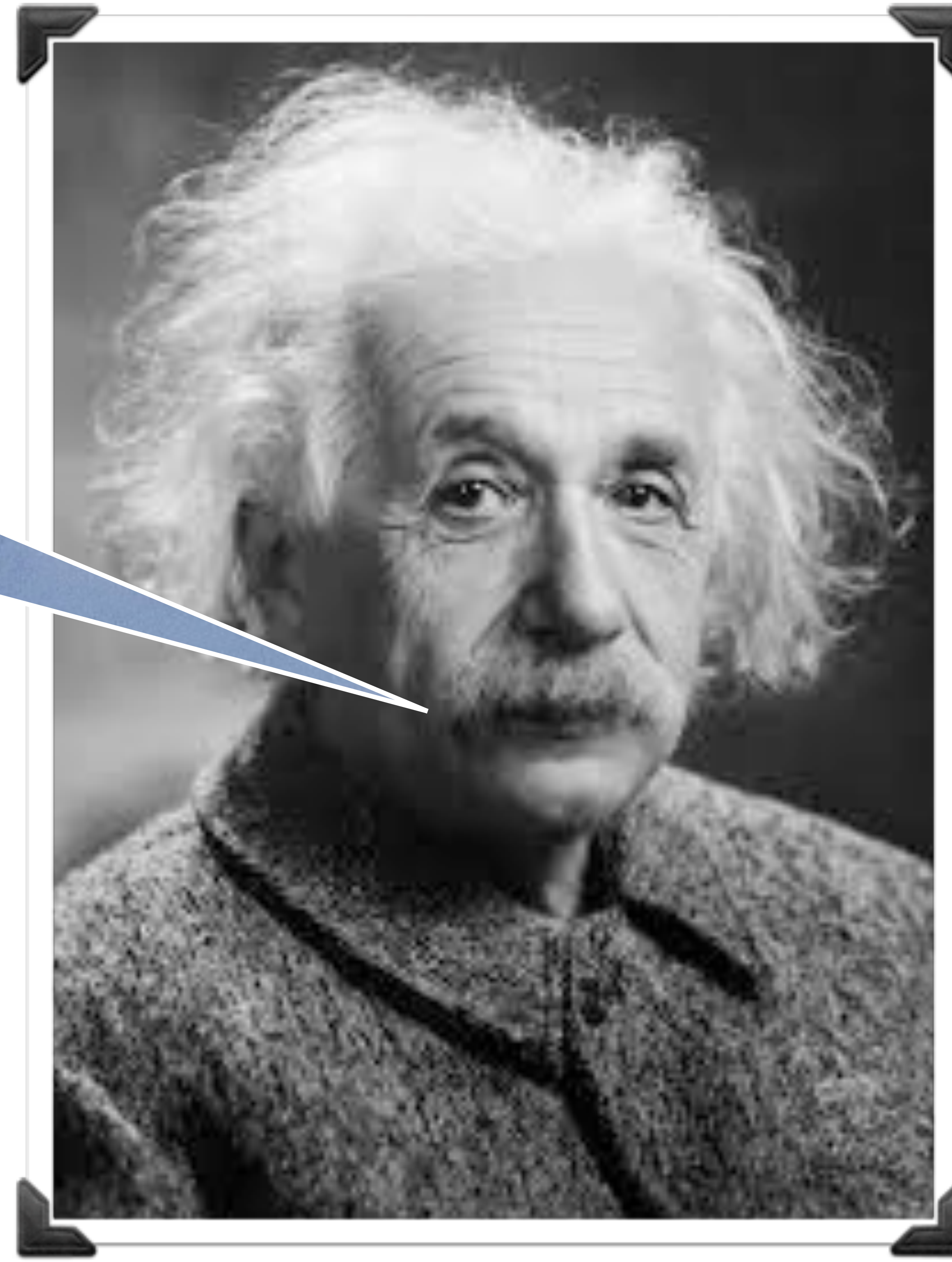
Principles of Astrophysics and Cosmology

Welcome Back to
PHYS 3368



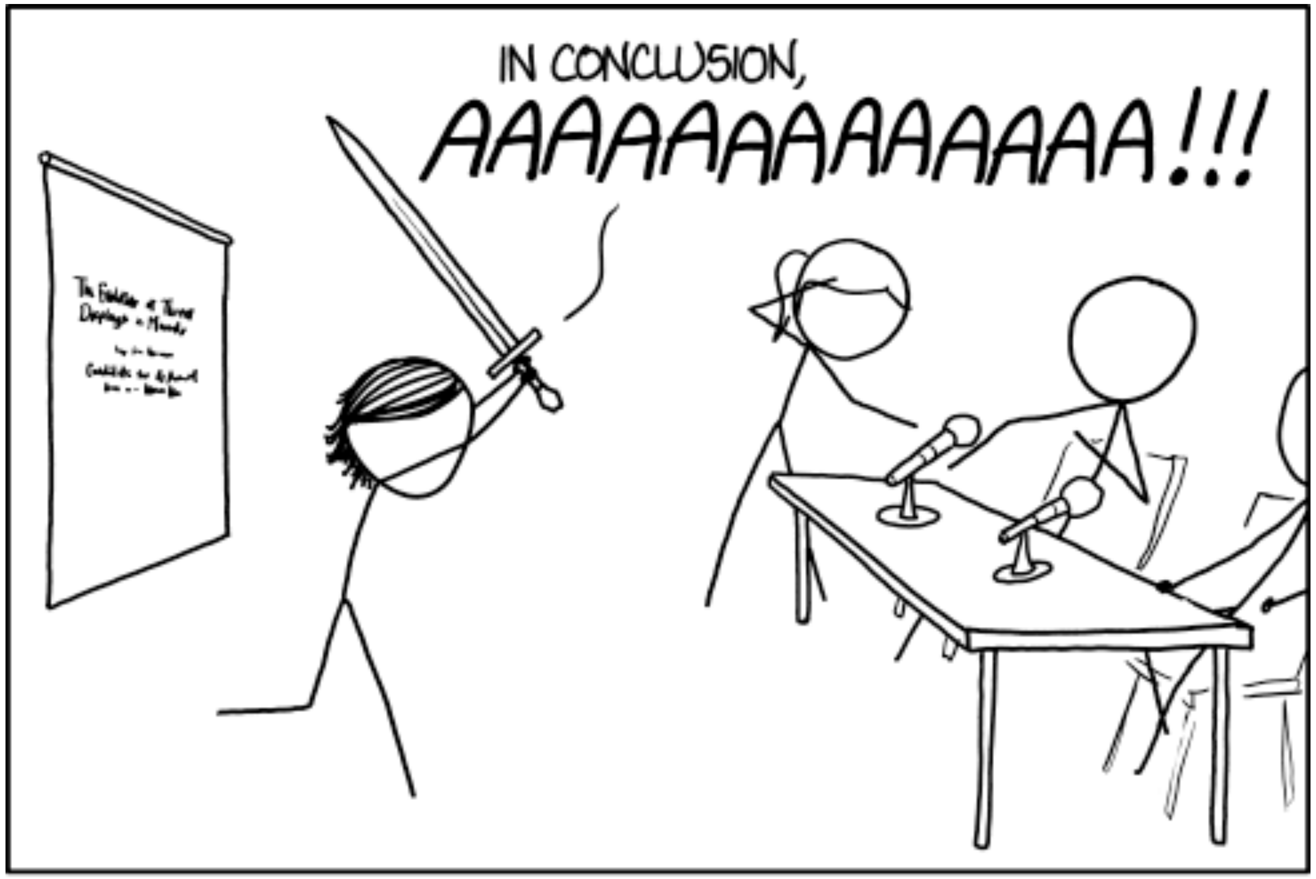
Albert Einstein
March 14, 1879 -
April 18, 1955

Announcements

- Reading Assignments: Chapters 8.3, 8.5 & 9.1 - 9.2.
- Problem Set 11 is due in class on Wednesday, April 29th.
- Monday, April 27th in class lab. Be to report to FOSC 032 that day.
- Final paper re-writes are due in class on Monday, May 4th. A pdf must be emailed to Prof. Cooley before 6 pm and you must also submit a hard copy in class before 6 pm that day.
- Problem Set 12 is due by 4 pm on Tuesday, May 5th. You may turn your problem set into Lacey Porter in the main office during regular business hours. She will provide you a copy of the solutions.
- The final exam in this course will be on Wednesday, May 6th from 6:30 - 8:00 pm. It will cover the second half of the course.

Support Your Classmates

- Matthew and Mayisha will be giving their senior thesis defenses this Friday at 1:30 pm in FOSC 157.



THE BEST THESIS DEFENSE IS A GOOD THESIS OFFENSE.

Rough Lecture Outline

- April 22: History of the Universe and Dark Energy (8.3, 8.5)
- April 27: April 27: Last Lab Day!
- April 29: Redshift and the CMB as tests (9.1 - 9.2) and CMB Anisotropy (9.3)
- May 4: Nucleosynthesis & Quasars (9.4 - 9.5)

Today's Lecture

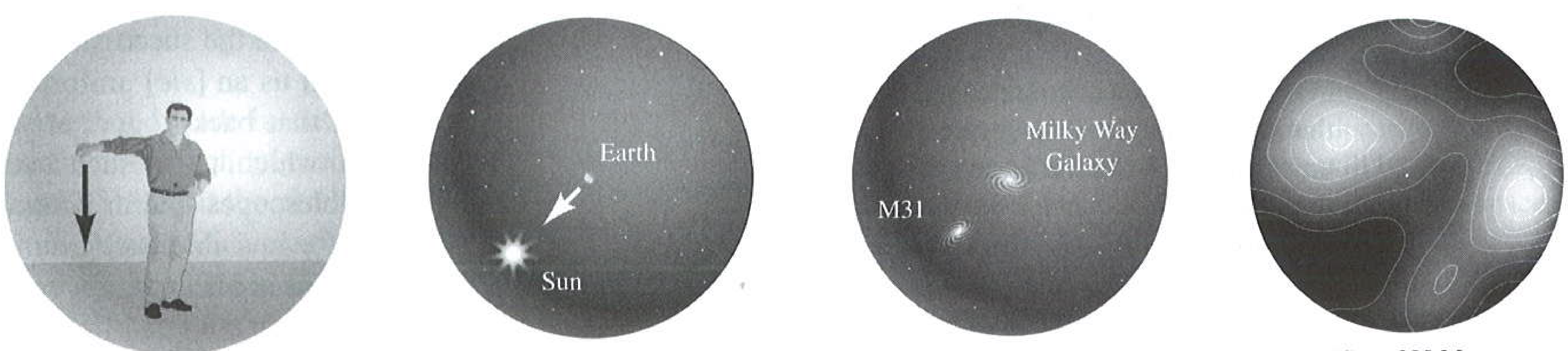
- How did it all begin?
- Where will it all end?
- What is dark energy?
  (Hint: No one knows….)

Last Time:

The **Cosmological Principle** postulates that the Universe is **isotropic** and **homogenous**.

- Isotropic: At large enough scales the Universe is the same in all directions. There is no preferred direction in space.

- Homogenous: Every observer sees the same expansion in space. There is no preferred location.



(a) — 3 m (b) — 3 AU (c) — 3 Mpc (d) — 200 Mpc

If space is homogenous, it must have the same curvature everywhere. Three possibilities: flat, positive curved or negative curved.

Last Time:

We have the FRW metric:

$$ds^2 = c^2 dt^2 - R^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

We have 3 Friedmann Equations:

$$\left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

sum of potential and kinetic energy is conserved locally

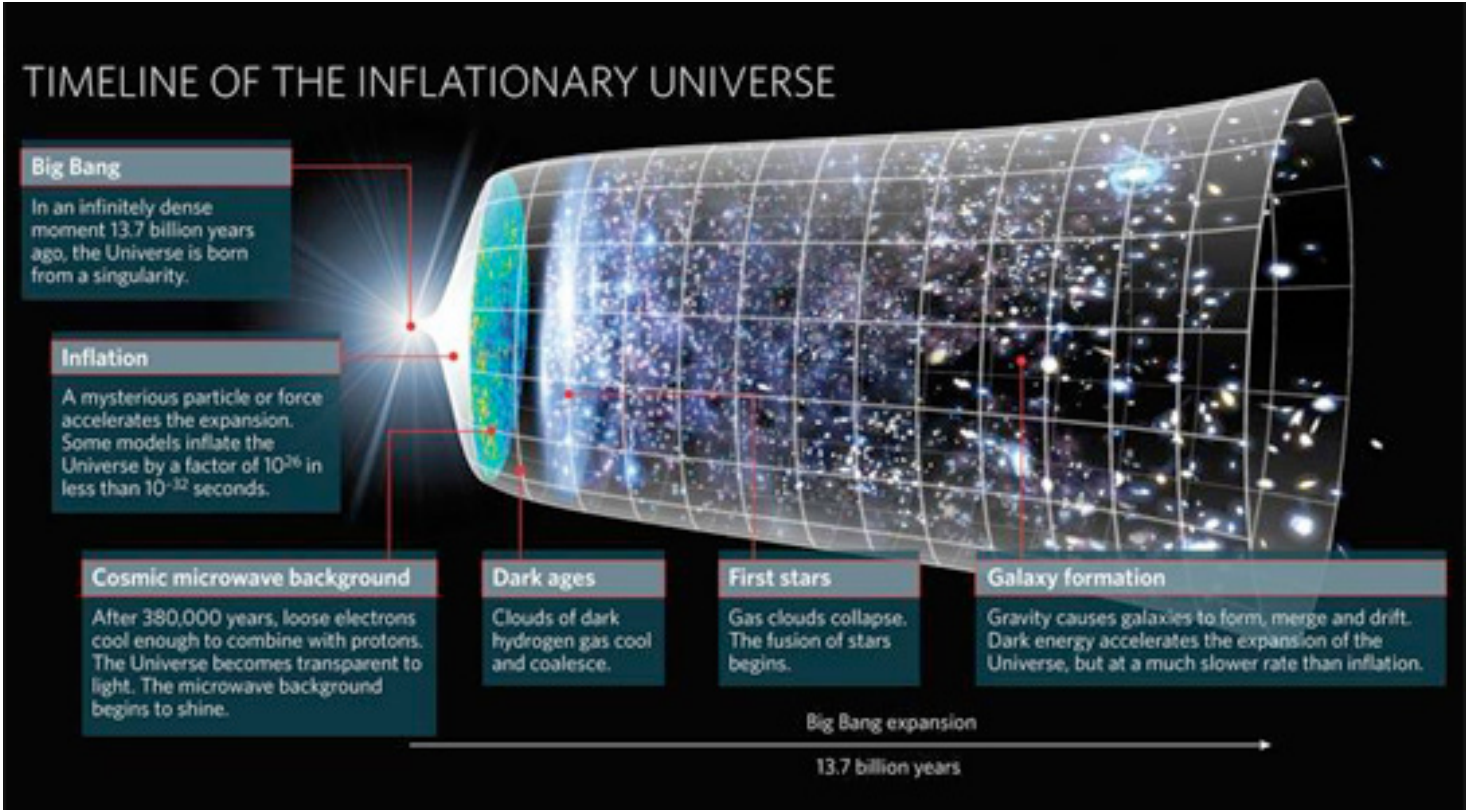
$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2} (\rho c^2 + 3P)$$

equation of motion under the influence of gravity

$$\dot{\rho} c^2 = -3 \frac{\dot{R}}{R} (\rho c^2 + P)$$

the fluid equation

History of the Universe



Mathematical Description

Solutions to the Friedmann equations give us a mathematical description of the history and fate of the universe.

Re-examine the equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P)$$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)$$

Only two of these equations are independent. The second can be derived from the first and third.

We have 3 unknowns: R (scale factor), ρc^2 (density), and P (pressure). These unknowns are each a function of time.

We need an ***equation of state***: a mathematical relationship between the pressure and the energy density of "stuff" that fills up the universe.

We will consider 2 cases:

Case 1: Matter dominated - pressure << matter density

What happens to the third equation in this case?

$$\dot{\rho}\cancel{c^2} = -3\frac{\dot{R}}{R}(\rho\cancel{c^2} + \cancelto{0}{P})$$

$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{R}}{R}$$

This equation has the solution

$$\rho \propto R^{-3}$$

Case 2: Radiation dominated - dominant density comes from ultra-relativistic particles. In this case the pressure is 1/3 of the energy density.

$$P = \frac{1}{3}u = \frac{1}{3}\rho c^2$$

What happens to our Friedmann Equation in this case?

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)$$

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + \frac{1}{3}\rho c^2)$$

$$\dot{\rho}c^2 = -4\frac{\dot{R}}{R}\rho c^2$$

$$\frac{\dot{\rho}}{\rho} = -4\frac{\dot{R}}{R}$$

The solution to this equation is — $\rho \propto R^{-4}$

matter dominated: $\rho \propto R^{-3}$ radiation dominated: $\rho \propto R^{-4}$

Now let's consider the scale factor *R(t)*. Examine the first Friedmann equation.

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

Since ρ goes as both R^{-3} and R^{-4}, we can find a time where the R^{-2} term can be neglected.

$$\frac{8\pi}{3}G\rho \gg \left|\frac{kc^2}{R^2}\right|$$

Now consider the matter dominated era

$$\left(\frac{\dot{R}}{R}\right)^2 \sim \frac{1}{R^3} \longrightarrow R^{1/2}dR \sim dt$$

$$R^{3/2} \propto t \longrightarrow R(t) \propto t^{2/3}$$

matter dominated: $R(t) \propto t^{2/3}$ radiation dominated: $\rho \propto R^{-4}$

Now consider the radiation dominated era

$$\left(\frac{\dot{R}}{R}\right)^2 \sim \frac{1}{R^4} \longrightarrow R\,dR \propto dt$$

$$R^2 \propto t \longrightarrow R(t) \propto t^{1/2}$$

Compare the scale factors between the two eras. The expansion is slower during the radiation dominated era. Any ideas why?

The gravitating effect of radiation pressure slows the expansion down in the radiation dominated era.

Examine what happens as we go back to the earliest times:

$$\lim_{t \to 0} R(t) = 0$$

$$\lim_{t \to 0} \rho(t) = \lim_{t \to 0} R^{-4} = \infty$$

The singularity in the density is the Big Bang!

Examine the Future

We can no longer ignore the curvature term, k. So, we have 3 cases: k = -1, 0 ,+1

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

Case: Flat Universe, k = 0

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho$$

Recall the Hubble parameter: $H \equiv \frac{\dot{R}}{R}$

$$\rho = \frac{3H^2}{8\pi G}$$

This implies a critical density for each moment in time.

What is the current critical density? In cgs units, H_0 is presently 2.3×10^{-18} s^{-1}.

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = \frac{3(2.3 \times 10^{-18}\ \mathrm{s}^{-1})^2}{8\pi \times 6.7 \times 10^{-8}\ \mathrm{cgs}} = 9.2 \times 10^{-30}\ \mathrm{g\ cm}^{-3} = 1.4 \times 10^{11} M_\odot\ \mathrm{Mpc}^{-3}$$

The matter density of the Universe is often quoted in terms of the critical density.

$$\Omega_m \equiv \frac{\rho}{\rho_c}$$

$\Omega_m = 1$ for a flat universe.

Assume that we are in the matter dominated era, then

$$R(t) \propto t^{2/3}$$

$$\dot{R} \propto t^{-1/3}$$

The scale factor grows as a function of time (forever) and gradually slows down until time = infinity!

The fate of the universe: It expands forever.

Case: Positively Curved Universe, k = +1

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

As R grows, the density ρ goes down. At some point we will have

$$\frac{8\pi}{3}G\rho = \frac{kc^2}{R^2}$$

Solving this equation for R gives us the radius at which this occurs.

$$R = \left(\frac{3c^2}{8\pi G\rho}\right)^{1/2}$$

At that time the expansion halts.

$$\left(\frac{\dot{R}}{R}\right)^2 = 0$$

$$\left(\frac{\dot{R}}{R}\right)^2 = 0 \qquad \frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P)$$

Note: The negative term in the third Friedmann equation does not change. What happens next?

The expansion stops, and then contraction begins.

The fate of the Universe: **The Big Crunch**!

For a positively curved universe the volume is finite but unbound. It is called a **closed** universe.

Case: Negatively Curved Universe, k = -1

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2}$$

Given sufficient time, the curvature term will dominate over the density term.

$$\left(\frac{\dot{R}}{R}\right)^2 \sim \frac{c^2}{R^2} \longrightarrow \dot{R} = c.$$

What does that mean?

The universe is **open** and will expand forever.

Radiation-to-Matter Domination

When does the Universe transition from radiation dominated to matter dominate?

The radiation density at any time is related to the radiation density today by

$$\rho_{rad} R^4 = \rho_{rad,0} R_0^4$$

$$\rho_{\rm rad} = \rho_{\rm rad,0} \frac{R_0^4}{R^4}$$

Similarly, for matter density

$$\rho_{\rm m} c^2 = \rho_{\rm m,0} c^2 \frac{R_0^3}{R^3}$$

Setting the two equal, we find

$$\frac{R_0}{R} = \frac{\rho_{\rm m,0} c^2}{\rho_{\rm rad,0}}$$

What are the densities today?

We can find the radiation density in the universe today using the cosmic microwave background. We know the CMB has a blackbody spectrum at temperature $T_0 = 2.73K$.

$$\begin{aligned}\rho_{\mathrm{rad},0} &= aT_0^4 \\ &= 7.6\times 10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}\times(2.73\,\mathrm{K})^4 \\ &= \underline{4.2\times 10^{-13}\,\mathrm{erg\,cm^{-3}}}\end{aligned}$$

The matter density of the Universes is 0.3 times the critical density.

$$\begin{aligned}\rho_{\mathrm{m},0}c^2 \approx 0.3\rho_{\mathrm{c},0}c^2 &= 0.3\times 9.2\times 10^{-30}\,\mathrm{g\,cm^{-3}}\times(3\times 10^{10}\,\mathrm{cm\,s^{-1}})^2 \\ &= \underline{2.5\times 10^{-9}\,\mathrm{erg\,cm^{-3}}}\end{aligned}$$

Clearly, we are in a matter dominated era.

When did the transition occur?

The transition occurred when the scale factor R was smaller than its present value R_0 by

$$\frac{R_0}{R} = \frac{\rho_{\mathrm{m},0} c^2}{1.7 \rho_{\mathrm{rad},0}} = 3500$$

Accounts for the energy density due to the cosmic neutrino background.

Thus, the time of the transition was

$$t = \frac{t_0}{(3500)^{3/2}}$$

Now, we need to calculate the age of the Universe, t_0.

Since, all but a very small fraction of the age of the universe, matter has dominated, we will make the simplification that at all times

$$\rho \sim R^{-3}$$

<u>Case: $\rho_0 = \rho_{c,0}$ and k = 0</u>

$$H^2(t) \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho \propto R^{-3}$$

Then, at the present time, we have

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{c,0} \longrightarrow \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\frac{3H_0^2}{8\pi G}\frac{R_0^3}{R^3}$$

$$\left(\frac{\dot{R}}{R}\right)^2 = H_0^2\frac{R_0^3}{R^3} \longrightarrow \boxed{t_0 = \frac{2}{3}H_0^{-1}}$$

Case: $\rho_0 = 0$ and k = -1

$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{kc^2}{R^2} \longrightarrow \ddot{R} = 0 \longrightarrow \dot{R} = \mathrm{const}$$

We know,

$$H = \frac{\dot{R}}{R} \longrightarrow \dot{R} = HR = H_0 R_0 = \frac{dR}{dt}$$

Solving,

$$\int_0^{R_0} \frac{dR}{H_0 R_0} = \int_0^{t_0} dt \longrightarrow \boxed{t_0 = H_0^{-1}}$$

Thus, the age of the Universe is in the range:

$$\frac{2}{3}H_0^{-1} < t_0 < H_0^{-1}$$

Since $H_0 = 70$ km s^{-1} Mpc^{-1}, $H_0^{-1} = 14$ Gyr. The age of the Universe is thus, between 9 and 14 Gyr for this range of the density parameter $\Omega_{m,0}$.

The transition from radiation-dominated to matter-dominated expansion occurred at the time:

$$t = \frac{t_0}{(3500)^{3/2}} = \frac{14 \times 10^9}{(3500)^{3/2}} \sim 68,000\, yr$$

Learning to Love Λ

Einstein added a term to his theory of general relativity to "hold back" gravity and achieve a static universe. This term is called the **cosmological constant**.

The addition of this constant changes the first two Friedmann Equations.

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\rho c^2 + 3P) + \frac{\Lambda}{3}$$

Note: This equation implies that deceleration must always occur — unless Λ is large enough. Then acceleration can occur.

$$\dot{\rho}c^2 = -3\frac{\dot{R}}{R}(\rho c^2 + P)$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

The cosmological constant acts as an energy density:

$$\epsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda$$

units of Λ are $[t^{-2}]$

If Λ is constant, ε_Λ must also be constant as R grows with time. What happens to the first Friedmann equation?

Once R is big enough, the cosmological constant will dominate the RHS.

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 \approx \frac{\Lambda}{3} \longrightarrow \dot{R} \approx \left(\frac{\Lambda}{3}\right)^{1/2} R.$$

$$R(t) \propto \exp\left[\left(\frac{\Lambda}{3}\right)^{1/2} t\right] = \exp(Ht)$$

$$R(t) \propto \exp\left[\left(\frac{\Lambda}{3}\right)^{1/2} t\right] = \exp(Ht)$$

Implications:

1. The universe will enter a phase where it's expand exponentially and this expansion is accelerating.

2. During the exponential expansion phase, the particle horizon tends to a constant coordinate r_h. This means that there is a fixed limit, beyond which light will never reach us.

3. Observers in an exponentially expanding Universe are surrounded by an **event horizon**. This bounds the space by which they can interact causally.

So, Where do we Live?

We live in a universe where space is flat (k = 0) and the cosmological constant is non-zero. This is described by

$$\frac{H^2}{H_0^2} = \frac{8\pi}{3H_0^2}G\rho + \frac{\Lambda}{3H_0^2}$$

Recall

$$\rho_{\mathrm{c},0} = \frac{3H_0^2}{8\pi G}$$

Substitute

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{\mathrm{c},0}} + \frac{\Lambda}{3H_0^2}$$

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{c,0}} + \frac{\Lambda}{3H_0^2}$$

We can rewrite using the density parameter and an analogous parameter for Λ

$$\Omega_m \equiv \frac{\rho}{\rho_c} \qquad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}$$

Substitute

$$1 = \Omega_{\rm m,0} + \Omega_{\Lambda,0}$$

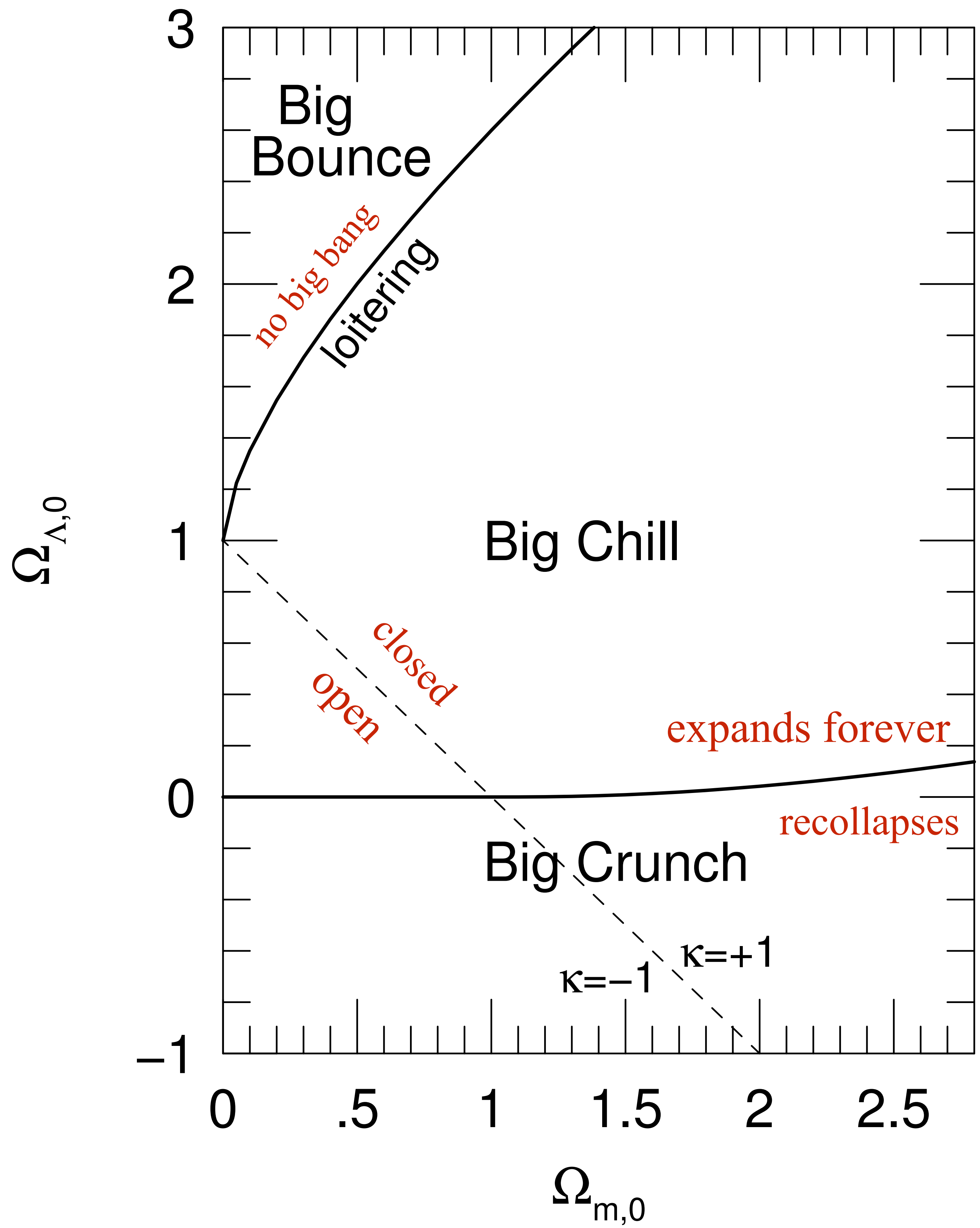
Since the same argument can be made at anytime, for k=0 we always have

$$\Omega_{\rm m} + \Omega_\Lambda = 1$$ for open, flat universe

What if we had a different geometry?

$\Omega_m + \Omega_\Lambda > 1$ for closed, positive curvature

$\Omega_m + \Omega_\Lambda < 1$ for open, negative curvature


3
2
1
0
−1
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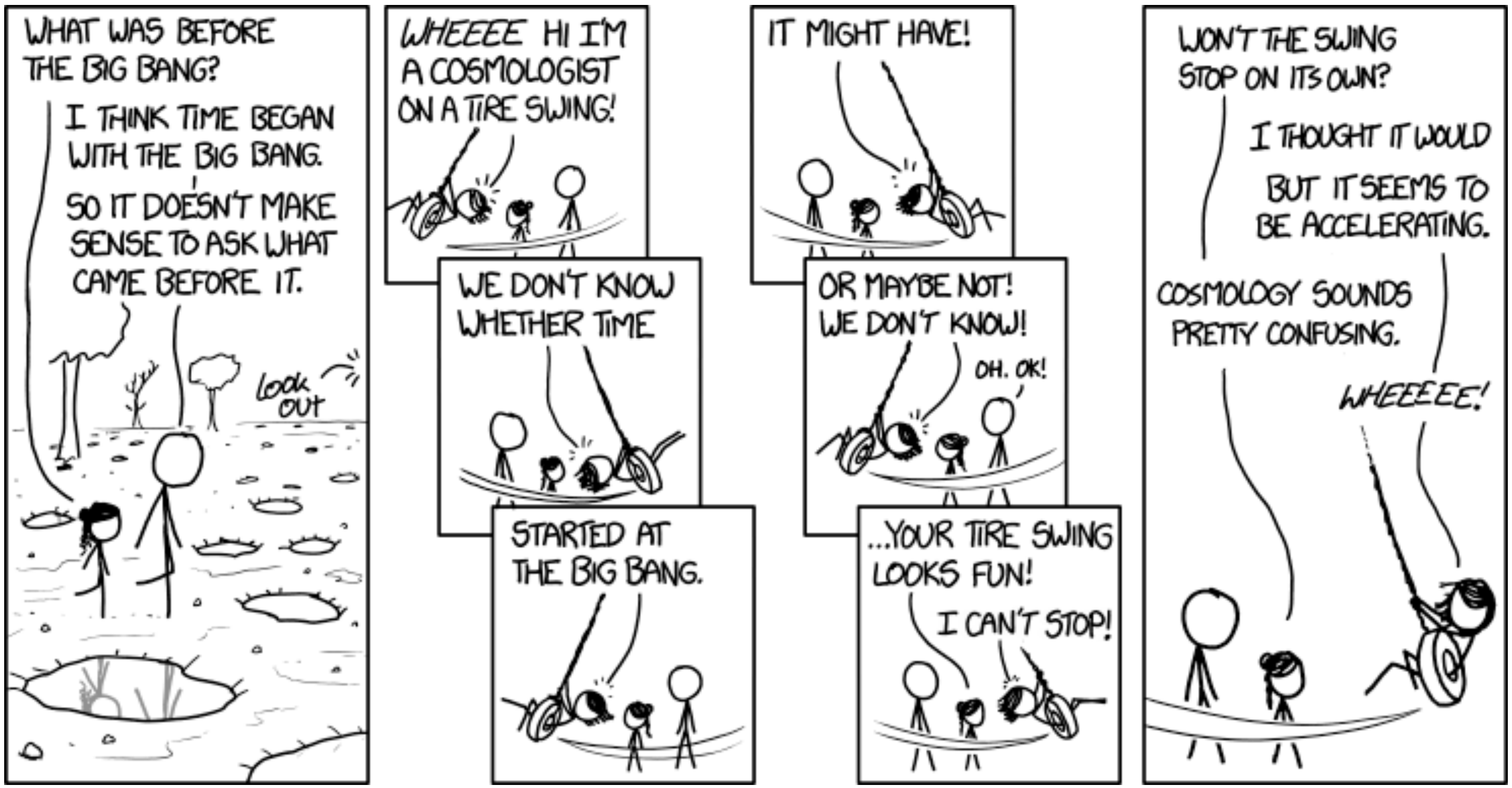
.5
1
1.5
2
2.5
Big Bounce
no big bang
loitering
Big Chill
closed
open
expands forever
recollapses
Big Crunch
κ=−1
κ=+1
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$


WHAT WAS BEFORE THE BIG BANG?
I THINK TIME BEGAN WITH THE BIG BANG.
SO IT DOESN'T MAKE SENSE TO ASK WHAT CAME BEFORE IT.
LOOK OUT
WHEEEE HI I'M A COSMOLOGIST ON A TIRE SWING!
WE DON'T KNOW WHETHER TIME
STARTED AT THE BIG BANG.
IT MIGHT HAVE!
OR MAYBE NOT! WE DON'T KNOW!
OH. OK!
...YOUR TIRE SWING LOOKS FUN!
I CAN'T STOP!
WON'T THE SWING STOP ON ITS OWN?
I THOUGHT IT WOULD
BUT IT SEEMS TO BE ACCELERATING.
COSMOLOGY SOUNDS PRETTY CONFUSING.
WHEEEEE!