Principles of Astrophysics and Cosmology

Welcome Back to PHYS 3368

Arno Allan Penzias
April 26, 1933 -

Robert Woodrow Wilson
January 10, 1936 -
Announcements

- Reading Assignments: Chapter 9.

- Final paper re-writes are due in class on Monday, May 4th. A pdf must be emailed to Prof. Cooley before 6 pm and you must also submit a hard copy in class before 6 pm that day.

- Problem Set 12 is due by 4 pm on Tuesday, May 5th. You may turn your problem set into Lacey Porter in the main office during regular business hours. She will provide you a copy of the solutions.

- The final exam in this course will be on Wednesday, May 6th from 6:30 - 8:00 pm. It will cover the second half of the course.
Rough Lecture Outline

- April 29:  Redshift and the CMB as tests (9.1 - 9.2) and CMB Anisotropy (9.3)
- May 4:  CMB Anisotropy, Nucleosynthesis & Quasars (9.3 - 9.5)
Today’s Lecture

- What is the cosmological redshift and how does it differ from what we know?
- What is the CMB and where did it come from?
- How and why is the CMB so isotropic?
Review Question

True or False. Justify your answer.
The inclusion of the cosmological constant in the Friedmann equations guarantees that the Universe will eventually collapse upon itself.

FALSE: Examine the first two Friedmann Equations.

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G\rho - \frac{k c^2}{R^2} + \frac{\Lambda}{3}
\]

Expansion or contraction depends upon density, curvature and \( \Lambda \).

\[
\frac{\dot{R}}{R} = -\frac{4\pi G}{3 c^2} (\rho c^2 + 3P) + \frac{\Lambda}{3}
\]

If \( \Lambda \) is large enough, deceleration of the expansion will occur.
0.5 \Omega_m,0, 0.5 \Omega_\Lambda,0

\kappa = -1, \kappa = +1

expands forever
recollapses
lottering
Big Chill
Big Crunch
Big Bounce

Last Time:
Last Time:

**History of the Universe**

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**TIMELINE OF THE INFLATIONARY UNIVERSE**

**Big Bang**
In an infinitely dense moment 13.7 billion years ago, the Universe is born from a singularity.

**Inflation**
A mysterious particle or force accelerates the expansion. Some models inflate the Universe by a factor of $10^{26}$ in less than $10^{-32}$ seconds.

**Cosmic microwave background**
After 380,000 years, loose electrons cool enough to combine with protons. The Universe becomes transparent to light. The microwave background begins to shine.

**Dark ages**
Clouds of dark hydrogen gas cool and coalesce.

**First stars**
Gas clouds collapse. The fusion of stars begins.

**Galaxy formation**
Gravity causes galaxies to form, merge and drift. Dark energy accelerates the expansion of the Universe, but at a much slower rate than inflation.

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Principles of Astrophysics & Cosmology - Professor Jodi Cooley
Experimental Predictions

- We will study three experimental predictions of the cosmological model and their observational verification.
  - Cosmological Redshift
  - The Cosmic Microwave Background (CMB)
  - Nucleosynthesis
Review: Hubble Diagram

Galaxies are recessing away with speed \( v \), proportional to distance \( D \).

Hubble’s Law:

\[
v = H_0 D
\]

H_0 = Hubble’s Constant

= 70 km s\(^{-1}\) Mpc\(^{-1}\)
Cosmological Redshift

- Not the traditional Doppler shift.
- As light travels from distant galaxy, space expands.
- Result: Wavelength of light increases (frequency decreases).
- The light is redshifted.
Consider light from a distant galaxy at comoving radial coordinate \( r_e \).

**Wavefront 1:**
- Emission time: \( t_e \)
- Arrival time: \( t_0 \)

**Wavefront 2:**
- Emission time: \( t_e + \Delta t_e \)
- Arrival time: \( t_0 + \Delta t_0 \)

**FWR Metric:**
\[
0 = c^2 dt^2 - R(t)^2 \frac{dr^2}{1 - kr^2}
\]

The cosmological redshift is the ratio
\[
\frac{\Delta t_0}{\Delta t_e} = \frac{\lambda_0}{\lambda_e} = \frac{\nu_e}{\nu_0} = \frac{R(t_0)}{R(t_e)} \equiv 1 + z
\]
Comments on Redshift

- Cosmological redshift is due to expansion of the universe.
  - Redshift can also be due to relative motions (Doppler effect) or gravitation (gravitational redshift).
- Redshift can be measured using spectral lines (absorption or emission).
- In an expanding universe, $R(t_0) > R(t_e)$. Thus, $z$ is always redshifted (rather than blueshifted).
Optical Spectra

![Graph showing optical spectra with different redshifts (z=0.087, z=0.165, z=0.300, z=0.371). The graph includes lines for Hβ (λ4861), [O III] (λλ4959, 5007), and Hα (λ6563). The observed wavelength is plotted on the x-axis, and the flux density is plotted on the y-axis.]
- The scale factor $R(t)$ depends upon $H_0$, $k$, $\Omega_m$, and $\Omega_\Lambda$.
  - So, if we can measure $R(t)$ at different times in the universe’s history, we can deduce information about our universe.
  - Impossible to measure $R(t)$ directly.

- However, we can measure the cosmological redshift $z$ and that gives the ratio between the scale factor of today and when the light was emitted.
  - So, we can deduce cosmological parameters by measuring properties that depend on $R(t)$ of distant objects through $z$.
  - Models with different cosmological parameters make different predictions of how flux and angular size change with distance.

- Difficulty, the cosmological redshift is distinct from Doppler velocity.
Distance is calculated from the measured flux assuming a known luminosity. Is the relation between flux, luminosity and distance still valid?

\[ f_\nu(d) = \frac{L_\nu}{4\pi d^2} \]

- **Flux** is the energy per unit surface area per unit time.
  - Decreases as light spreads into space.
  - Inversely proportional to the surface area of a sphere with radius equal to the distance between the observer and source.

- Appropriate distance is called the ‘comoving distance’ or ‘proper motion distance’ which is the product of \((R_0 \times r)\). It is the ratio of the transverse velocity (distance per unit time) of the object to its proper motion (in radian per unit time). It factors out the expansion of the universe.
  - Contrast: Proper Distance ~corresponds to the location of an object at a specific moment in time. It changes over time.
What else affects the observed flux?

- The light will be redshifted.
  - decreased frequency $\rightarrow$ decreased energy $\rightarrow$ decreased flux
  - flux decreased by factor $1/(1+z)$

- Cosmological time dilation
  - observed duration of an event is longer by a factor
    \[
    \frac{\Delta t_0}{\Delta t_e} = \frac{R(t_0)}{R(t_e)}
    \]
    - energy is spread over greater time interval
    - decreases energy per unit time $\rightarrow$ decreases flux
    - flux decreased by factor $1/(1+z)$

Overall, the flux decreases by a factor $(1+z)^{-2}$.

Luminosity distance:

\[
D_L = rR_0(1 + z) \quad \Rightarrow \quad f = \frac{L}{4\pi D_L^2}
\]
Hubble Diagram

Note: $z$ replaces velocity

$\Omega_m = 0.3, \Omega_\Lambda = 0.7$

$\Omega_m = 1, \Omega_\Lambda = 0$
The Earliest Times

- Go back to an early time when the density of high enough that the mean free path of photons was small and baryonic matter and radiation were in thermal equilibrium.

- We have already shown that during this radiation dominated time

\[ \rho \propto R^{-4} \]

- Recall, the energy density relates to temperature:

\[ \rho = \alpha T^4 \]

which implies

\[ T \propto \frac{1}{R} \]

At an early time the universe was in a “dense, hot state”. 
Recombination

a  Before recombination

b  After recombination
Surface of Last Scattering: The set of points in space beyond which the Universe becomes opaque. Photons emerging from this surface undergo negligible scattering and absorption until they reach us.

- Before recombination, atoms and radiation were in equilibrium.
  - Atoms and radiation at same temperature.
  - Radiation had a black body spectrum.

- After recombination, radiation decoupled from matter.
  - Radiation still had a black body spectrum.
  - Radiation continued to evolve:
    - Number density $n \sim R^{-3}$
    - Photon energy $E \sim R^{-1}$
    - Energy density $\rho \sim R^{-4}$
  - Radiation keeps its blackbody spectrum
How does radiation maintain a blackbody spectrum?

Consider that every photon gets redshifted from emitted frequency \( \nu \) to an observed frequency \( \nu' \) according to the transformation

\[
\nu' = \frac{\nu}{1+z} \quad \text{and} \quad d\nu' = \frac{d\nu}{1+z}
\]

Recall from chapter 2, the Planck spectrum:

\[
B_\nu = \frac{2h\nu^3}{c^2} \frac{d\nu}{e^{h\nu/kT} - 1}
\]

To obtain number density, divide by photon energy, \( h\nu \)

\[
n_\nu = \frac{2\nu^2}{c^2} \frac{d\nu}{e^{h\nu/kT} - 1}
\]

Since number density is conserved,

\[
n_{\nu'} = \frac{2\nu'^2}{c^2} \frac{d\nu'}{e^{h\nu'/kT'} - 1}
\]

where

\[
T' \equiv \frac{T}{1+z} \quad \text{and} \quad T_{\text{cmb}} = \frac{T_{\text{rec}}}{1+z_{\text{rec}}}
\]
CMB History

- 1940s: Gamow predicts (based on nucleosynthesis) that recombination occurs at zero $\sim 1000$. Thus, the thermal spectrum $\sim$ few 10s degree K, with peak $\lambda \sim 1$ mm (microwave region of EM spectrum).

- 1965: This Cosmic Microwave Background (CMB) radiation was accidentally discovered by Penzias and Wilson.
  - They assumed the radiation had a Planck spectrum and translated the intensity to a temperature, $T_{\text{cmb}} \sim 3$ K.

- 1990s and beyond: More precise measurements confirm a black body spectrum and temperature of $2.72548 \pm 0.00057$ K.
The CMB photon number density is

\[ n_{\gamma,\text{CMB}} \sim \frac{aT^4}{2.8kT} = 400 \text{ cm}^{-3} \]

How does this compare to the number density of photons originating from stars?

\[ n_{\gamma,*} \approx 4 \times 10^{-3} \text{ cm}^{-3} \]

Thus, there are \(10^5 \) CMB photons for every stellar photon!

How does this compare to the baryon number density?

\[ n_B \approx \frac{0.04\rho_c}{m_p} = 2 \times 10^{-7} \text{ cm}^{-3} \]

Thus, the baryon-to-photon ratio is

\[ \eta \equiv \frac{n_B}{n_\gamma} \approx 5 \times 10^{-10} \]

Energy density due to matter is much larger than radiation, but the number density of photons is much larger than the mean number of baryons!
Anisotropy of the CMB

After removing a small dipole in the CMB sky from the motion of the Local Group (which we belong), the CMB is very uniform across the sky. There are small temperature fluctuations of $\delta T = 29 \, \mu K$ or $\delta T / T \sim 10^{-5}$. 
How and why can the Universe appear so isotropic?

- At recombination, the horizon size was 2º. So different regions separated by more than ~ 2º could not be causally related.
- If they were not in casual contact, how is it that they have the same temperature to within $10^{-5}$?

Explanation for the ‘horizon problem’ is inflation.

- During the first fraction of a second of the evolution of the Universe, a rapid expansion of the Universe occurred.
- Resulted in casually connected regions to expand beyond the size of the horizon at that time.
- Different parts of the microwave sky that we see today were, in fact, causally connected before inflation.
- Details of how this happened is still an area of debate in research today.
- Prediction of this theory is that the Universe is almost exactly flat.
Inhomogeneities in the nearly uniform cosmic mass distribution are set up at the end of the inflationary era.

- Most of the mass at this time is non-baryonic, pressure-less dark matter.
- Also there is a relativistic gas of baryons and radiation.
- Equation of state describing the photon-baryon gas is

\[ P = \frac{1}{3} \rho c^2 \]

The speed of sound is then

\[ c_s = \sqrt{\frac{dP}{d\rho}} = \frac{c}{\sqrt{3}} \]

Note: Ordinary sound waves are driven solely by gas pressure. This is not the case in the early universe.
Aside: Fourier Transform

- Fourier’s theorem: Any periodic function can be described as a sum of sines and cosines.

- The ‘power spectrum’ shows strength of signal fluctuations at a particular frequency.

![Graphs showing Fourier expansions and power spectra for different periodic functions.](Image)
Consider a region of over-density in the early universe. The gravitational potential will attract the baryon-photon fluid. This fluid is compressed in denser regions.

Pressure in the baryon-photon fluid opposes the compression and causes an expansion that stops after the density exceeds the equilibrium density. The gas becomes under-dense.

These counteracting forces create oscillations, analogous to sound waves created in air by pressure differences.

Standing sound waves of all wavelengths (represented in the spatial Fourier spectrum) are formed.

\[ \tau = \frac{\lambda}{c_s} \]

As the Universe emerges from the inflationary era, these **acoustic oscillations** are stationary everywhere and hence **in phase**.
- After inflation, regions of denser dark matter pull in the baryon-photon fluid via gravitational attraction.
- The Fourier mode that composes either an over dense or under dense region has a wavelength

\[ \lambda = 2c_s t_{\text{rec}} = \frac{2c t_{\text{rec}}}{\sqrt{3}} \]

- At the time of recombination, baryons and photons decouple, allowing photons to stream away.
- Leaves a frozen fingerprint of cool (rarified) and hot (compressed) regions of space on the sky.
- Higher modes may have undergone one compression and one rarefraction or two compression and one rarefraction, etc.

- Hence, they too will be at their hottest or coldest at $t_{\text{rec}}$.

- This will correspond to display spots of particular sizes → the fluctuation power spectrum should have discrete peaks at the favored spatial scales.

In reality, the picture is complicated by several processes other than those we considered. However, they can be calculated to make predictions for particular cosmological models.
Next time:
- Examine the angular scale of the first acoustic peak as a measure of the global curvature of space.
- Nucleosynthesis of Light Elements
- Quasars ans Cosmological Probes (time permitting)
Stay ed!

\[ I(f) = \left( \frac{2hf^5}{c^2} \right) \left( \frac{1}{e^{hf/kt} - 1} \right) \]

Energy Density

0 160.4 GHz

SCIENCE.

IT WORKS, BITCHES.