^4 = \sigma T^4$$

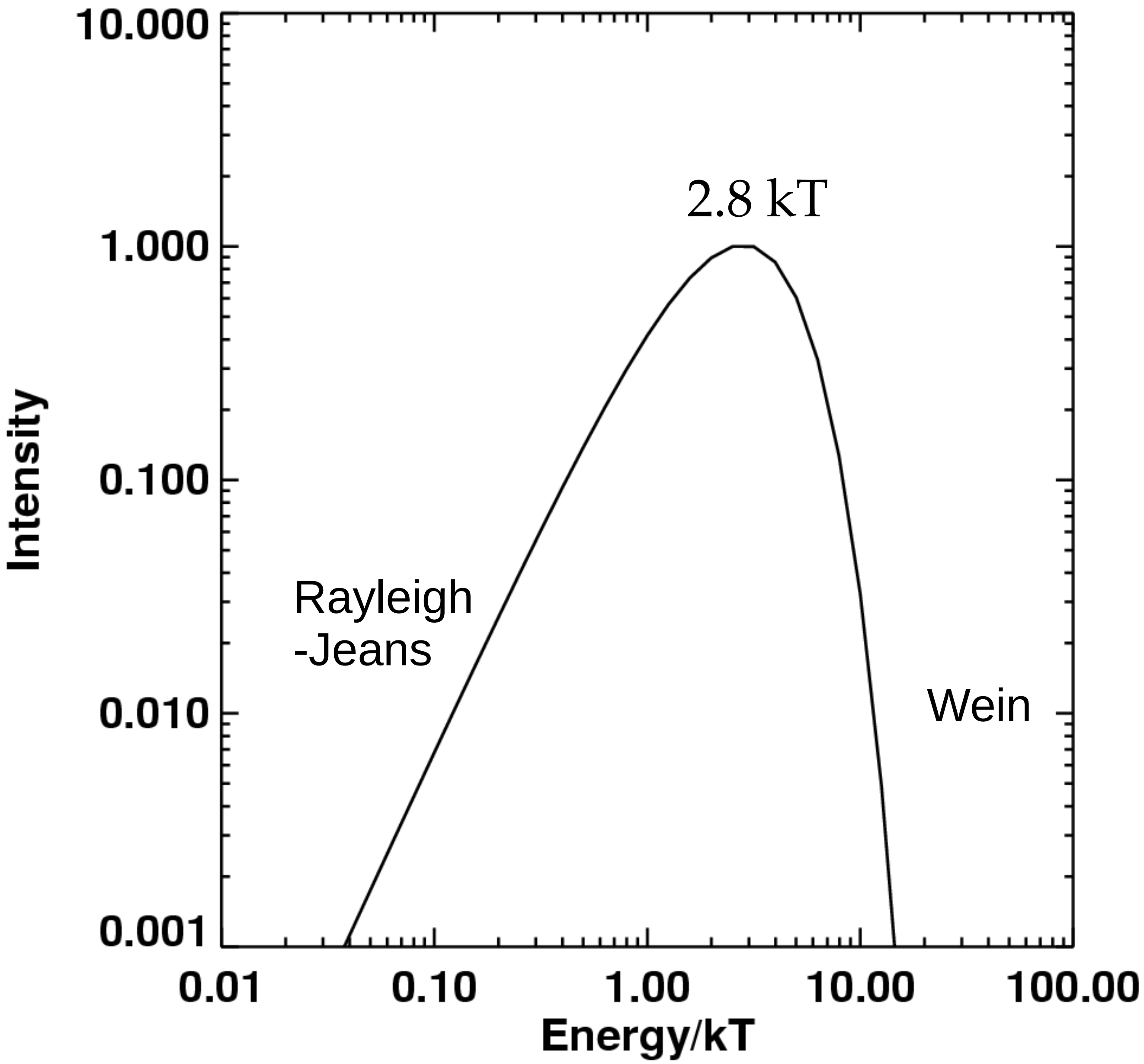
Radiation constant

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3} = 7.6 \times 10^{-15} \text{ erg cm}^{-3}\text{K}^{-4}$$

Stefan-Boltzmann constant

$$\sigma = \frac{c}{4} a = 5.7 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2}\text{K}^{-4}$$

Blackbody Radiation



- Intensity of blackbody radiation is

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

- Falls off at high energy = high frequencies as exp(-hv/kT). Known as **Wein tail**
- Falls off at low energy = low frequencies as ν^2. Known as **Rayleigh-Jeans law**.
- Note: spectrum peaks at

$$h\nu_{max} = 2.8\,kT$$

Recall: E = hν

It is often more convenient to look at quantities such as energy density, intensity and flux per photon wavelength interval.

Recall that the energy in an interval must be the same whether it is measured in wavelength or frequency.

$$B_\lambda d\lambda = B_\nu d\nu$$

Thus,

$$B_\lambda = B_\nu \left| \frac{d\nu}{d\lambda} \right| = B_\nu \frac{c}{\lambda^2} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Recall: $\nu = c/\lambda$

How can we find the peak of the blackbody spectrum?

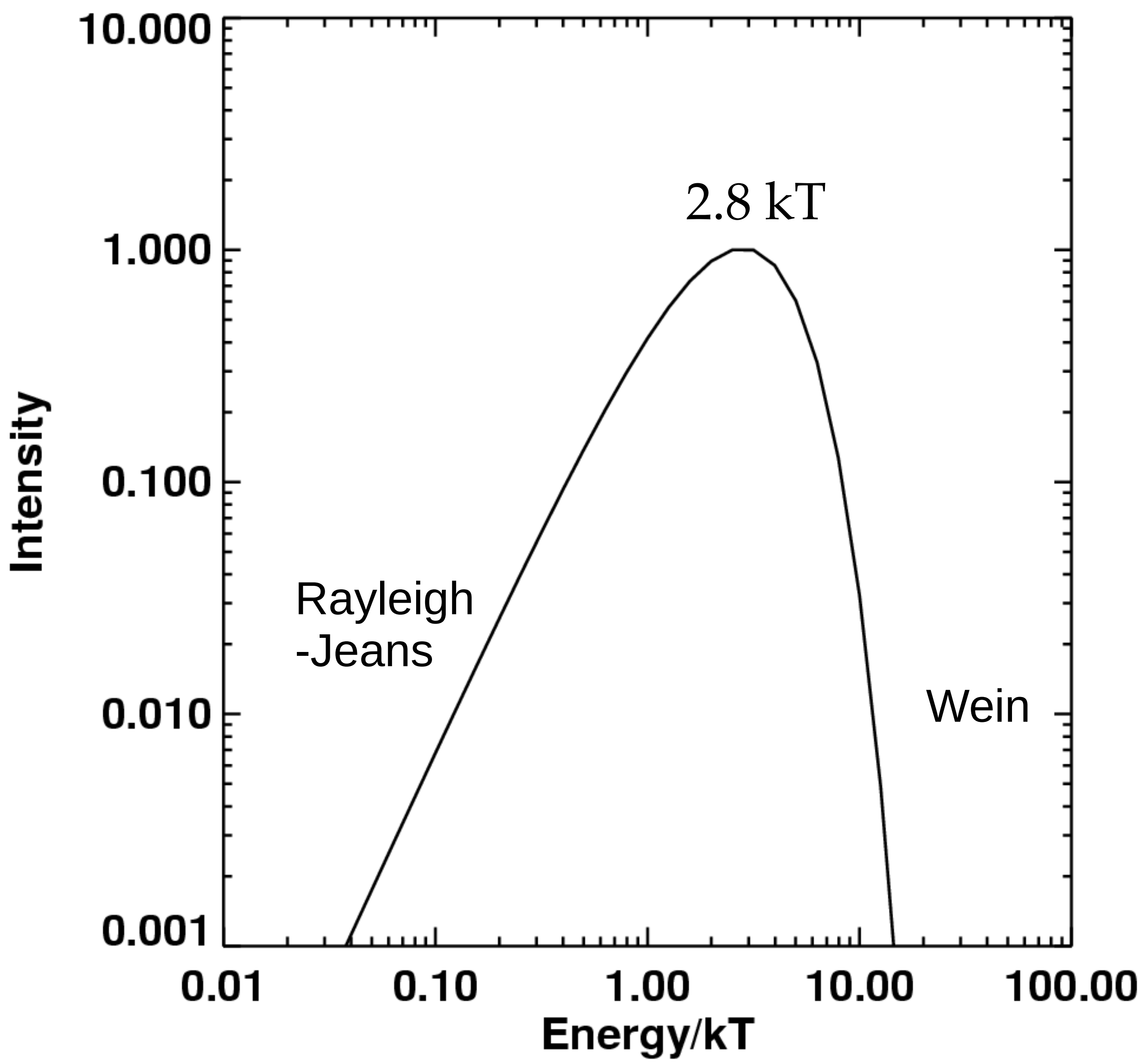
Take the derivative of the blackbody spectrum and set it equal to zero.

$$\frac{dB_\nu}{d\nu} = 0 \qquad \Bigg| \qquad \frac{dB_\lambda}{d\lambda} = 0$$

Leads to

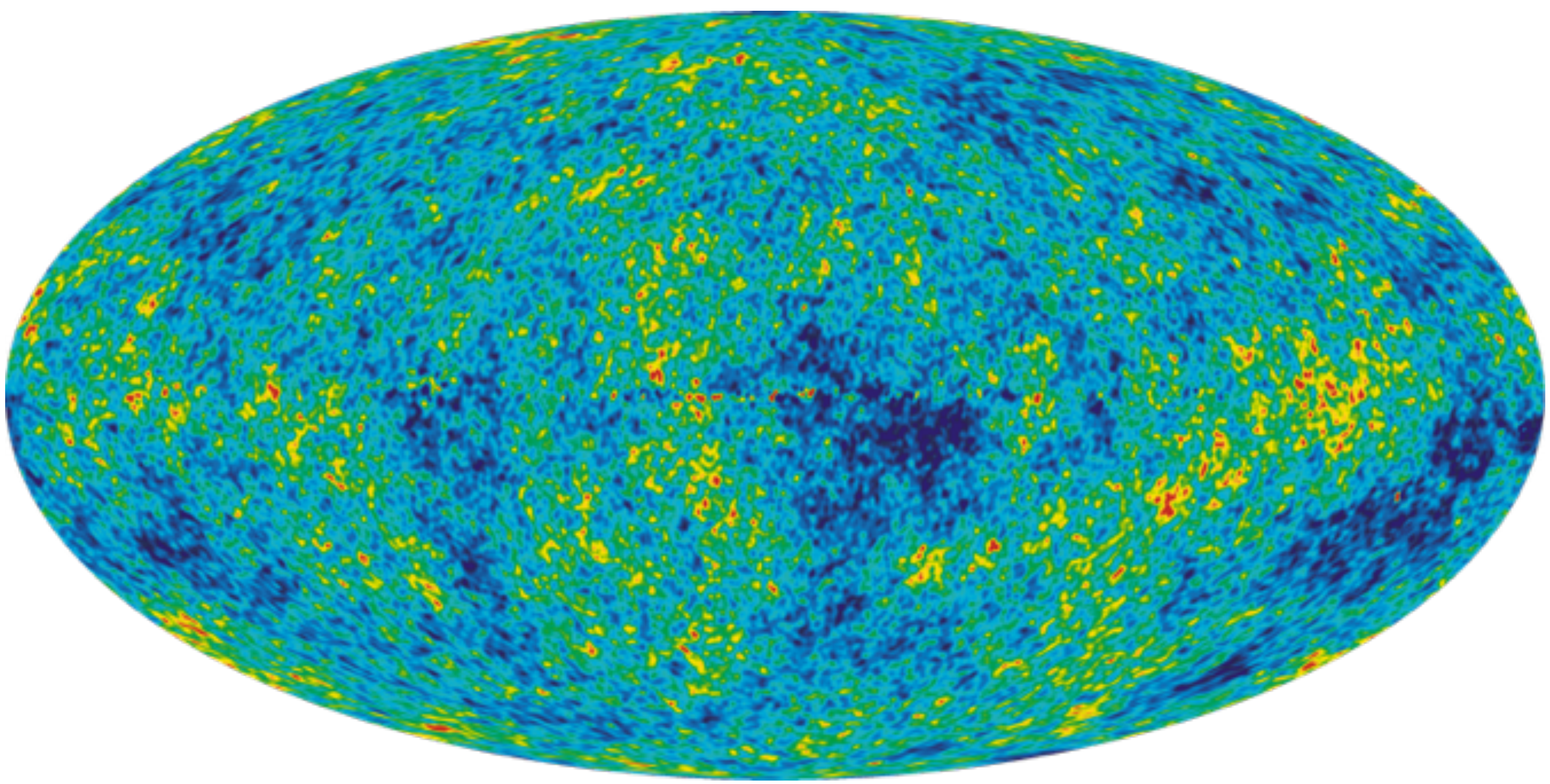
$$\lambda_{\max} T = 0.29 \text{ cm K}$$

$$h\nu_{\max} = 2.8\, kT$$



Example: CMB

The cosmic microwave background (CMB) radiation is a thermal radiation left over from the big bang. It fills the observable universe almost uniformly. The CMB has a thermal blackbody spectrum at a temperature of 2.7 K.



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What is the wavelength at the maximum intensity of the spectrum of this radiation?

$$\lambda_{max} T = 0.29\ cm\ K$$

$$\lambda_{max} = \frac{0.29\ cm\ K}{T} = \frac{0.29\ cm\ K}{2.7\ K}$$

$$\boxed{\lambda_{max} = 0.11\ cm}$$

Example: CMB

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What is the total power incident on Earth from this radiation?

$$\frac{P}{A} = f = \sigma T^4$$

$$P = 4\pi r_E^2 \sigma T^4$$

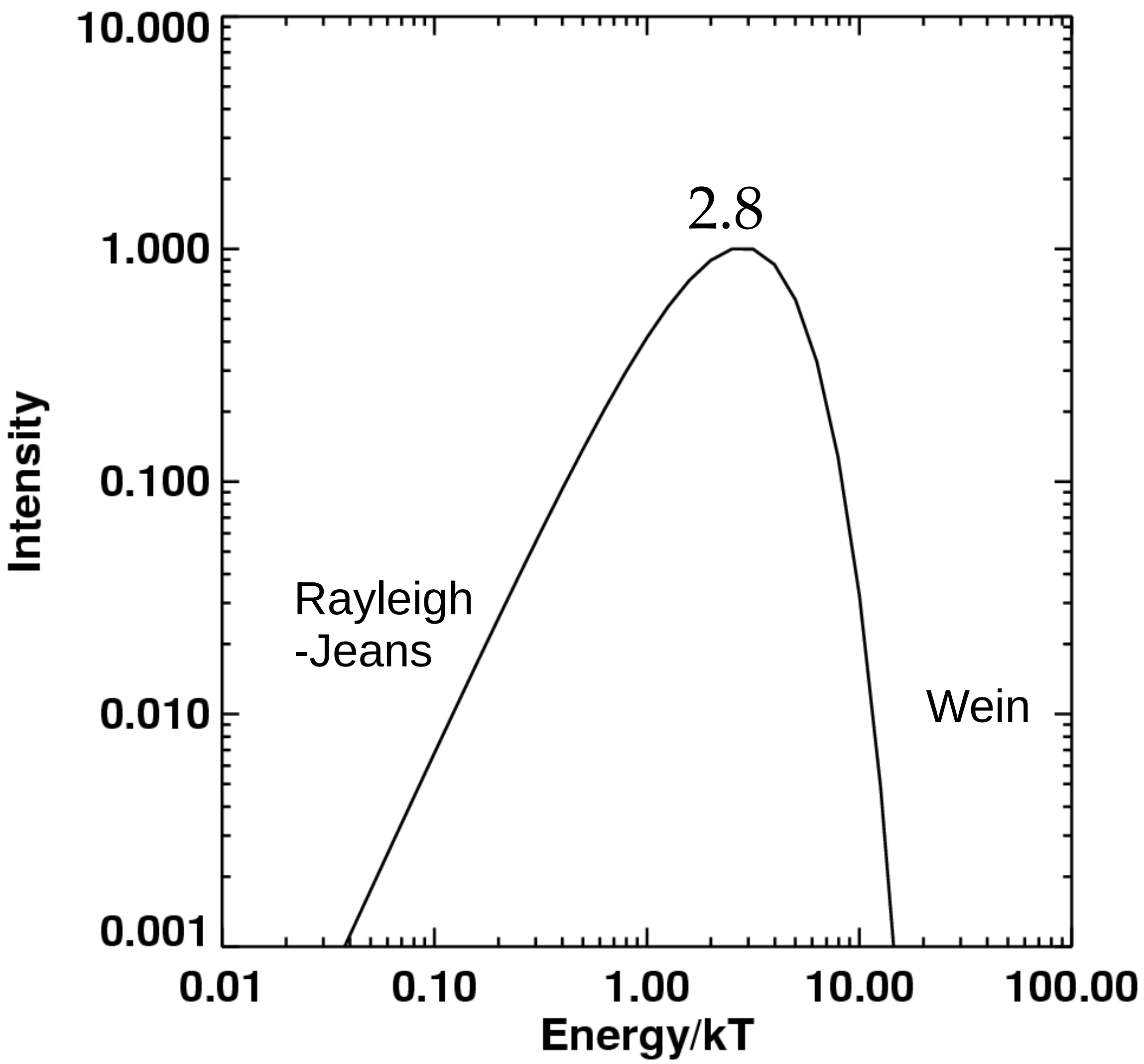
$$P = 4\pi (6.4 \times 10^8\ cm)^2 (5.7 \times 10^{-5}\ erg\ cm^{-2}\ s^{-1}\ K^{-4})\ (2.7\ K)^4$$

$$\boxed{P = 1.6 \text{ x } 10^{16} \text{ erg s}^{-1}}$$

Final Notes on Blackbody Radiation

The functions u_ν, B_λ, etc. are determined uniquely by one parameter, T (temperature).

Far from their peaks, the spectra assume tow simple forms:



$\underline{\nu \ll \text{peak}}$: *Rayleigh Jeans side*

$$B_\nu \approx \frac{2\nu^2}{c^2} kT$$

$$B_\lambda \approx 2ckT\lambda^{-4}$$

$\underline{\nu \gg \text{peak:}}$ *Wein tail side*

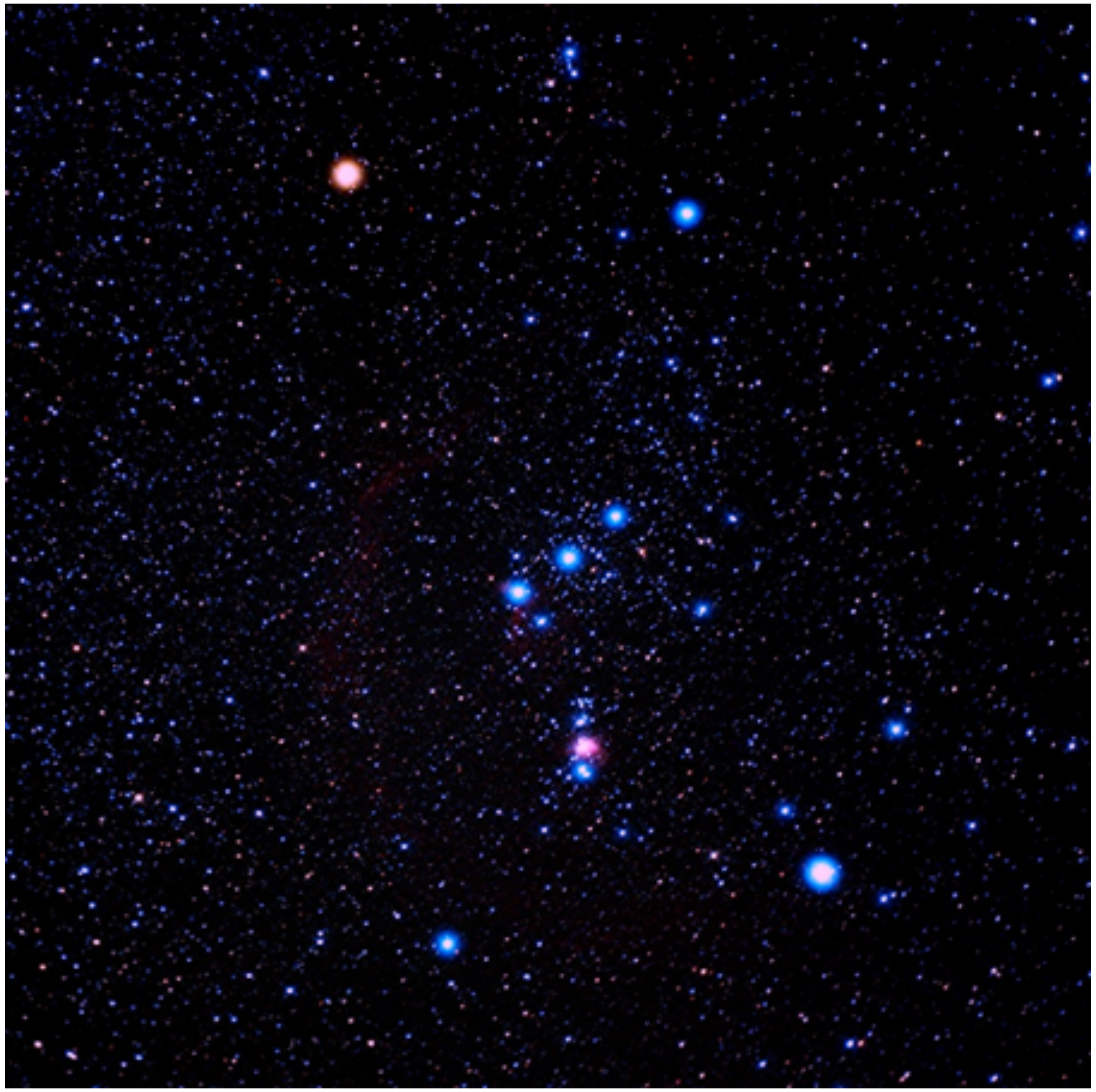
$$B_\nu \sim e^{-(h\nu/kT)}$$

$$B_\lambda \sim e^{-(hc/\lambda kT)}$$

The constellation Orion has two well known stars - Betelgeuse and Rigel.

Betelgeuse appears red.

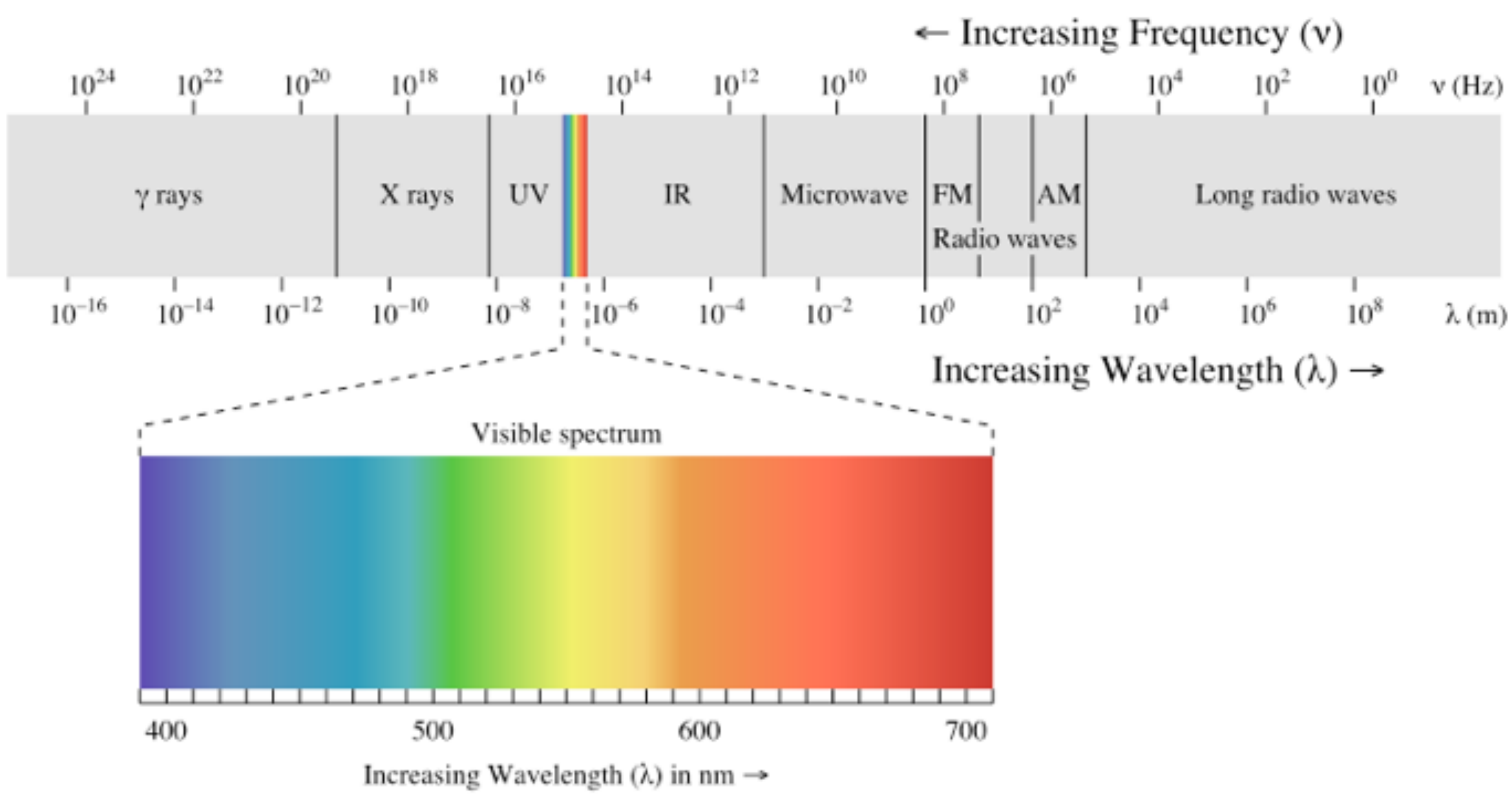
Rigel appears blue-whitish.



Which has the greater surface temperature?

Remember, as the temperature increases the wavelength decreases.

$$\lambda_{max} \propto \frac{1}{T}$$



Constellation Orion

Betelgeuse has a surface temperature of 3600 K and Rigel has a surface temperature of 13,000 K. Treat both stars as blackbodies and calculate the peak of their spectrums.

Betelgeuse:

$$\lambda_{max} = \frac{2.898 \times 10^{-3} m \cdot K}{3600K} = 8.05 \times 10^{-7} m = 805nm$$

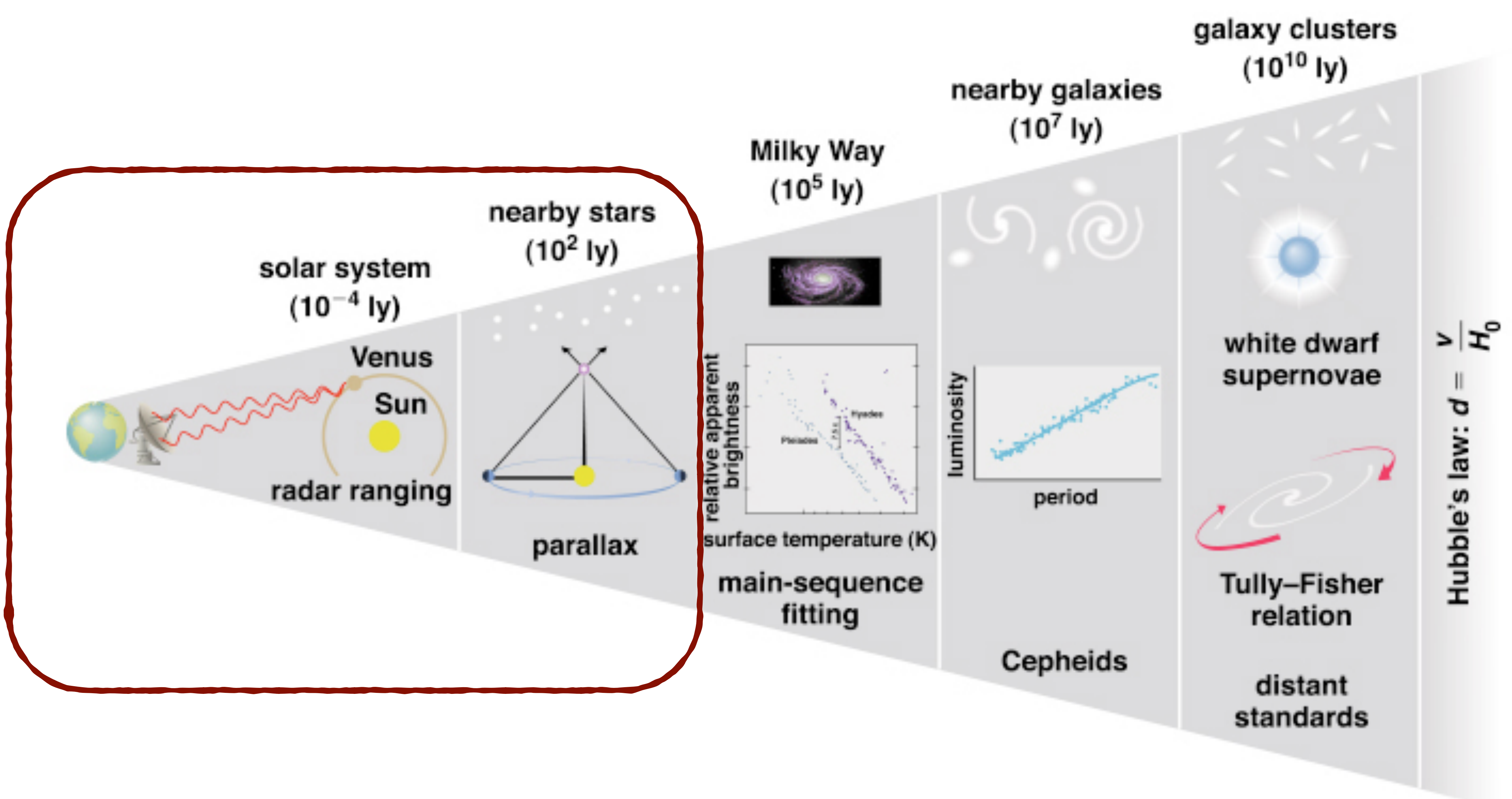
infared

Rigel:

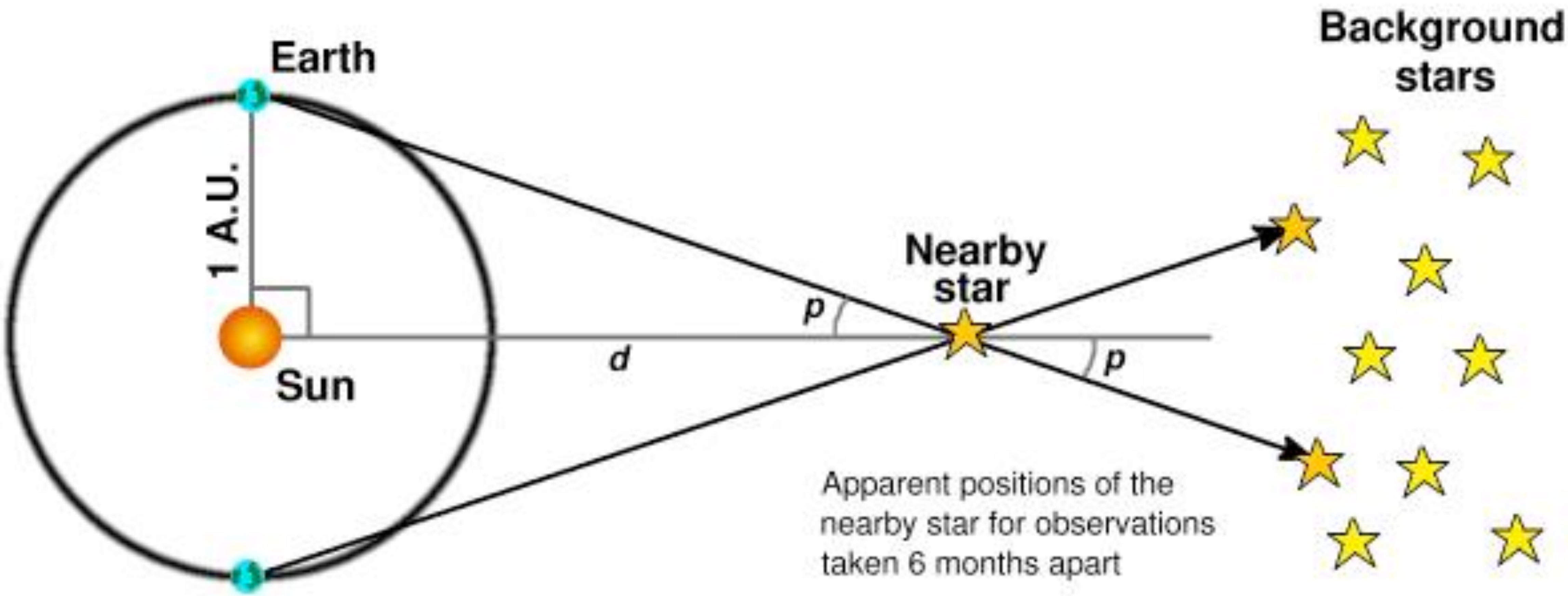
$$\lambda_{max} = \frac{2.898 \times 10^{-3} m \cdot K}{13,000K} = 2.23 \times 10^{-7} m = 223nm$$

ultraviolet

Distance Ladder



Trigonometric Parallax



Ecliptic Plane: Plane perpendicular to the Earth's orbit.

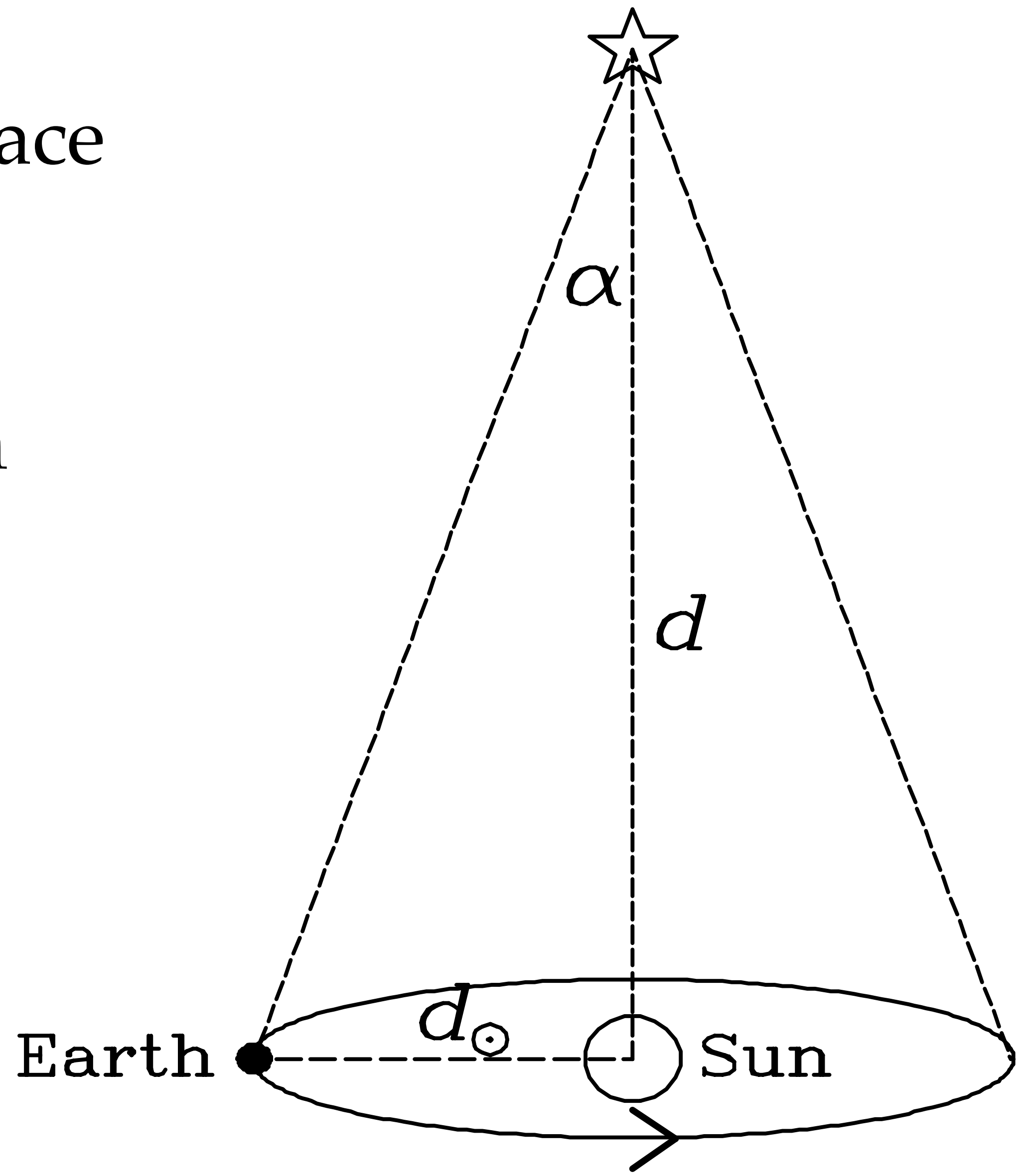
Stars perpendicular to the Ecliptic plane trace a circle in the sky.

Stars parallel to the Ecliptic plane trace a line segment in the sky.

Stars in other directions trace out an ellipse on the sky:

$$\alpha = \frac{d_\odot}{d}$$

this subscript marks properties of the sun



More Units

AU:

astronomical unit; 1 AU is the distance between the Earth and the sun

$$1\ AU = 1.5 \times 10^{8}\ km$$

pc:

parsec; 1 pc is defined as the distance for which the parallax is 1 arcsecond (1/3600 of a degree of arc).

$$1\mathrm{pc} = 2.1 \times 10^{5}\ \mathrm{AU} = 3.1 \times 10^{18}\mathrm{cm} = 3.3\ \mathrm{ly}$$

ly:

lightyear; distance light travels in vacuum in one year.

$$1\ ly = 9.46 \times 10^{17}\ cm$$

The End (for today)!