# Principles of Astrophysics and Cosmology

Welcome Back
to PHYS 3368

**George Gamow**
**March 4, 1904 – August 19, 1968**

# Announcements

- Reading Assignments:  Chapter 3.10 - 3.12 and 4.1.
- Problem Set 6 is due Wednesday, March 4th, 2015.
- Next lab is Monday, March 16th.  Be sure to report to FOSC 032 that day.
- Midterm Exam 1 is in class on Wednesday, March 18th.  It will be open book, open note and cover chapters 1 - 3.
- Dark Sky viewing has been postponed due to weather.  Check your email for details on rescheduling possibilities.

## Last Time:

We discussed different contributions to opacity.

Kramer's Law:

$$\bar{\kappa}_{\rm bf,ff} \sim \frac{\rho}{T^{3.5}}$$

Electron Scattering:

$$\kappa_{es} = \frac{n_e \sigma_T}{\rho}$$

We derived several scaling relationships for Main Sequence stars.

We discussed the p-p chain for fusion in the sun and we discussed the conditions that make fusion possible in the sun.

The probability for 2 nuclear particles to overcome the Coulomb barrier is given by the function

$$g(E) = e^{-\sqrt{E_G/E}}$$

where $E_G$ is the **Gamow Energy** and g(E) is the **Gamow Factor**.

$$E_G = (\pi \alpha \mathcal{Z}_A \mathcal{Z}_B)^2 2\mu c^2$$

# Goals for Today's Class

- Calculate how long it will take to deplete the Sun's core of hydrogen.
- Are there other nuclear reactions we should be concerned with?
- What's up with the neutrino solar flux? In 1968, Ray Davis and colleagues had an experiment that measured only ~30% of the predicted solar neutrino flux.

# Nuclear Reaction Rates

For a nuclear reaction to occur, the two nuclei must be within the strong force's interaction range.

The probability of a nuclear reaction to occur will still depend upon a nuclear cross section.

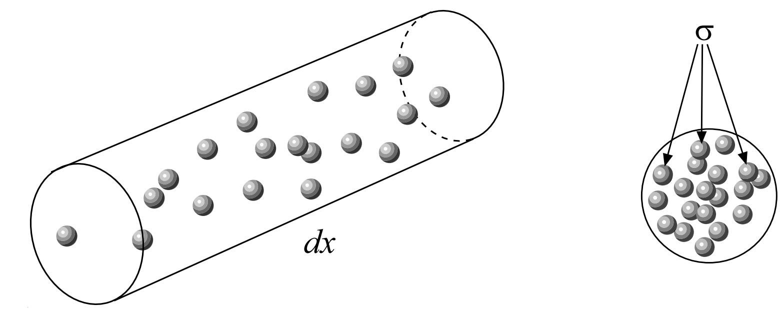
$$\sigma_{AB}(E) = \frac{S_0}{E} e^{-\sqrt{E_G/E}}$$

$S_0$ = a constant units of [area] x [energy]

$E_G$ = Gamow Energy

**Recall:**

The number of reactions per nucleus A as it traverses distance dx in a field of $n_B$ targets is




$$dN_A = n_B \sigma_{AB} dx$$

"Divide" both sides by dt.

$$\frac{dN_A}{dt} = n_B \sigma_{AB} \frac{dx}{dt} = n_B \sigma_{AB} v_{AB}$$

Our goal is to find the power density function ε(ρ, T, X, Y, Z). So, we multiply by density $n_A$ to get reactions per unit time per unit volume.

$$\mathcal{R}_{AB} = n_A n_B \sigma_{AB} v_{AB}$$

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Each reaction releases energy Q.

Q x R = Power per unit volume

Divide by density (ρ = M/V) gives the power per unit mass.

$$\epsilon = n_A n_B \sigma_{AB} v_{AB} Q/\rho.$$

Now we can use our knowledge of mass abundance to write

$$n_A = \frac{\rho X_A}{A_A m_H} \qquad n_B = \frac{\rho X_B}{A_B m_H}$$

Substituting into our equation for power density:

$$\epsilon = \frac{\rho X_A X_B}{m_H^2 A_A A_B} \sigma_{AB} v_{AB} Q$$

We need to modify this equation. Can anyone guess how?

$$\epsilon = \frac{\rho X_A X_B}{m_H^2 A_A A_B} \sigma_{AB} v_{AB} Q$$

We need to account for the fact that there is a distribution of velocities. We need to average over all velocities and weight the distribution appropriately.

$$\epsilon = \frac{\rho X_A X_B}{m_H^2 A_A A_B} \langle \sigma_{AB} v_{AB} \rangle Q$$

with

$$\langle \sigma_{AB} v_{AB} \rangle = \int_0^\infty \sigma_{AB} v_{AB} P(v_{AB}) dv_{AB}$$

Since the gas is classical and non-relativistic, the relative velocities ($v_A$, $v_B$) will follow a Maxwell-Boltzman distribution.

$$P(v)dv = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT}\right) dv$$

notation change: $v_{AB} = v$

We have the following 2 equations:

$$\sigma_{AB}(E) = \frac{S_0}{E} e^{-\sqrt{E_G/E}}$$

$$P(v)dv = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT}\right) dv$$

which need to be substituted into

$$\langle \sigma_{AB} v_{AB} \rangle = \int_0^\infty \sigma_{AB} v_{AB} P(v_{AB}) dv_{AB}$$

which gives —

$$< \sigma_{AB} v_{AB} >= \int_0^\infty \frac{S_0}{E} e^{-\sqrt{E_G/E}} \, v 4\pi (\frac{\mu}{2\pi kT})^{\frac{3}{2}} v^2 e^{-\frac{\mu v^2}{2kT}} dv$$

$$< \sigma_{AB} v_{AB} >= \int_0^\infty \frac{S_0}{E} e^{-\sqrt{E_G/E}} \, v 4\pi (\frac{\mu}{2\pi kT})^{\frac{3}{2}} v^2 e^{-\frac{\mu v^2}{2kT}} dv$$

Simplify:

$$< \sigma_{AB} v_{AB} >= \frac{S_0}{(kT)^{3/2}} \frac{4\pi}{(2\pi)^{3/2}} \int_0^\infty \frac{\mu^{\frac{1}{2}} v^2}{E} e^{-\sqrt{E_0/E}} e^{-\frac{\mu v^2}{2kT}} \mu v dv$$

U - Substitution:

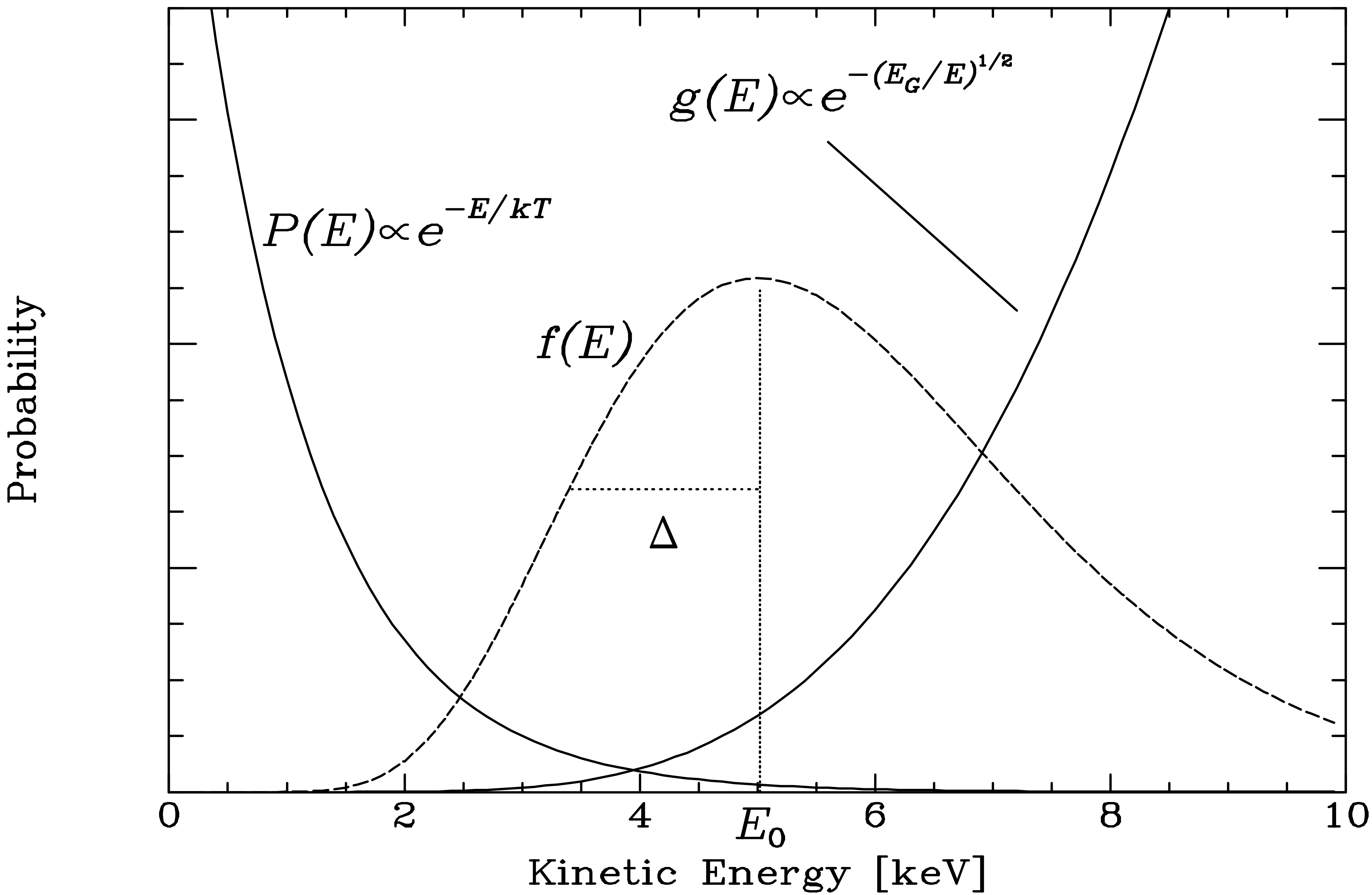
$$E = \frac{1}{2}\mu v^2 \qquad \frac{dE}{dv} = \mu v \qquad dE = \mu v dv$$

$$\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty e^{-E/kT} e^{-\sqrt{E_G/E}} dE$$

from Boltzman distribution falls with energy

from coulomb repulsion rises with energy

$$\langle \sigma v \rangle = \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{S_0}{(kT)^{3/2}} \int_0^\infty e^{-E/kT} e^{-\sqrt{E_G/E}} dE$$



To find the maximum of the integrand we do what?

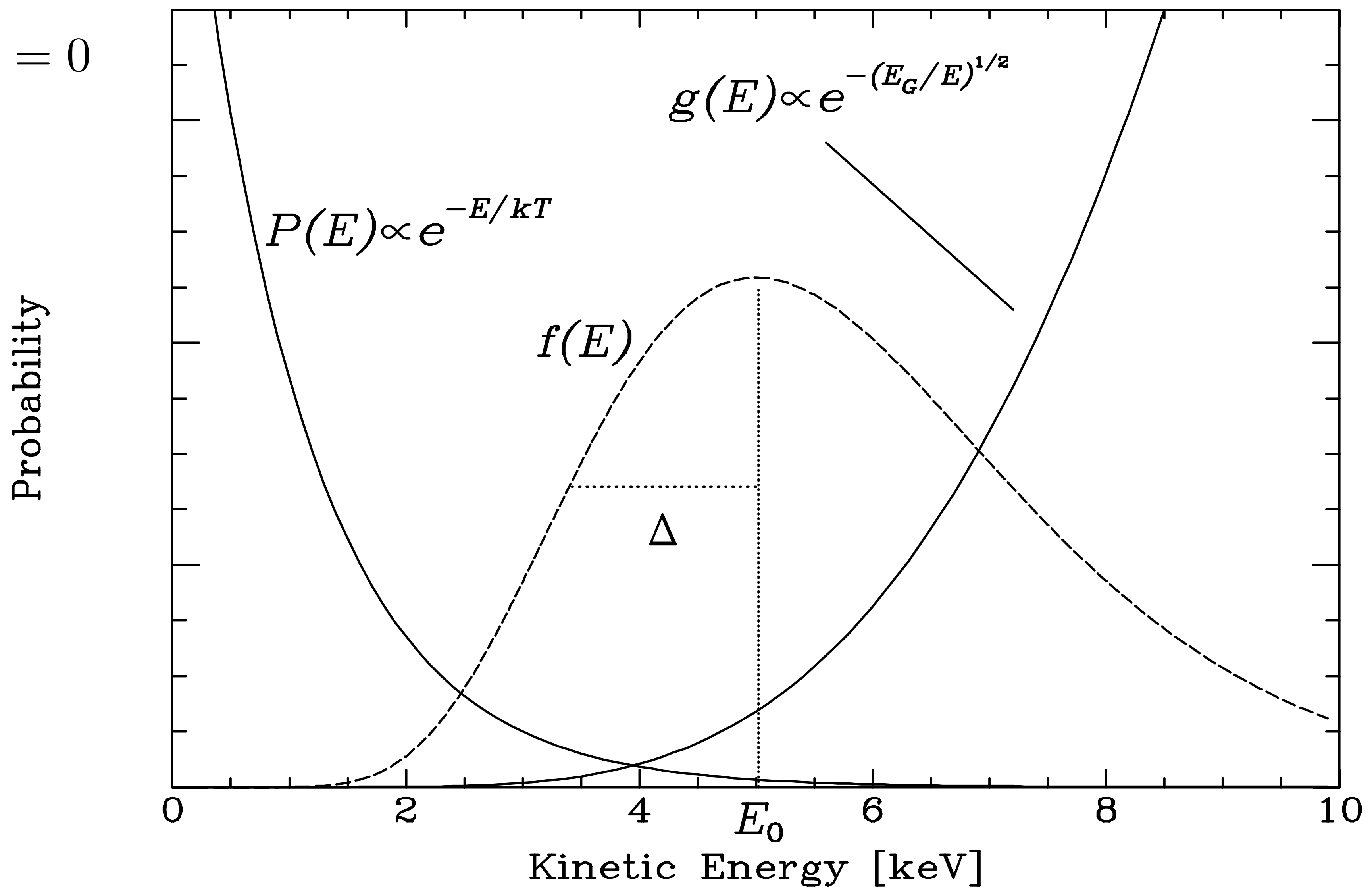
Take derivative and set it equal to zero. Solve for E.

**homework**

$$\frac{df(E)}{dE} = \frac{d}{dE}\left(e^{-E/kT} e^{-\sqrt{E_G E}}\right) = 0$$

$$E_0 = \left(\frac{kT}{2}\right)^{2/3} E_G^{1/3}$$

Notice that f(E) can be approximated by a Gaussian centered at $E_0$ with width (standard deviation) of $\Delta$.




Power Density (in full glory)

$$\epsilon = \frac{2^{5/3}\sqrt{2}}{\sqrt{3}} \frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

$$\epsilon = \frac{2^{5/3}\sqrt{2}}{\sqrt{3}} \frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

Use the power density to estimate the luminosity of the sun.

$\rho = 150\ g\ cm^3$ (mass density in core of Sun)

$X_A = X_B = 0.5$ (hydrogen abundance in core)

$$p + p \longrightarrow d + e^+ + \nu_e$$

$Q = 26.74\ MeV - 0.52\ MeV = 26.2\ MeV$ (energy of chain - ν energy)

$S_0 = 4 \times 10^{-46}\ cm^2\ keV$ (theoretical calculation)

$E_G = 500\ keV$ (value for 2 protons)

$kT = 1\ keV$ (typical core temperature)

$A_A = A_B = 1$ (atomic number of hydrogen)

$\mu = \frac{m_p}{2}$ (reduced mass)

**Final note:** Since the reaction is between identical particles, divide collision rate by 2.

These numbers yield a power density of

$$\epsilon = 10 \text{ erg } \text{ s}^{-1}\text{g}^{-1}$$

Multiply by the mass of the core of the sun (0.2 $M_{sun}$).3

$$L \sim 4 \times 10^{33} \, erg \, s^{-1}$$

Compare to actual value:

$$L \sim 3.8 \times 10^{33} \, erg \, s^{-1}$$

Notice this:

$$\frac{dN_A}{dt} = n_B \sigma_{AB} v_{AB} \qquad \epsilon = \frac{\rho X_A X_B}{m_H^2 A_A A_B} \sigma_{AB} v_{AB} Q$$

$$n_A = \frac{\rho X_A}{A_A m_H}, \;\; n_B = \frac{\rho X_B}{A_B m_H}$$

Multiply power density by a clever factor:

$$\epsilon \times \frac{m_H A_A}{X_A Q} = \frac{\rho X_A X_B}{m_H^2 A_A A_B \sigma_{AB} v_{AB} Q} \times \frac{m_H A_A}{X_A Q}$$

$$= n_B \sigma_{AB} v_{AB} = \frac{dN_A}{dt}$$

← number of reactions per nucleus A per unit time

For the p + p reaction, this gives a rate of 1.6 x $10^{-18}$ $s^{-1}$ per proton. The inverse of this number is the time a proton waits until it reacts with another proton.

$$\tau_{pp} \sim 6 \times 10^{17} \text{ s} \sim 2 \times 10^{10} \text{ yr}$$

Sun's age ~ 5 billion years

$$\epsilon = \frac{2^{5/3}\sqrt{2}}{\sqrt{3}} \frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

The total power density at a point in a star is the sum of power densities due to all nuclear reactions. Each reaction will be described by the power density equation.

What does the exponential factor in the power density equation tell us about which type of species will be most favorable?

Species with a small $E_G$ (and hence low atomic number) will be preferred.

**Example:**

$$p + d \rightarrow {}^3\text{He} + \gamma, \ \ (E_G = 0.66\text{MeV})$$

$$p + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma, \ \ (E_G = 35.5\text{MeV})$$

These reactions have comparable nuclear cross sections $S_0$.

$$\epsilon = \frac{2^{5/3}\sqrt{2}}{\sqrt{3}} \frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

**Assume:** Comparable abundances of d and $^{12}$C and take the typical kinetic energy of 1 keV.

**Recall:** The reaction rates are related to the power density by

$$\mathcal{R} = \epsilon \frac{\rho}{Q}$$

Taking the ratio of $^{12}$C to d, we find

$$\frac{\mathcal{R}(p^{12}\mathrm{C})}{\mathcal{R}(pd)} \sim \exp\left[-3\frac{35.5^{1/3} - 0.66^{1/3}}{(4 \times 0.001)^{1/3}}\right] \sim e^{-46} \sim 10^{-20}$$

So, we see that the species with the lower atomic number indeed has a higher reaction rate.

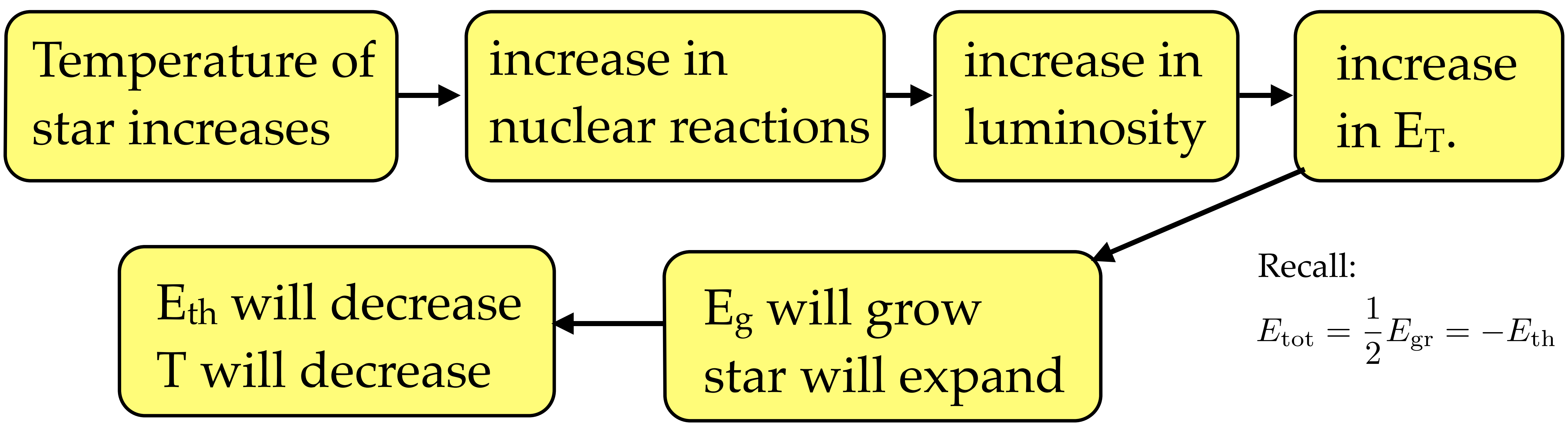
$$\epsilon = \frac{2^{5/3}\sqrt{2}}{\sqrt{3}} \frac{\rho X_A X_B}{m_H^2 A_A A_B \sqrt{\mu}} Q S_0 \frac{E_G^{1/6}}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

The higher the **Gamow energy**, the more strongly the reaction depends on temperature.

$$p + p \rightarrow d + e^+ + \nu_e \qquad (T^4)$$

$$p +^{12}\mathrm{C} \rightarrow^{13}\mathrm{N} + \gamma \qquad (T^{18})$$

This temperature dependency combined with viral theorem keeps stars stable.




Recall:

$$E_{\rm tot} = \frac{1}{2}E_{\rm gr} = -E_{\rm th}$$

A key prediction of this picture is that there is a constant flux of neutrinos coming out of the Sun.

Since neutrinos interact by the weak interaction, it guarantees that they escape the Sun's core almost unobstructed. (unlike photons).

The flux of neutrinos seen on Earth is given by:

$$f_{\rm neutrino} = \frac{2 f_{\odot}}{26.2 \text{ MeV}} = \frac{2 \times 1.4 \times 10^{6} \text{ erg s}^{-1} \text{ cm}^{-2}}{26.2 \times 1.6 \times 10^{-6} \text{ erg}} = 6.7 \times 10^{10} \text{ s}^{-1} \text{ cm}^{-2}$$

This solar flux was first measured in the 1960's in experiments at the Homestake Mine in South Dakota.

# Ray Davis Solar Neutrino Experiment

http://www.bnl.gov/bnlweb/raydavis/pictures.htm

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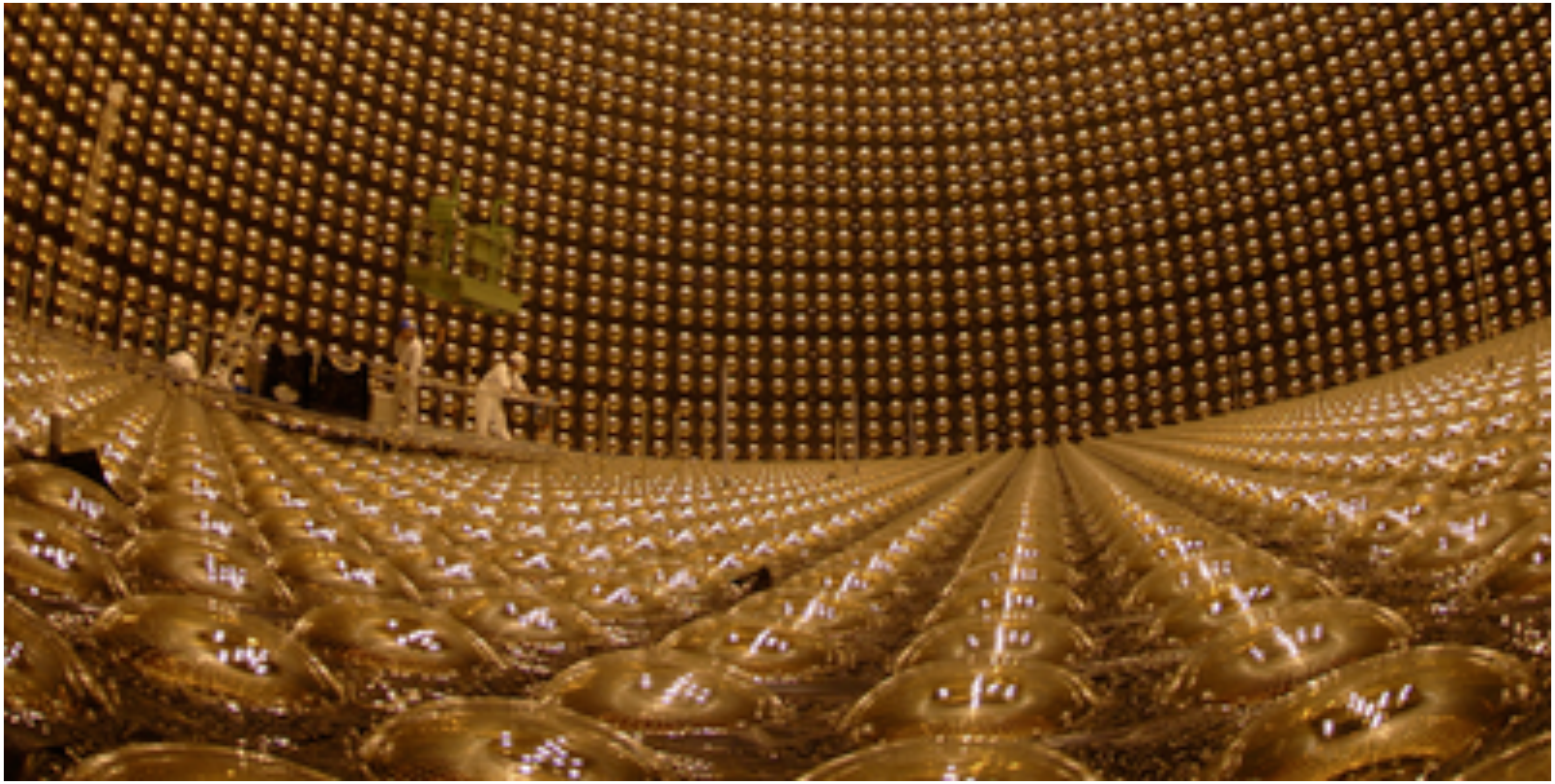
The Homestake mine tank

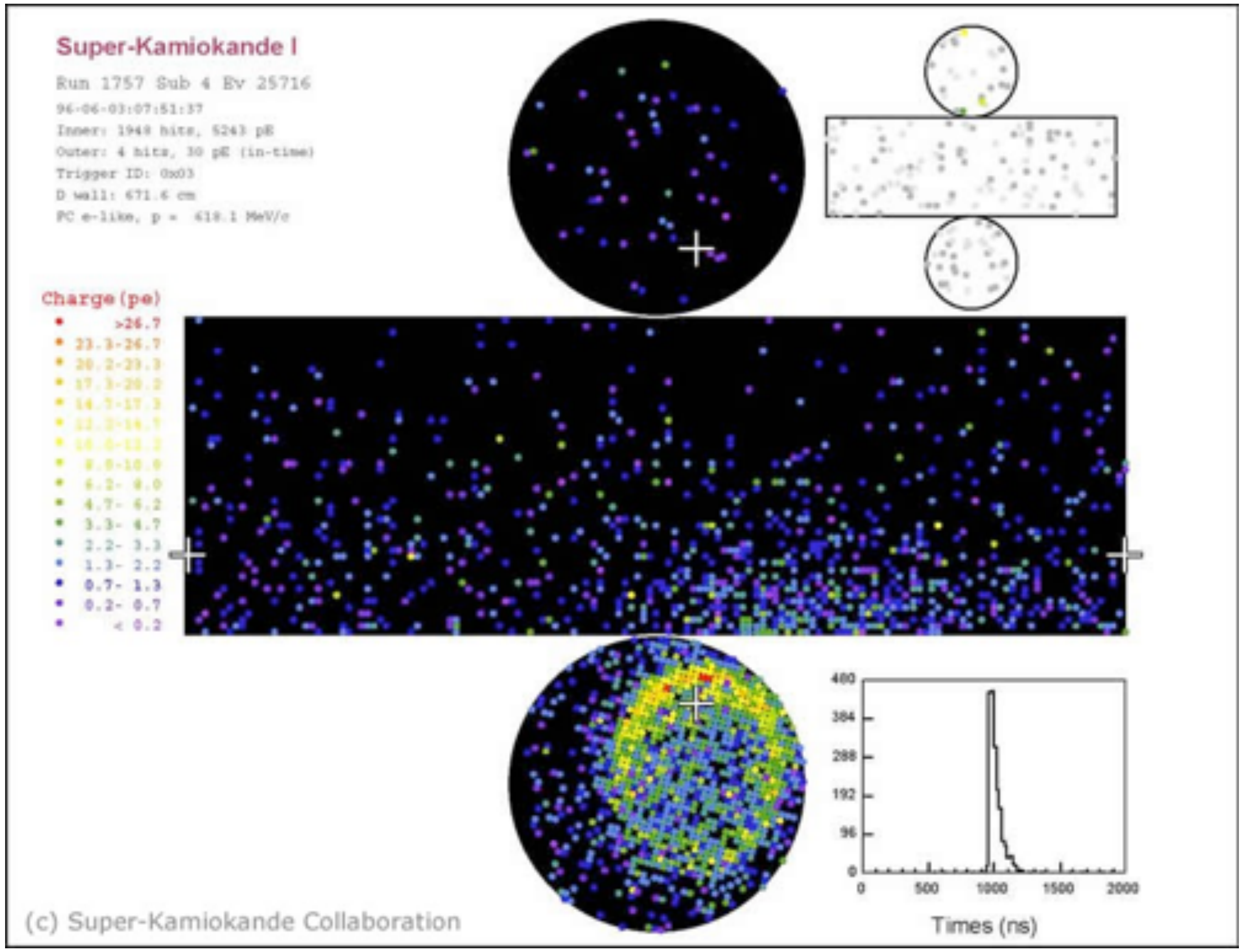
- 20 ft diameter x 48 ft long
- 100,000 gallons of perchloroethylene
- located 4,900 ft below ground
- Homestake Mine, SD

Experiment detected a deficit of solar neutrinos (~30% of what was expected).

# Super Kamiokande

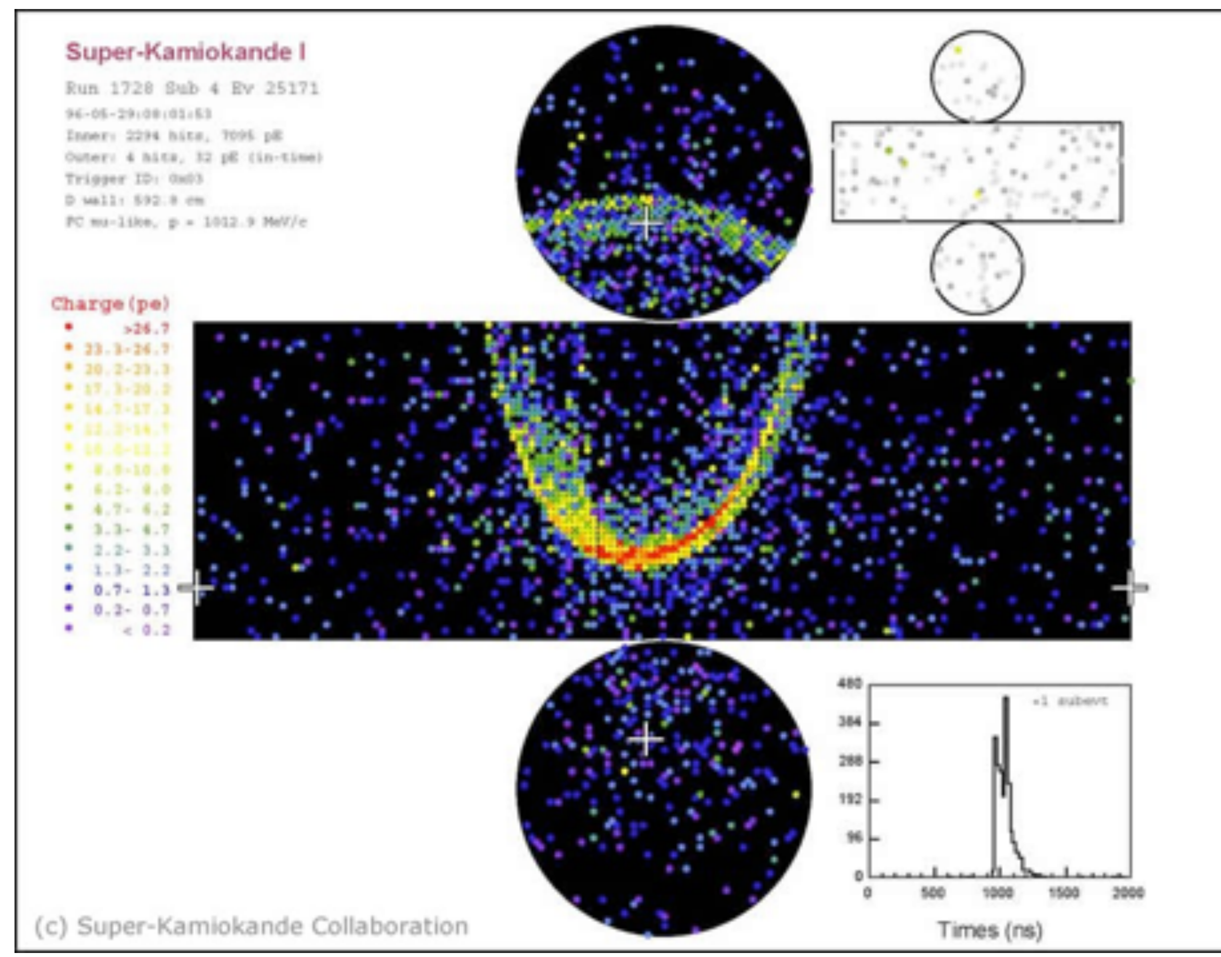


- 3,300 ft underground in Kamioka, Japan in the Mozumi Mine.
- 50,000 tons of ultra-pure water.
- Over 11,000 PMTs
- 111 ft diameter x 119 ft height




The emitted electron from a $\nu_e$ interaction generates an EM shower which produces an image with a fuzzy Cherenkov ring.

The emitted muon from a $\nu_\mu$ interaction generates an image with a crisp Cherenkov ring.



Next up - CNO Cycle, Convection and Stellar Evolution!

**Stay Tuned!**