

# Principles of Astrophysics and Cosmology

Welcome Back  
to PHYS 3368



**Nikolai Shakura**  
**October 7, 1945 -**

**Rashid Sunyaev**  
**March 1, 1943 -**

# Announcements

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- Reading Assignments: Chapter 4.6, & 8.1 - 8.2.
- Problem Set 9 is due in class on Wednesday, April 8th.
- Monday, April 13th: Special lecture about something awesome by Matt Stein. If you are participating in Honors Convocation, you are excused from class that day.
- Wednesday, April 15th in class lab. Be to report to FOSC 032 that day.
- Wednesday, April 15th your final paper is due (hard copy and electronic pdf). Be sure to review the paper guidelines.
- Dr. Cooley will be out of town April 14th - April 17th.
- The final exam in this course will be on Wednesday, May 6th from 6:30 - 8:00 pm. It will cover the second half of the course.

Dear Physics Folks,

We have two special events coming up.

- 1) [The 2015 SMU Physics Symposium: Sunday 12 April](#)
- 2) [Our SPS Awards Celebration Dinner: Monday 13 April](#)

All are welcome at both events. Details are below,  
but please sign up by **Wednesday noon** at the above links.

Thanks,  
Prof. Olness

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### Through a Cosmos Darkly: 2015 SMU Physics Symposium.

We are pleased to invite you to our 2015 SMU Physics Symposium.

**Sunday 12 April 2015** 6:00pm -- 8:30pm,  
Umphrey Lee, Mack Ballroom

Reservations are required.

**THE DEADLINE IS WEDNESDAY NOON!!!**

You may register from the link posted on the main physics page at: <http://www.physics.smu.edu>

The Department is pleased to host all the physics folks (majors/minors/grads/faculty/staff)  
for this event as it also serves as our annual awards ceremony.  
Guests are welcome for the price of the dinner, \$20/person.

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### SPS/Physics Dinner: Peggy Sue's BBQ

We are pleased to invite you to our SPS/Physics Dinner.

**Monday 13 April 2015** 7:30pm  
Peggy Sue's BBQ

All are welcome, but we need a count so please sign up at: <http://doodle.com/d3r3dizg4zpc2vti>

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Links to sign up!



# Goals for Today's Class

- What are the properties of accretion disks?
- How do accretion disks evolve?
- What is a black-widow pulsar?

# Semi-detached Binary

In the case of a semi-detached binary, there is always mass transfer from the Roche-lobe-filling star to its companion.

Binary Type	Receiving Star
Algol-type	main sequence
cataclysmic variables	white dwarf
type Ia supernova	white dwarf
x-ray binary	neutron star or black hole



# Accretion Disks

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To model accretion disks we will assume:

- particles move on an approximate circular orbit
- they lose energy and angular momentum due to viscous interactions with particles on nearby orbits
- frictional heat is radiated away with each disk annulus acting as a blackbody of a given temperature.

Note: The nature of viscosity is still not well known

How does energy change when a mass  $dM$  in an accretion disk around a star of mass  $M$  change when it's orbit goes from radius  $r+dr$  to radius  $r$ ?

$$dE_g = GMdM \left( \frac{1}{r} - \frac{1}{r+dr} \right) \sim \frac{GMdMdr}{r^2}$$

This is only the gravitational potential energy.

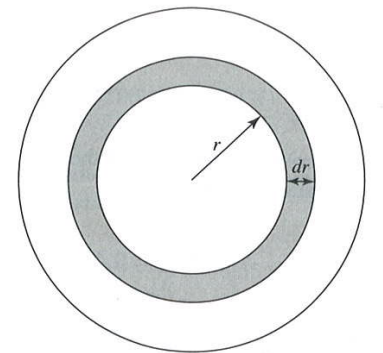
Half of the total energy is the potential energy. We must also consider the thermal energy.

Recall that the viral theorem gives

$$E_{total} = E_{th} + E_{gr} = \frac{E_{gr}}{2}$$

Thus,

$$dE_{th} = \frac{1}{2} \left( \frac{GMdM}{r} - \frac{GMdM}{r+dr} \right) \sim \frac{GMdMdr}{r^2}$$



What is the luminosity?

surface area annulus

accretion rate of  
particular annulus

$$2(2\pi r)dr\sigma T^4 = dL = \frac{dE}{dt} = \frac{1}{2}GM\frac{dM}{dt} \left( \frac{1}{r} - \frac{1}{r+dr} \right) = \frac{1}{2}GM\dot{M}\frac{dr}{r^2}$$

surface area annulus  
includes top and bottom

Solve for T

$$2(2\pi r)dr\sigma T^4 = \frac{1}{2}GM\dot{M}\frac{dr}{r^2}$$

$$T(r) = \left( \frac{GM\dot{M}}{8\pi\sigma} \right)^{1/4} r^{-3/4}$$

Notice,  $T \propto r^{-3/4}$ . This means that the inner regions of the disk are hottest and thus most luminous.

To find the total luminosity of the disk, we integrate over all annuli.

$$L = \int_{r_{\text{in}}}^{r_{\text{out}}} 2(2\pi r)\sigma T^4(r)dr = \frac{1}{2}GM\dot{M} \left( \frac{1}{r_{\text{in}}} - \frac{1}{r_{\text{out}}} \right)$$

In the case the  $r_{\text{out}} \gg r_{\text{in}}$

$$L = \frac{1}{2} \frac{GM\dot{M}}{r_{\text{in}}}$$



## Radiative efficiency:

The fraction of rest mass energy of accreted material that is radiated.

$$\eta = \frac{L}{\dot{M}c^2} = \frac{1}{2} \frac{GM}{c^2 r_{\text{in}}}$$

Accreting Object	Inner Radius of Disk	Radiative Efficiency
neutron star $1.4 M_{\text{sun}}$	10 km	0.10
non-rotating black hole	$3r_s$	0.057
maximally rotating black hole	$0.5r_s$	0.42

The radiative efficiency of nuclear burning is 0.007 or less in main sequence stars.

# Example: White Dwarf

Calculate the typical luminosity of an accretion disc where the accretor is a white dwarf with a mass of  $M_{\text{sun}}$  and radius  $10^4$  km. The typical accretion rate is  $10^{-9} M_{\text{sun}} \text{ yr}^{-1}$ .

$$L = \frac{1}{2} \frac{GM\dot{M}}{r_{\text{in}}} = \frac{6.7 \times 10^{-8} \text{ cgs} \times 2 \times 10^{33} \text{ g} \times 10^{-9} \times 2 \times 10^{33} \text{ g}}{2 \times 3.15 \times 10^7 \text{ s} \times 10^9 \text{ cm}}$$

$$L = 4 \times 10^{33} \text{ erg s}^{-1} \approx L_{\odot}.$$

Calculate the temperature at the inner radius (which dominates the disk).

$$T(r) = \left( \frac{GM\dot{M}}{8\pi\sigma} \right)^{1/4} r^{-3/4} = \left( \frac{6.7 \times 10^{-8} \text{ cgs} \times 2 \times 10^{33} \text{ g} \times 10^{-9} \times 2 \times 10^{33} \text{ g}}{3.15 \times 10^7 \text{ s} \times 8\pi \times 5.7 \times 10^{-5} \text{ cgs}} \right)^{1/4} (10^9 \text{ cm})^{-3/4}$$

$$T(r) = 5 \times 10^4 \text{ K}$$

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In which part of the EM spectrum does this star peak?

$$\lambda_{max} = \frac{0.29 \text{ cm K}}{T} = \frac{0.29 \text{ cm K}}{5 \times 10^4 \text{ K}} = 5.8 \times 10^{-6} \text{ cm} = 58 \text{ nm}$$

This is well into the UV part of the EM spectrum.

Compare this to a neutron star accretor. For a typical neutron star of  $1.4 M_{\text{sun}}$  with radius 10 km we have

- $L \sim 10^{37}$  ergs (vs  $\sim 10^{33}$ )
- $T \sim 10^7 \text{ K}$  (vs  $\sim 10^4$ )
- $\lambda_{\text{max}} \sim 0.58 \text{ nm}$  (x-ray spectrum)

# Accreting White Dwarfs

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**Novae** are a class of cataclysmic variable binary stars. Mass transfers through disk, builds up on WD surface and eventually undergoes nuclear fusion.

- Typical energy  $\sim 10^{46}$  erg.
- Duration  $\sim 1$  month, typical luminosity  $\sim 4 \times 10^{39} \text{ erg s}^{-1}$ .

**Type Ia supernovae** are the runaway version of the nova eruption. Mass builds up on the WD until mass exceeds the Chandrasekhar limit. WD fuses to iron-group elements and explodes.

- Typical energy  $\sim 10^{51-52}$  erg.
- Duration  $\sim 1$  month, typical luminosity  $\sim 10^{43-44} \text{ erg s}^{-1} \sim 10 L_{\text{sun}}$ .
- 99% of energy is carried away by neutrinos (thus, core-collapse SN are far more energetic)
- have a narrow range of observed optical luminosities.
- useful as “standard candles” for measuring distances.

# Eddington Limit

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Consider radiation pressure from an object of luminosity  $L$  acting on ionized inflowing gas. The dominant interaction will be Thomson scatter.

The rate at which an electron scatters photons depends on the # photons per unit area is the energy flux at that frequency divided by the energy of an individual photon.

$$\Sigma = \frac{f_\nu}{h\nu} = \frac{L_\nu}{4\pi r^2 h\nu}$$

The electron will scatter via Thomson scatter at a rate

$$R_{scat} = \sigma_T \frac{L_\nu}{4\pi r^2 h\nu}$$

Each scattering event transfers, on average a momentum to the electron given by

$$p = \frac{h\nu}{c}$$

The force exerted on the electron is then

$$F_\nu = \frac{dp}{dt} = R_{\text{scat}} \frac{h\nu}{c} = \frac{L_\nu \sigma_T}{4\pi r^2 c}$$

The total force is found by integrating over all frequencies,  $\nu$ .

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi r^2 c}$$

Gravitational attraction prevents the electron from being repelled by the accreting source of luminosity. The gravitational force will be felt more strongly by protons, but electrons are attracted to the protons by the coulomb attraction. Thus,

$$F_{\text{grav}} = \frac{GMm_p}{r^2}$$



$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi r^2 c}$$

$$F_{\text{grav}} = \frac{GMm_p}{r^2}$$

The accretion flow will stop if  $F_{\text{rad}} > F_{\text{grav}}$  since the net force on matter in the flow would then be outward. The maximum accretion rate and maximum luminosity occurs when the radiation pressure exactly balances gravity. This is the **Eddington Luminosity**.

$$\frac{L_E \sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2}$$

$$L_E = \frac{4\pi c G M m_p}{\sigma_T}$$

Calculate this limit in terms of the  $M/M_{\text{sun}}$ .

$$L_E = \frac{4\pi \times 3 \times 10^{10} \times 6.7 \times 10^{-8} \text{ cgs} \times 2 \times 10^{33} \text{ g} \times 1.7 \times 10^{-24} \text{ g}}{6.7 \times 10^{-25} \text{ cm}^2} \frac{M}{M_{\odot}}$$

$$= 1.3 \times 10^{38} \text{ erg s}^{-1} \frac{M}{M_{\odot}} = 6.5 \times 10^4 L_{\odot} \frac{M}{M_{\odot}}$$

**Limiting luminosity is called the Eddington Luminosity**

# Notes:

- Our calculations of luminosities for accretion onto neutron stars implies we would get luminosities higher than the Eddington limit. This is not really true. We made an assumptions/simplifications of spherical accretion and an isotropically radiating source.
- Matter is taken in along an equatorial plan and radiates preferentially in directions perpendicular to the plane..
- Detailed calculations show that accretion disks become unstable when radiating near  $L_E$ .
- $L_E$  applies to systems undergoing steady-state accretion.

# Evolution of Interacting Binary Systems

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Recall that isolated neutron stars power their pulsar emission and their surrounding SN remnant emission at the expense of their rotational energy.

Neutron stars in binary systems that are accreting matter from a companion can GAIN angular momentum.

The jets and beams present in pulsars can hit one side of the donor star, heat it, ablate it or completely destroy it. These pulsars are known as *black-widow pulsars*. Example: a millisecond pulsars with no companion.

<http://www.nasa.gov/content/goddard/with-a-deadly-embrace-spider-pulsars-consume-their-mates/#.VSKgpEaRqC8>

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Let's examine the changes in evolution to of the parameters in a binary system.

The orbital angular momentum of a circular binary composed of  $M_1$  and  $M_2$  with separation distance  $a$ .

$$J = I\omega = \mu a^2 \omega$$

where  $I$  is the moment of inertia and  $\mu$  is the reduced mass,

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

Recall Kepler's law (from chapter 2).

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$

Substituting yields

$$J = \mu a^2 \frac{\sqrt{G(M_1 + M_2)}}{a^{3/2}} = \mu \sqrt{G(M_1 + M_2)a}$$

$$J = \mu \sqrt{G(M_1 + M_2)a}$$


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Conservation of total mass and angular momentum require

$$\frac{dJ}{dt} = 0$$

Which term(s) are chaining with time? Need to invoke the chain rule.

$$\frac{dJ}{dt} = \sqrt{G(M_1 + M_2)} \left( \frac{d\mu}{dt} \sqrt{a} + \frac{\mu}{2\sqrt{a}} \frac{da}{dt} \right) = 0$$

$$-\sqrt{a} \frac{d\mu}{dt} = \frac{\mu}{2\sqrt{a}} \frac{da}{dt}$$

$$-\frac{2}{\mu} \frac{d\mu}{dt} = \frac{1}{a} \frac{da}{dt}$$



$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

$$-\frac{2}{\mu} \frac{d\mu}{dt} = \frac{1}{a} \frac{da}{dt}$$


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Examine  $d\mu/dt$

$$\frac{d\mu}{dt} = \frac{1}{M_1 + M_2} \left( \frac{dM_1}{dt} M_2 + M_1 \frac{dM_2}{dt} \right)$$

However, conservation mass requires that

$$\dot{M}_1 = -\dot{M}_2$$

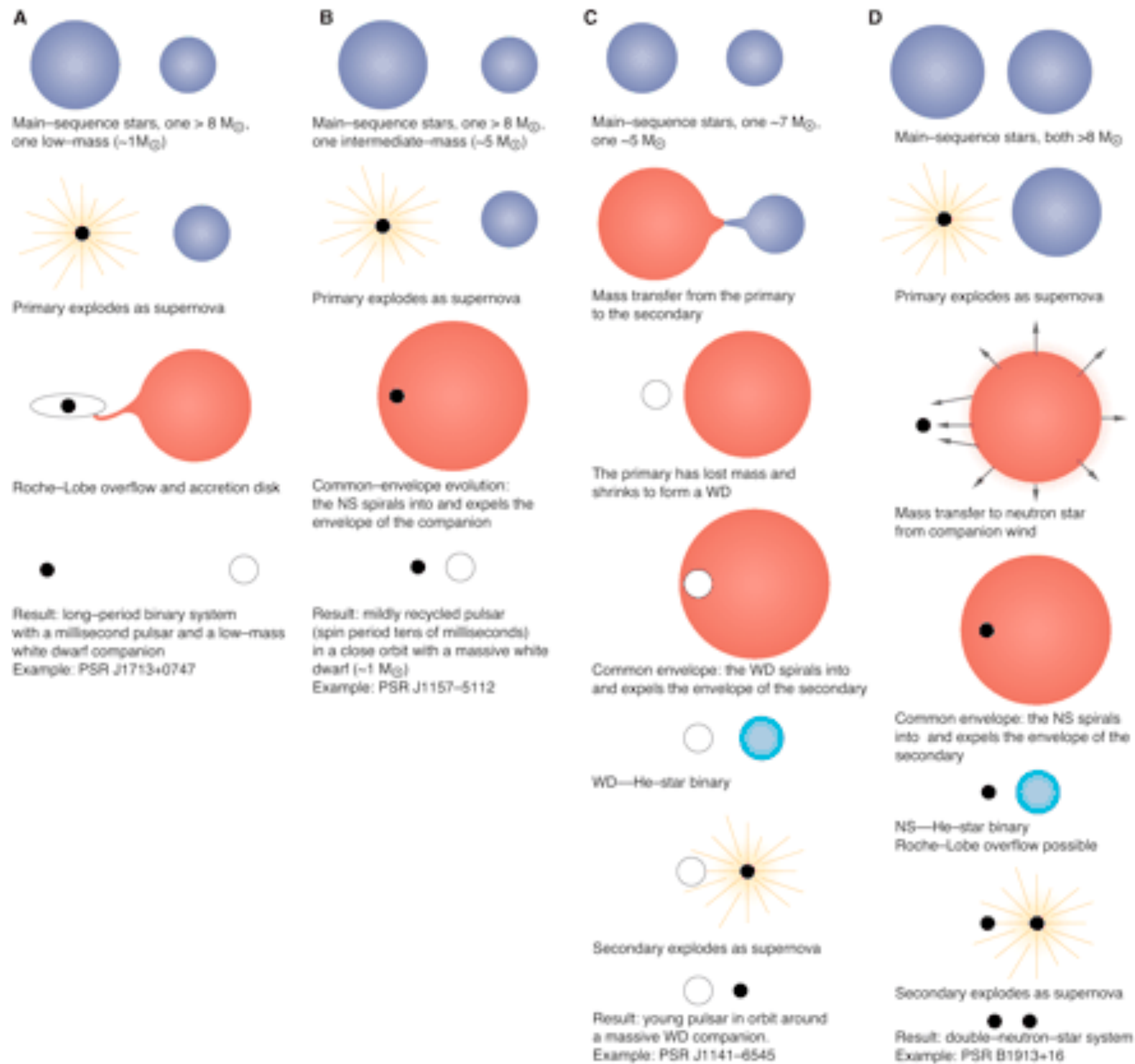
Thus, we can write

$$\frac{d\mu}{dt} = \frac{\dot{M}_1}{M_1 + M_2} (M_2 - M_1)$$

Substituting yields

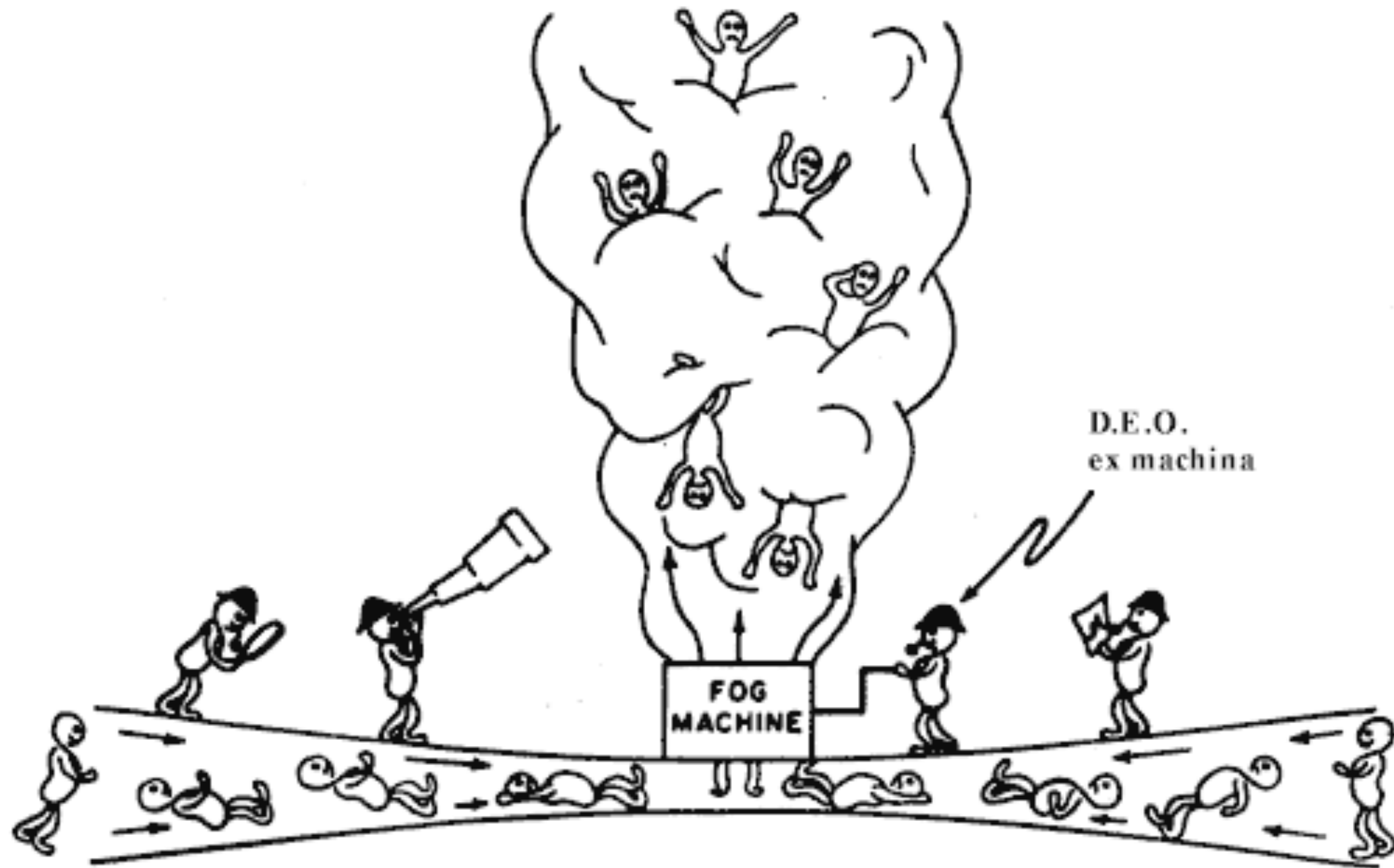
$$2\dot{M}_1 \frac{M_1 - M_2}{M_1 M_2} = \frac{1}{a} \frac{da}{dt}$$

This equation determines how period and separation evolve.



<http://www.sciencemag.org/content/304/5670/547/F1.expansion>

# Stay Tuned ....



<http://laserstars.org/humour/Fashionable.html>