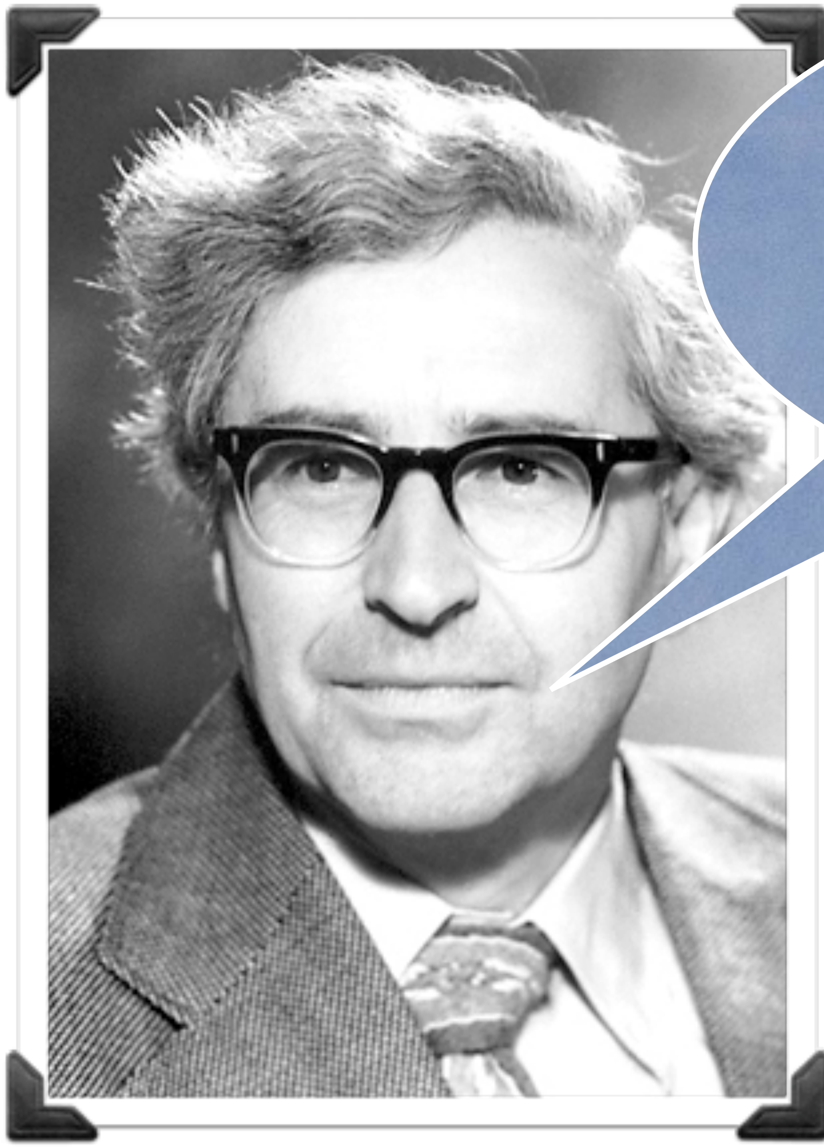
Principles of Astrophysics and Cosmology



Welcome Back to PHYS 3368

Anthony Hewish
May 11, 1924 -

Joceyln Bell
July 15, 1943 -

Announcements

- Reading Assignments: Chapter 4.4 - 4.6.
- Problem Set 9 is due in class on Wednesday, April 8th.
- No Classes Friday, April 3. Thus, no office hours on Friday.
- Monday, April 13th: Special lecture about something awesome by Matt Stein. If you are participating in Honors Convocation, you are excused from class that day.
- Wednesday, April 15th in class lab. Be to report to FOSC 032 that day.
- Wednesday, April 15th your final paper is due (hard copy and electronic pdf). Be sure to review the paper guidelines.
- Dr. Cooley will be out of town April 14th - April 17th.
- The final exam in this course will be on Wednesday, May 6th from 6:30 - 8:00 pm. It will cover the second half of the course.

Rough Lecture Outline

- April 6: Accretion and evolution of binary systems (4.6)
- April 8: Friedman Equation (8.1 - 8.2)
- April 20: History of the Universe and Dark Energy (8.3, 8.5)
- April 22: Redshift and the CMB as tests (9.1 - 9.2)
- April 27: CMB Anisotropy (9.3)
- April 29: Nucleosynthesis & Quasars (9.4 - 9.5)
- May 4: Special Topic Lecture (TBD)

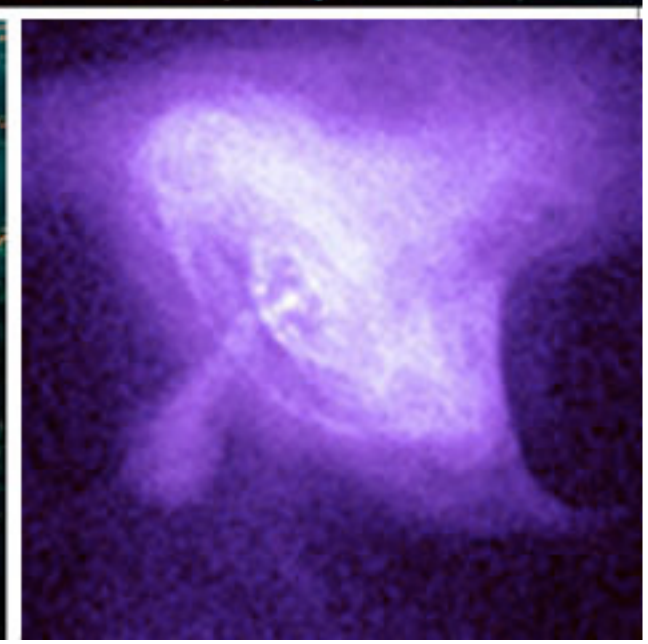
Goals for Today's Class

- What is up with the Crab?
- Explore Black Holes (which will require a tiny bit of GR)
- Are nuclear reactions the only way to power stars?

The Crab Nebula

Optical image - scale 4 pc per side.

Zoom in of marked area in optical.



Zoom in of marked area in x-rays.

Last Time:

What are possible mechanisms for producing the periodicity of the observed magnitude and regularity in these stars?

1. ~~binaries~~
2. ~~stellar pulsations~~
3. stellar rotation

1. The separation distance required between 2 binaries is 200 km. Normal stars and WD are too large, neutrons stars are okay. However, GR requires that stars in tight binary lose energy, spiral inward and orbital velocity increases. Observations indicate that pulsars slow over time.

2. No known class of stars produces a pulsation period of ~0.8 s. Normal stars and WD pulsate at 100 - 1000 s. Neutrons stars pulsate at 0.1 s.

Stellar Rotation:

Assume anisotropic emission from a rotating star. What is the fastest a star can spin?

Angular frequency at which centrifugal forces do not break it apart.

$$\frac{GMm}{r^2} > m\omega^2 r.$$

$$\frac{M}{r^3} > \frac{\omega^2}{G}$$

$$\bar{\rho} = \frac{3M}{4\pi r^3} > \frac{3\omega^2}{4\pi G} = 1.3 \times 10^{11} \text{ g cm}^{-3}$$

If the Crab is a spinning star and not flying apart, it's mean density must be 5x WD, but consistent with neutron star.

Assume that the luminosity of the Crab nebula is powered by the pulsar's rotational energy loss as it spins down.

What is the formula for rotational energy?

$$E_{\rm rot} = \frac{1}{2} I\omega^2$$

How would I get the total luminosity due to rotational energy?

$$L_{\rm tot} = -\frac{dE_{\rm rot}}{dt} = -I\omega\frac{d\omega}{dt}$$

What is the moment of inertia of a sphere?

$$I = \tfrac{2}{5} Mr^2$$

$$L_{tot} = -I\omega\frac{d\omega}{dt} \qquad I = \frac{2}{5}Mr^2$$

Substitute and solve for Mr^2

$$L_{tot} = -\frac{2}{5}Mr^2\omega\frac{d\omega}{dt}$$

$$Mr^2 = -\frac{5}{2}\frac{L_{tot}}{\omega\frac{d\omega}{dt}} = -\frac{5 \times 5 \times 10^{38}\ \mathrm{erg\ s^{-1}}}{2 \times 190\ \mathrm{s^{-1}}(-2.4 \times 10^{-9}\ \mathrm{s^{-2}})} = 3 \times 10^{45}\ \mathrm{g\ cm^2}$$

Compare this to a 1.4 M_{sun} neutron star of radius 10 km.

$$Mr^2 = 1.4 \times 2 \times 10^{33}\ \mathrm{g} \times (10^6\ \mathrm{cm})^2 = 2.8 \times 10^{45}\ \mathrm{g\ cm^2}$$

How does this compare to our sun?

$$Mr^2 = (2 \times 10^{33}\ g) \times (7 \times 10^{10}\ cm)^2 = 9.8 \times 10^{54}\ g\ cm^2$$

How fast would the sun spin if it were to collapse to a neutron star of radius 10 km? The rotation period of the sun is 25 days and the sun's radius is 7×10^{10} cm.

Use conservation of momentum to solve.

$$I_i \omega_i = I_f \omega_f$$

$$\frac{2}{5} M R_i^2 \omega_i = \frac{2}{5} M R_f^2 \omega_f$$

$$\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2 = \underline{3 \times 10^{-6}\ s^{-1}} \times \left(\frac{7 \times 10^{10}\ cm}{10 \times 10^{5}\ cm}\right)^2 = \underline{1.5 \times 10^{4}\ s^{-1}}$$

Spin up rates on order of 10^9. Collapse of main sequence stars are expected to produce objects with a spin on the order of ms.

Is the nail in the coffin?

- The spin rate of pulsars is that expected from the collapse of the cores of main sequence stars.
- The mean densities are those of neutron stars
- Their rotational energy accounts for the luminosity of supernova ejecta in which the stars are embedded.
- Location of pulsars at the sites of historical SN is expected to accompany the formation of a neutron star.

Magnetic Fields

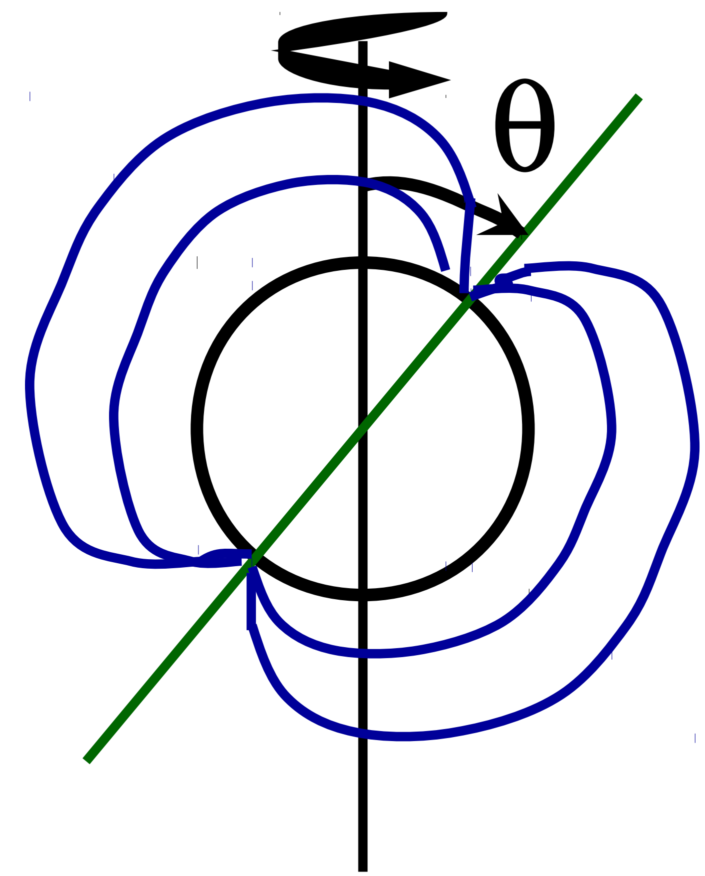
If a solar type star collapses to form a neutron star, while conserving magnetic flux, we would expect

$$R_{sun}^2 B_{sun} = R_{NS}^2 B_{NS}$$

$$\frac{B_{NS}}{B_{sun}} = (\frac{7 \times 10^{10}\ cm}{10^6\ cm})^2 \quad \sim 5 \times 10^9$$

For the sun, B ~ 100 G, so we would expect a NS to have a field of magnitude ~10^{12} G.

Consider a NS having a magnetic field axis misaligned w.r.t. the stars rotation axis by some angle θ.



A spinning magnetic dipole radiates an EM luminosity of

$$L = \frac{1}{6c^3} B^2 r^6 \omega^4 \sin^2\theta \quad \propto \omega^4$$

Solving for B, and substituting in observed values of the Crab gives B ~ 8 x 10^{12} Gauss.

If the EM radiation is leading to the pulsar's rotation energy, then

$$\frac{dE_{\mathrm{rot}}}{dt} = I\omega\frac{d\omega}{dt} \propto \omega^4 \longrightarrow \frac{d\omega}{dt} = C\omega^3$$

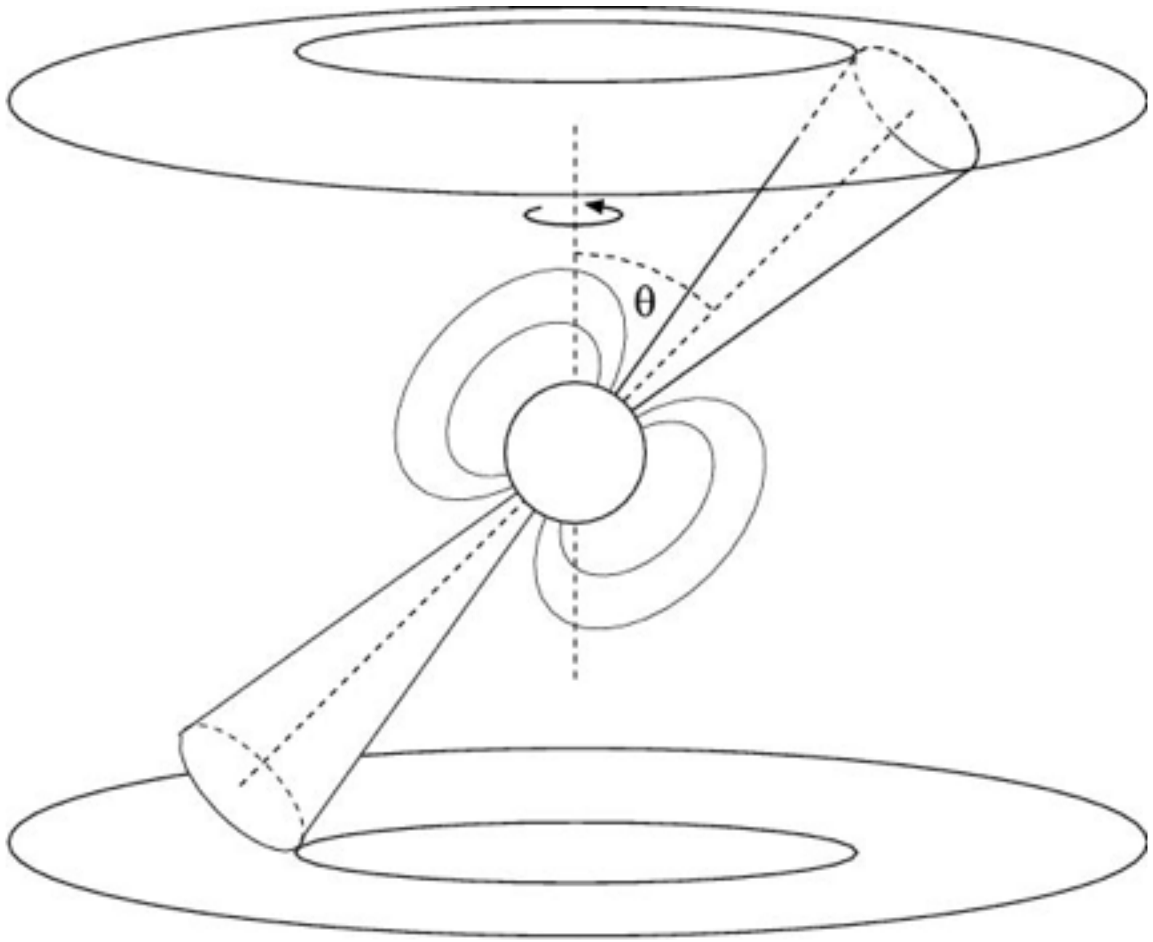
Separating variables and solving yields the age of a pulsar :

$$t_{\mathrm{pulsar}} = \frac{\omega_0^3}{2\dot{\omega}_0}\left(\frac{1}{\omega^2} - \frac{1}{\omega_i^2}\right)$$

For the Crab, pulsar = 1260 years. This is consistent with the historical age of ~960 years.

Neutron Star Cooling

- Only a small fraction of neutron stars are observable from Earth.
- As NS slow down and lose rotational energy, they become undetectable as pulsars.
- Detailed calculations of NS cooling are much less certain than those for WD.
- Poorly constrained EOS for nuclear matter leads to uncertainty in the structure and composition of a neutron star.



Black Holes

- In the case of a stellar remnant with mass > the allowed mass of a neutron star, no known mechanism can prevent complete gravitational collapse.
- GR predicts that even if a new form of pressure kicks in at high densities, it will not be strong enough to overcome gravity.
- The star will collapse to a black hole from where no radiation or matter can escape.

Let's find the "radius" of a black hole.

$$\frac{GMm}{r} = U > E_k = \frac{1}{2}mv_e^2$$

$$\frac{GM}{r} > \frac{1}{2}v_{\rm e}^2 = \frac{1}{2}c^2$$

Rearranging terms

$$\boxed{r_s = \frac{2GM}{c^2} = 3\ \mathrm{km}\frac{M}{M_\odot}}$$

This is the Schwarzschild radius

Note that the equation on the previous page is incorrect for two reasons.

1. The KE of a photon is not $mc^2/2$
2. The gravitational PE is not described by Newton's limit.

We will outline the correct derivation.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein's tensor energy-momentum tensor

energy-momentum tensor:

Represented a 4x4 matrix, each of the indices runs over the 4 space-time coordinates. This term in the equations includes mass-energy density and pressure.

Einstein's tensor:

Consists of combinations of 1st & 2nd PDEs wrt spacetime coordinates of the metric $g_{\mu\nu}$.

The metric tells us how to calculate the interval ds is the interval between two spacetime events.

$$(ds)^2 = \sum_{\mu,\nu} g_{\mu\nu} dx_{\mu} dx_{\nu}$$

normally 1 time and 3 space elements.

In the absence of matter, spacetime is flat. In that case, we can use the **Minkowski metric**.

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

The 4 x 4 matrix describing $g_{\mu\nu}$ then look like

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

What happens if $dt = 0$?

$$(ds)^2 = -[(dx)^2 + (dy)^2 + (dz)^2] \ = |ds|$$

This is just the distance between 2 points.

What happens if $dx = dy = dz = 0$?

$$(ds)^2 = (cdt)^2$$

ds/c = time between two points. Proper time $\tau = ds/c$ is the time elapsed on a clock moved between two points.

Light travels along "null geodesics" for which $ds = 0$.

What if we use spherical coordinates?

$$(ds)^2 = (cdt)^2 - (dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

The 4 x 4 matrix describing $g_{\mu\nu}$ then look like

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

In the case of spacetime in a vacuum surrounding a static, spherically symmetric, mass distribution we get the **Schwarzchild metric:**

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

$$(ds)^2 = \left(1 - \frac{r_s}{r}\right)(cdt)^2 - \left(1 - \frac{r_s}{r}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

where r_s is the Schwarzchild radius

$$r_s = \frac{2GM}{c^2}$$

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2 \qquad r_s = \frac{2GM}{c^2}$$

For a clock at rest, what is the proper time?

$$d\tau \equiv \frac{ds}{c} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} dt = \left(1 - \frac{r_s}{r}\right)^{1/2} dt$$

Consider a stellar remnant so compact that its radius fits within r_s.

- As $r \rightarrow r_s$, $d\tau \rightarrow 0$. Gravitational time dilation becomes infinite.
- EM and magnetic fields will appear to oscillate more slowly, leading to a gravitational redshift

$$\frac{\lambda}{\lambda_0} = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} = \left(1 - \frac{r_s}{r}\right)^{-1/2}$$

where λ_0 = emitted wavelength, λ = observed (at infinity).

Note, the redshift becomes infinite as $r \rightarrow r_s$.

Event Horizon

Recall that light moves along null geodesics. If we set *ds = 0* the coordinate speed of a light beam moving radially becomes

$$\frac{dr}{dt} = \pm c\left(1 - \frac{2GM}{rc^2}\right) = \pm c\left(1 - \frac{r_s}{r}\right)$$

- What happens if $r >> r_s$?

The speed is c, as expected.

- What happens if $r_s >> r$?

The speed appears to approach 0.

No information can emerge from a radius smaller than r_s, which constitutes an **event horizon** around the black hole.

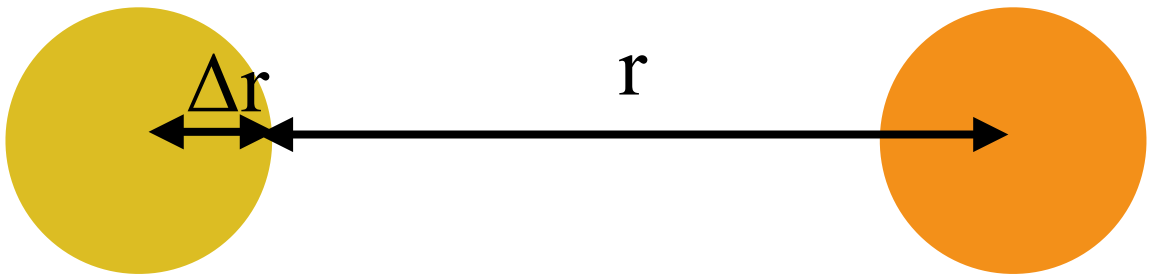
The collapse of matter to r_s takes an infinite amount of time for an observer at infinity (but finite amount of time for someone falling in). As such, the matter is "frozen" in time as it falls in. However, there is no observable differences in frozen stars and truly collapsed black holes. More details can be found on pages 97-98 of your textbook.

Interacting Binaries

- Many objects (including stars) are powered not by nuclear reactions, but by accretion of matter onto gravitational wells.
- We will focus on stars in binaries which will exert forces on each other.
    - Force on center of mass maintains binary orbit.
    - Force is stronger for parts of the star facing towards companion and weaker for parts facing away from companion. This is a result of **tidal forces** that stars exert at small distances.

Tidal Forces:

Forces that cause distortions of equipotential surfaces.



Consider mass element *m* in star 1 at a distance Δr from the center. What is the force on *m* due to star 1?

$$\frac{F_{\rm grav}}{m} = \frac{GM_1}{(\Delta r)^2}$$

What is the tidal force felt by *m* due to star 2 at distance r (assume Δr << r)

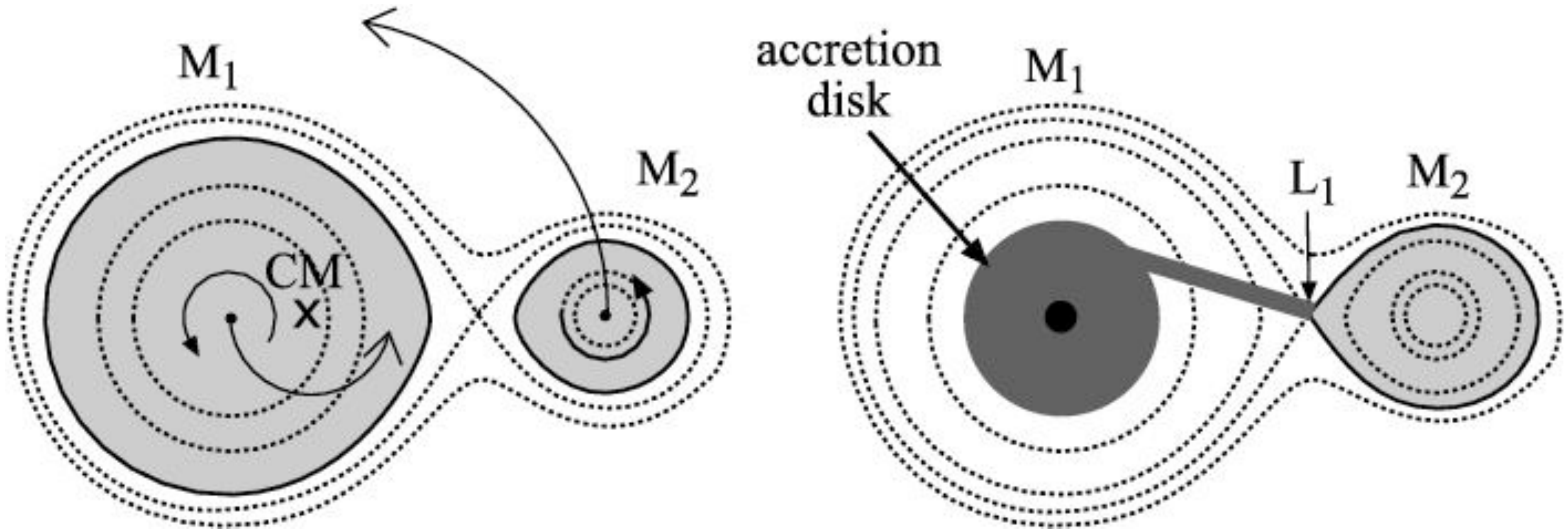
$$\frac{F_{\rm tide}}{m} = GM_2\left(\frac{1}{r^2} - \frac{1}{(r+\Delta r)^2}\right) \approx \frac{2GM_2\Delta r}{r^3}$$

Taking the ratio of forces yields:

$$\frac{F_{\rm tide}}{F_{\rm grav}} = \frac{2M_2}{M_1}\left(\frac{\Delta r}{r}\right)^3$$

Tidal forces are largest when Δr/r is biggest.

Once the stars achieve synchronized, circularized orbits **tidal locking** is achieved. Everything will appear stationary in a frame rotating at binary frequency.

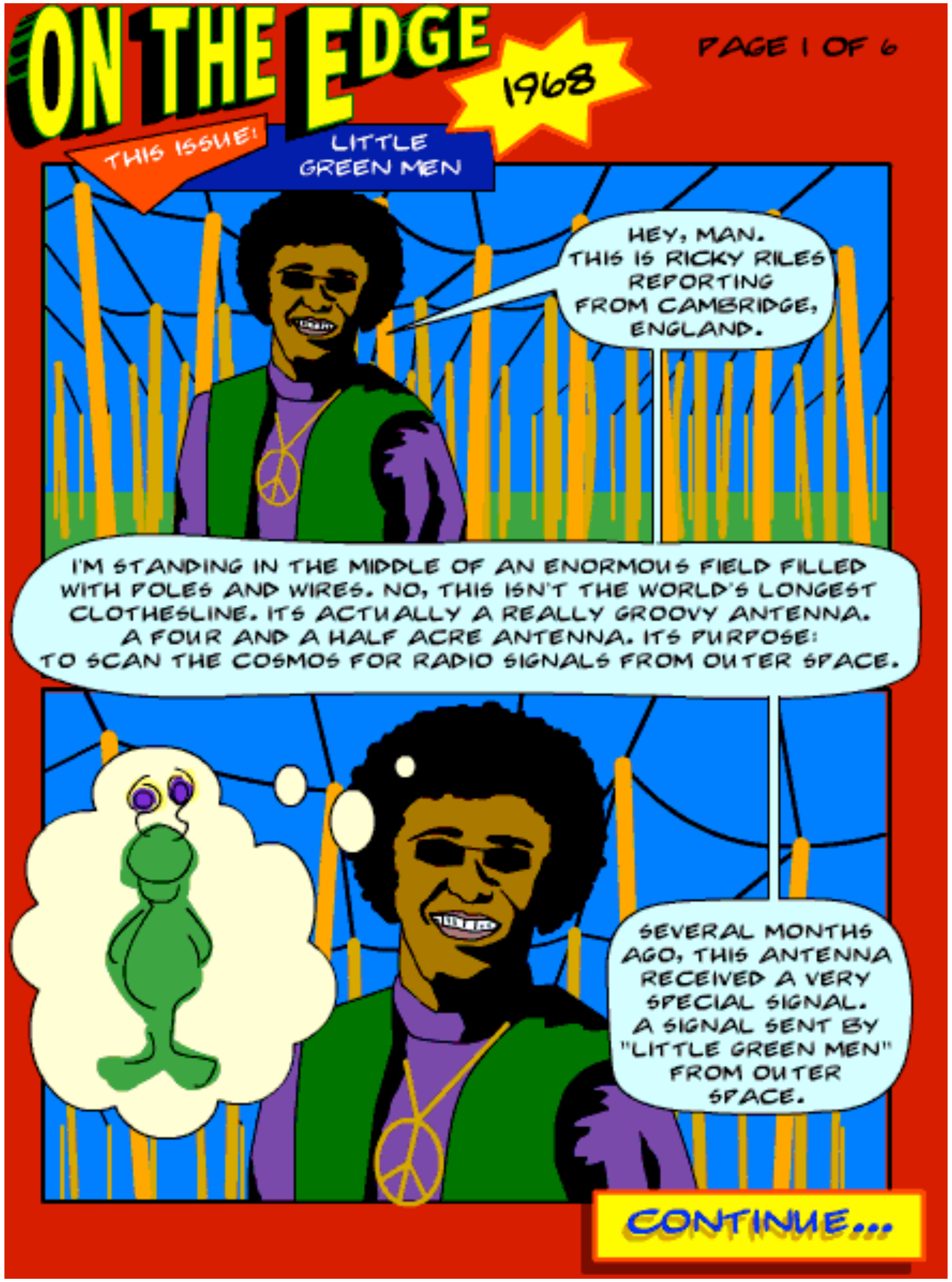


Roche lobes are the deepest non-disjoint equipotential surface in the rotating frame.

Binary systems can be:

- detached: neither star fills its Roche lobe
- semi-detached: one star fills its Roche lobe
- contact: both stars fill their Roche lobes.

If a star fills its Roche lobe, matter transfers via the first Lagrangian point. Matter will have angular momentum and form an accretion disk around the other star.



Stay Tuned ….

Continue reading at:

http://www.pbs.org/wgbh/aso/ontheedge/pulsar/index.html