Exercise 4.17
\[
\lambda = \frac{\hbar}{p} = \frac{\hbar}{mc} = \frac{6.63 \times 10^{-34} \text{J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{kg})(3 \times 10^8 \text{m/s})} = 2.43 \times 10^{-12} \text{m}
\]

Exercise 4.18
\[
E = \frac{p^2}{2m} = \frac{3}{2} k_B T \quad \text{with} \quad p = \frac{\hbar}{\lambda}
\]
\[
\left(\frac{\hbar}{\lambda}\right)^2 = \frac{3}{2} k_B T \Rightarrow \frac{\hbar^2}{\lambda^2} = 3 k_B T \Rightarrow
\]
\[
\lambda = \frac{\hbar}{\sqrt{3mk_B T}}
\]

Exercise 4.19
\[
\lambda = \frac{\hbar}{\sqrt{3mk_B T}}, \quad T(\text{K}) = T(\text{°C}) + 273
\]

a) For an electron:
\[
\lambda = \frac{6.63 \times 10^{-34} \text{J} \cdot \text{s}}{\sqrt{3}(9.11 \times 10^{-31}) (1.38 \times 10^{-23} \text{J/K})(295 \text{K})} = 6.29 \text{ nm}
\]

b) For a proton
\[
\lambda = \frac{6.63 \times 10^{-34} \text{J} \cdot \text{s}}{\sqrt{3}(1.67 \times 10^{-27} \text{kg})(1.38 \times 10^{-23} \text{J/K})(295 \text{K})} = 0.147 \text{ nm}
\]

Although the proton's speed would be smaller, its mass is so much larger that its momentum is large enough to produce a smaller wavelength.

In situations in which dimensions are smaller or comparable to \( \lambda \approx 6.29 \text{ nm} \), the electron will exhibit its wave nature. At the same time, the dimensions would have to be smaller by a factor of \( k^2 \) for a proton to exhibit a wave nature.
Exercise 2.3:

If the maximum nonrelativistic speed is taken to be \( c/10 \), the wavelength would be

\[
\lambda = \frac{\hbar}{P} = \frac{6.63 \times 10^{-34} \text{J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{kg})(2.5 \times 10^{-10} \text{m/s})} = 2.43 \times 10^{-\text{m}}
\]

Wavelengths this small or smaller would imply relativistic motion. Use formula example 4.3

\[
V = \frac{\frac{\hbar}{\sqrt{2m\lambda^2}}}{(2(9.11 \times 10^{-31} \text{kg})(1.6 \times 10^{-19} \text{m})(2.43 \times 10^{-10} \text{m})^2)} = 2.500 \text{ V}
\]

Exercise 2.4:

Find the speed of the moon

\[ F = ma \quad a = \frac{v^2}{R} \]

\[
F = G \frac{m_{\text{Earth}} m_{\text{Moon}}}{r^2} = m_{\text{Moon}} \frac{\frac{v^2}{R}}{r}
\]

\[ v = \sqrt{\frac{G m_{\text{Earth}}}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{kg})}{3.84 \times 10^8 \text{m}}} = 10.2 \times 10^3 \text{ m/s}
\]

Thus \( \lambda = \frac{\hbar}{P} = \frac{\hbar}{m \cdot \sqrt{2m\lambda^2}} = \frac{6.63 \times 10^{-34} \text{J}\cdot\text{s}}{(7.35 \times 10^{-26} \text{kg})(10.2 \times 10^3 \text{m/s})} = 8.85 \times 10^{-60} \text{ m}
\]

This is much smaller than the dimensions of the region in which it moves. In fact, it is much smaller than the atomic nucleus.

The moon certainly orbits as a classical particle.
Heisenberg principle: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Delta x = 1 \mu m = 10^{-6} m, \quad m = 0.145 \text{ kg}, \quad \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} J s$

$\Delta x \Delta p \geq \frac{\hbar}{2} \Rightarrow (10^{-6})(0.145 \text{ kg}) \Delta v \geq \frac{1}{2}(1.055 \times 10^{-34} J s)$

$\Delta v \geq 3.6 \times 10^{-20} m/s$

It is theoretically impossible to say whether it might not be moving at $\approx 10^{-22} m/s$.

**Exemple 4.43**

$\Delta x \Delta p \geq \frac{\hbar}{2} \Rightarrow (5 \times 10^{-15} m)(1.67 \times 10^{-27} \text{ kg}) \Delta v \geq \frac{1}{2}(1.055 \times 10^{-34} J)$

$\Rightarrow \Delta v \geq 6.3 \times 10^6 m/s$

its KE = $\frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(6.3 \times 10^6 m/s)^2 \approx 0.2 \text{ MeV}$

**Exercise 4.48**

$\Delta E = 150 \text{ MeV} = 150 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$

$\Delta t \geq \frac{\hbar}{2 \Delta E}$

$\Delta t \geq \frac{\frac{1}{2}(1.055 \times 10^{-34} J s)}{150 \times 10^{-13} J}$

$\Delta t \geq 2.2 \times 10^{-24} \text{ s}$
Exercise 4.62

\[ \text{Eq. 4-27} \quad \Delta \omega \Delta t > \frac{\hbar}{2} \Rightarrow \Delta \omega > \frac{1}{2 \Delta t} \]

Angular frequency:

\[ \Delta f = \frac{1}{2 \pi} \Delta \omega \quad \Delta t = 1 \text{ns} = 10^{-9} \]

\[ \Delta f = 7.96 \times 10^{7} \text{Hz} \]

(C) For the 1060 nm laser:

\[ f = \frac{c}{\lambda} = \frac{3 \times 10^{8} \text{m/s}}{1060 \times 10^{-9} \text{m}} = 2.83 \times 10^{14} \text{Hz} \]

The relative uncertainty is significant for 100 MHz radio wave, but is very small compared to the frequency of the light.

Exercise 4.63

\[ \Delta x = 0.3 \mu \text{m} \]

\[ \Delta k = \frac{1}{2} \frac{1}{\Delta x} = \frac{1}{2} \frac{1}{0.3 \times 10^{-6}} = 1.67 \times 10^{6} \text{ m}^{-1} \]

\[ k = \frac{2 \pi}{\lambda} \quad (\text{wave vector}) \Rightarrow \Delta k = \frac{2 \pi \Delta \lambda}{\lambda^2} \]

\[ \Delta \lambda = \frac{\lambda^2 \Delta k}{2 \pi} = (6 \times 10^{-7} \text{m})^2 \frac{1.67 \times 10^6}{2 \pi} \]

\[ \Delta \lambda = 95 \text{ nm} \]

A 1 femtosecond pulse of 600 nm light is not just 600 nm light.