By the end of the 1800's, classical physics had many successes. One prominent physicist even had suggested that all that remained was to further increase the significant digits for measurements. However, at this same time, several observations were directly in conflict with the classical understanding. We describe three of this first. The existence and behavior of absorption and emission lines in light spectra was one such observation. A very crucial one was the way light was emitted from a heated object, called 'blackbody radiation'. Lastly, when light is incident on a conducting surface, electric charge was observed to be released which was impossible to understand classically. Understanding these observations led to a wholly different model of matter and energy we call 'quantum mechanics.

Atomic Spectra *L4*

A difficulty with the classical paradigm of physics arose when measurements of white light were passed thru gases. The spectrum of this light was observed to be missing very specific wavelengths, and the set of wavelengths missing (or 'absorbed') was unique to each chemical element in nature. Additionally, these elements could be made to emit light and the set of wavelengths of that emitted light was specific to each element and the same as the set of absorbed wavelengths. Since there was no model of an atom aside from a point-like particle, there was no way to understand the specificity of this behavior of light.

- if you take a low pressure gas & cause an electric discharge (like a neon light)
  - You will observe bright narrow lines, and dark elsewhere
    - lines at specific λs
    - "emission spectrum"

- if you pass white light through a gas
    - dark narrow lines on a bright continuum
    - "Absorption spectrum"

- each element has unique set of lines
  - how Helium was discovered
  - lines of emission at same λ^s as those of absorption
- no way to understand or predict patterns of lines classically

Black Body Radiation

In attempting to understand the properties of light as emitted by matter, the concept of a 'blackbody' has become useful. Such an object absorbs all electromagnetic radiation that is incident on it and therefore heats up. The heated body then re-radiates this electromagnetic radiation in a characteristic spectrum called a 'blackbody spectrum'. The observed properties of this spectrum were that at small and large wavelengths, the intensity of emission were tiny. However, for some characteristic wavelength, the intensity of emission was a maximum. This 'peak wavelength' is correlated directly with the temperature of the blackbody – warm bodies radiate with a smaller peak wavelength than cooler bodies. Classically, the spectrum of blackbody radiation could only be calculated on the assumption that all wavelengths of light were possible in the emission. This led to the calculation that the observed intensity should be infinite when approaching wavelengths of 0 size. This unphysical result was a major blow to classical physics.

- an object at temperature, T
  - emits thermal radiation. This is not reflected light, or light generated by chemical or nuclear reactions in the material. It is originating from the motion of the atoms in the material.
  - e.g. room temperature produces infrared light (i.e. longer wavelength than red)
    - characteristics depend on T
      - The object will emit λs from the whole spectrum but the intensity will peak at some λ
      - as the object gets hotter the peak moves to higher energies: IR-R-Y-G-B-V-UV

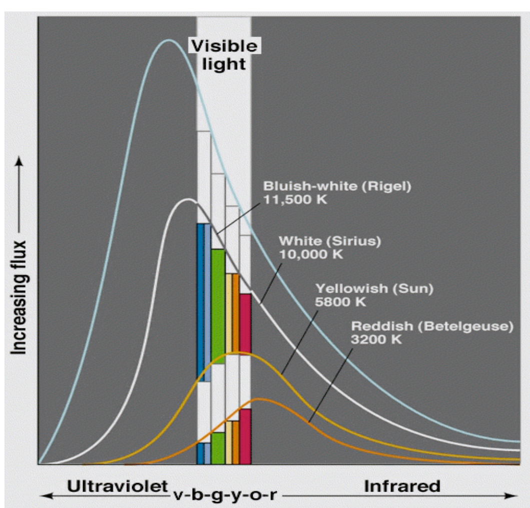
  classically
  - radiation from accelerated charged particles in atoms (vibrating) near surface of object
    - like small antennae
  - range of λs from range of energies of accelerating particles

- A black body
  - ideal system absorbing all radiation incident on it

  Example: red glowing coals deep in a cooking grill is a decent approximation

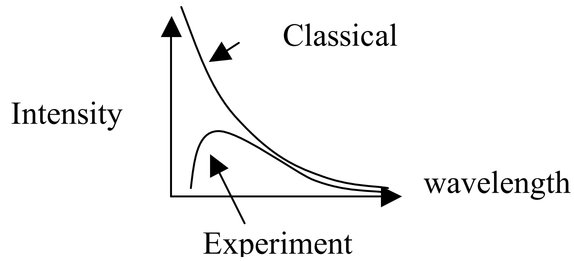
Observations

- plot intensity of light vs. λ



- as T up, peak λ down (Wien's Law)
- total power, P
  - related to area under the curve
  - $P \propto T^4$ (Stefan's Law) which means total power increases very rapidly as temperature increases

- classical expectation
  - have continuum of λs and energies for oscillation need to split energy evenly among all wavelengths

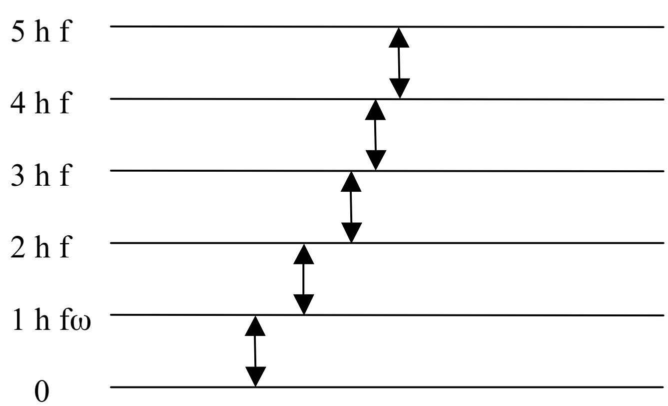


Intensity of light emitted, I, is calculated classically to be infinite when λ near 0 (classically)
- violates conservation of energy (can't have infinite energy when put in finite amount of energy)
- this is not what is observed

"Quanta" L9p1

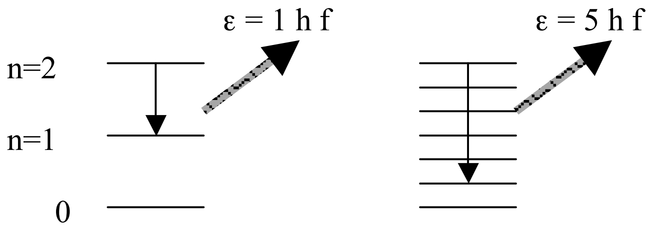
- M. Planck, 1900
- Assume radiation within the cavity is from "atomic oscillators"
  - remember, moving charge produces electromagnetic waves
  - also, heated objects have rapidly vibrating (oscillating) atoms
  - so oscillating atoms produce light we see as blackbody radiation
    - still no model of the atom, yet

Then make 2 assumptions
- The Energy of each oscillator can only have certain discrete values
  $E = n\,h\,f$ n = quantum number; h = Planck's constant, f = frequency
  - So each oscillator can have quantum states {0, 1hω, 2hω, 3hω, …}
- Oscillators emit or absorb energy only when making transitions from one quantum state to another
  $\Delta E = m\,h\,f$ $(m = n_2 - n_1)$



- If we now calculate the energy emitted by a black-body
  - Don't equally distributed Energy accross various λ bins (equipartition)
  - We must weight each λ bin according to a well-defined distribution that governs the occupation of higher energy states (This is the Boltzman distribution law) The weighting factor is ($e^{-\varepsilon/kT}$).

Energy level diagram *L9p2*



Longest wave length   shorter wave length

<u>Short wave length</u>

- Large energy separation
- Low probability of excited states
- Few downward transitions

<u>Long wave length</u>

- Small energy separation
- High probability of excited states
- Many downward transitions

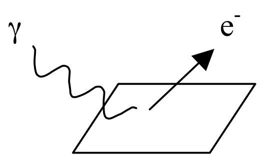
Planck model L9p3

- Average energy is associated with a given λ
  - Product of E of transitions & factor related to probability of transition occurring
  - at low frequencies, close together energy levels
    - high probability to go thru transition
    - many contributions, but each generates low energy (long wavelength)
  - As energy levels are further apart
    - would correspond to larger energy released in transition from one level to next
      - a shorter wavelength, high frequency
      - probability of excitation decreases
- Theoretical expression agrees with observation (the shape agrees)
  - intensity, I, is a function of wavelength and temperature (λ,T)
  - a parameter, “h”, called Planck’s constant enters the calculation
  - “h” is then adjusted to get a good fit
    “h” is fundamental constant of nature, h=6.626×10-34 J·s
  - Most people at the time (including Planck) thought this just a mechanical constant

Photoelectric Effect

A final fatal observation for classical physics came with the realization that light incident on a conducting metal plate caused electrical current to be released by the plate. This 'photoelectric' effect was expected to have a few properties based on the understanding of light, matter and current at the time. Essentially, the liberated current was believed to represent a substance which carried charge away from the material of the plate. To liberate the current required sufficient energy, and this energy was carried by the intensity of the light incident on the plate. If one turned up the light intensity, then the amount of energy being dumped into the plate would add up. Eventually, the current carrying material would have a high enough energy that it could escape the bonds that held it to the material of the plate. In this scheme, the intensity of the incident light should be the only thing dictating whether current flowed or not. The current should be independent of the frequency of the incident light. In actual experiment, it was observed that at high frequencies, the current did increase (to a point) with incident light intensity. However, at lower frequencies, the current was a strong function of the incident light frequency. More confusing, below some 'cutoff' frequency, no current was observed at all. The classical understanding was unable to come up with an explanation for this behavior.

- light made incident on certain metallic surfaces
  - electrons are emitted (flowing electrons called an ‘electrical current’)



Classical picture

- electrons should absorb energy continuously from electromagnetic waves
- as Intensity increases, energy is transferred to metal at a higher rate
  - e^- kinetic energy □

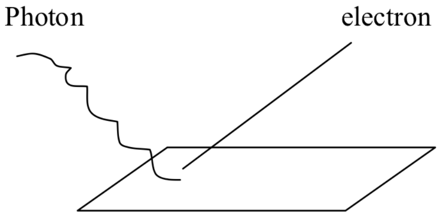
at low E, there should be a finite time, Δt, between turn-on of light & the ejection of electrons

But this is not what is observed. Below are several classical expectations that are not satisfied by the observations.

Observation	Classical expectation
Electrical current starts instantaneously (10^{-9} s) once light is turned on, even at low light yields	Takes time for energy from light to ‘build up’ in conductor. So electrical current should start only after some time when the energy accumulates enough.
No emission below some cutoff frequency	e- ejected regardless of light frequency
regardless of intensity of light source	Depend only on intensity which is what governs tranfer of energy to metal & e's
electron kinetic energy increases with light frequency	No relationship between frequency of light and kinetic energy of e's KE relates to intensity of light.

Photoelectric Effect Explanation L9p4

- Einstein extened the quantum concept to electromagnetic waves
- quantization is a fundamental property of light.
- Assume light is a stream of quanta: photons
- Each photon has energy E = hf



- Each photon, when incident on plate,
  - Gives all energy to a single electron in metal

  ∴ not continuous process, but discrete
    - You either liberate a photoelectron or you don't.
    - These electrons have maximum kinetic energy $KE_{max}=hf-\Phi$
      - (reminder: kinetic energy is energy of motion)
      - Phi is work function needed to get electron unbound
        Varies by material

  ∴ No dependence of KE on light intensity

  ∴ No time delay between time of light incident and electron ejection

  ∴ Dependence of electron ejection on light frequency since $E_\gamma>\Phi$.

  ∴ Dependence of KE on light frequency

Wave Properties of Particles L10p1

Photons and Electromagnetic waves

- light is made of particles, called photons
- but light behaves like a wave, as in the double-slit experiment and interference
- therefore, photons have frequency. f, so they are wave-like
    - How can they be waves & particles?
        - Have Energy & momentum for a particle
        - Interfere like a wave
    - Cannot use these “classical” pictures to describe light adequately
        - We use both complementarily (because it is all we have).
            - Principle of Complementarity: both wave and particle properties present

Wave-particle connection

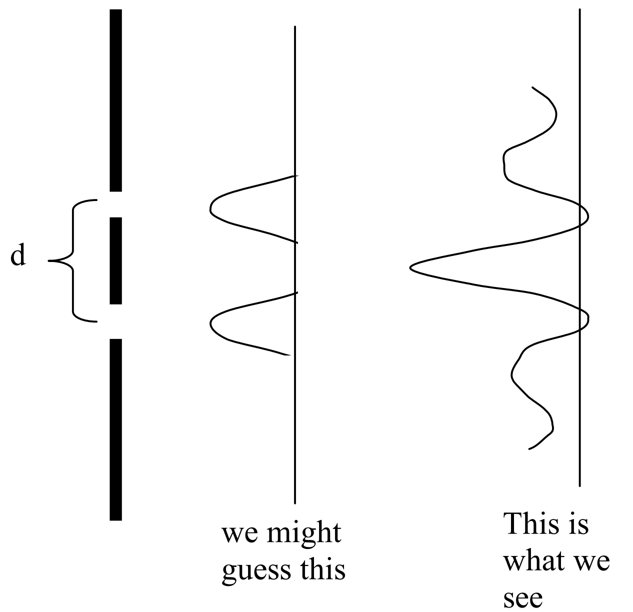
- De Broglie postulated
    - Perhaps particles have, in analogy with light, wavelike properties
      - for photons, expressions for energy and momentum are
        - $E = hf$; $p = h/\lambda$
        - E and p are particle properties
        - f and λ are wave properties (note: large f corresponds to large E)

    - could also hold for matter particles, like an electron
        - for example: $\lambda = h/p$ (here, ‘p’ is momentum)
            - $\lambda = h/mv$ (for a particle, momentum = mv, or mass time velocity)
                - called de Broglie wavelength of a particle
            - $f = E/h$ (E = total energy of the particle)
    - relating particle & wave properties
    - accidentally confirmed by observation of diffraction: Davisson-Germer experiment
    - used for electron microscope
        - high Energy electrons, better resolution
        - electron wavelengths much smaller than wavelengths of visible light

Electron Double Slit Experiment L10p3

electron passage thru matter

- observed interference effects accidentally at Bell Labs, 1928



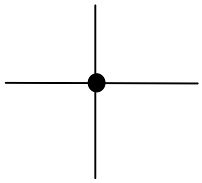
Consider a barrier with two slits in it in front of a screen. What happens when we send electrons at this barrier & screen? Some of the electrons will go through the barrier's slits & hit the screen & be recorded.
each electron goes through and registers at one location on screen.
If we send many electrons at the screen we'll begin to see them pile up in some places more than others (You might guess the pile up will be directly in front of the slits). but the probability of arriving at a given spot is determined by interference predictions! (this is a surprise, and it means each electron actually goes through BOTH SLITS!)

As we turn down the intensity of our electron source, the number of electrons will decrease. Each electron hits a spot randomly; there is no way to predict where it will hit. Each electron corresponds to a wave-function or wave-packet which knows about both slits. In other words, the single electron, being wave-like, interferes with itself when it goes thru the slit.

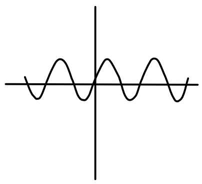
- if one slit covered
  - just one peak in front of the uncovered slit
  - we loose the interference pattern
- interpretation: an electron interacts simultaneously with both slits
  - if we try to determine which slit an electron goes through (by covering a slit) the interference pattern is destroyed. (Just like with Photons)

`But how can both be correct?  (waves & particles)`

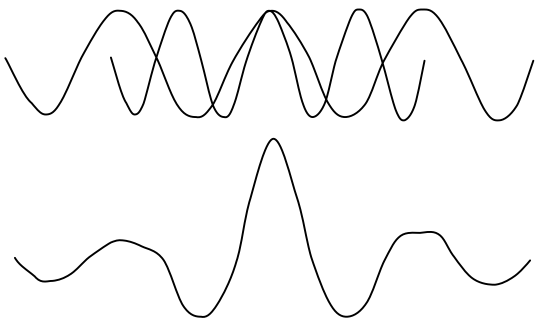
The Quantum Particle L10p2



- Ideal particle: localized in space, zero size



- Ideal wave: single frequency, infinitely long
    - Unlocalized in space



- Take two waves of different frequency.
    - A somewhat preferred locale (or set of locales at best)
- Can construct superposition such that destructive interference everywhere but at x=0
  so have a particle with a location.

position, x →

The result of summing up more than two waves with different frequency gives what we call a 'wave packet'. This is shown in the last figure above. A wave packet, unlike a simple wave, has a most likely position indicated by the peak location. Unlike a pure particle, the wave packet extends to large distances away from the most likely position.

The Uncertainty Principle L10p4

-
- To know momentum precisely, by de Broglie $\lambda = h/p$, means we know λ precisely, as a pure λ, not superposition of many λ needed to localize the particle.
- To know x precisely, requires a large number of λ's, so p cannot be known.

- We usually have experimental measurement uncertainties
  - Classically these are not a fundamental problem & could theoretically be reduced to zero
  - Quantum mechanics does not allow this.
-
- If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of it's x- direction momentum is made with uncertainty Δp_x, the product of these uncertainties will never be less than h/2
  $\Delta x\, \Delta p_x \geq h/2$.
  - Cannot measure momentum & position infinitely well, or well simultaneously
    - Not experimental uncertainty: from quantum state of matter.
  - similarly: $\Delta E\, \Delta t \geq h/2$
  - Later, we will see that particles that live a certain length of time, Δt, have a mass uncertainty which can be expressed as the resulting uncertainty in energy, given the above equation. This is true because mass and energy are equivalent – a topic we will explore more when we talk about relativity.

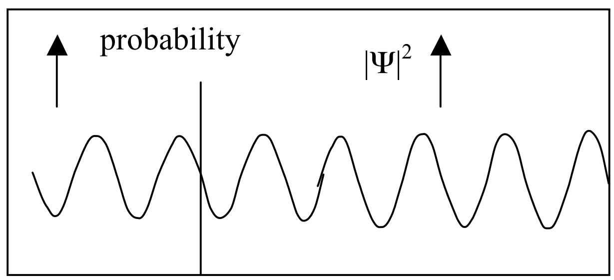
Quantum Vacuum or Sea

The uncertainty principle indicates that energy or momentum of a particle is uncertain by an amount ΔE or Δp, respectively. This also means that conservation of energy or conservation of momentum can be violated on a particle-by-particle basis, but only according to the uncertainty equations above. For instance, energy conservation can only be violated by ΔE for a length of time, Δt as given above. Large uncertainties in energy can only exist for very brief amounts of time, and small uncertainties can exist for longer times, consistent with $\Delta E\, \Delta t \geq h/2$.

The result of this behavior leads to some strange behavior, as we'll discuss later. But one concept we introduce here is that of the 'quantum vacuum' or 'quantum sea'. In classical physics, a vacuum is a region of space devoid of matter. In quantum mechanics, given the uncertainty principle, such a vacuum is seen not to exist. In it's place we have a region of space in which individual particles appear out of nowhere and disappear again. Since mass and energy are equivalent (we'll get to this when we talk about Relativity), the expression $\Delta E\, \Delta t \geq h/2$ means that a particle of a given mass can exist in the vacuum for a brief instant. Such a particle is called 'virtual' since it does not persist. So the vacuum is seeting with these particles constantly fluctuating.

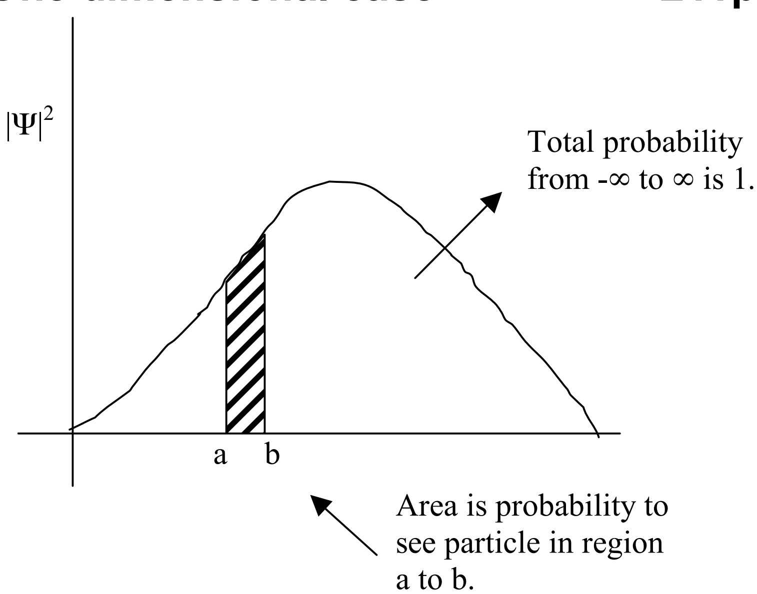
Probabilistic Interpretation in QM   L11p1

-
- Matter and EM radiation waves & particles
  - "implies" probabilistic nature to being a particle
    - Particles in places more often when intensity of wave highest
    - For de Broglie particles
      - have "probability function" or "wave amplitude" Ψ.
      - contains all information we can know about a particle
  - Wave Eq. (Schrödinger Equation) 1926
    - Wave function varies as a function of position in space, and time
      - the square of this function provides a measure of the probability for a particle to be at a particular position.
      - an abstract property of a wave-particle. We don't see this property. We can just make measurements of observable particle and/or wave properties.



  - When have particle with very well defined (precise) momentum, it's spread through space.
    - Probability density, ψ^2.
      - Probability to find particle in tiny region around some point.

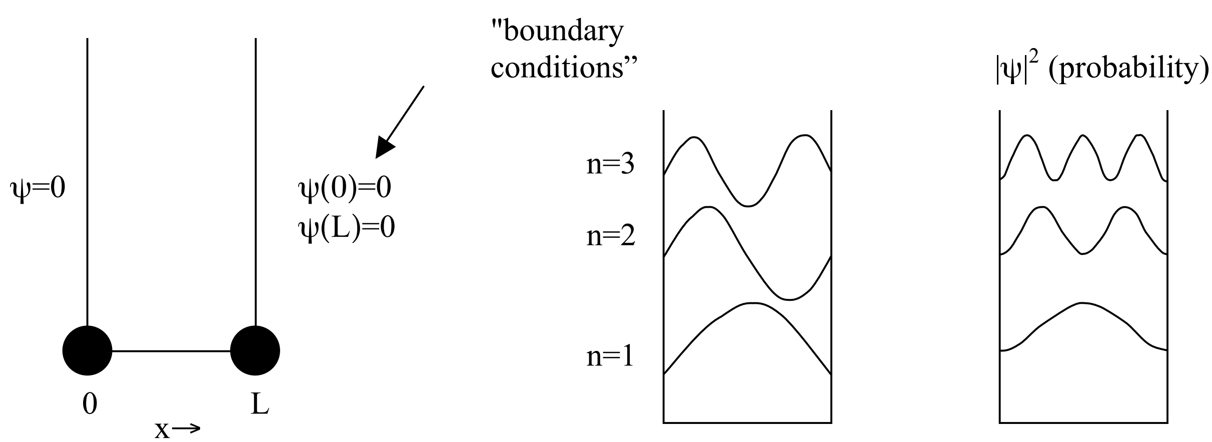
One dimensional case   L11p2



Particle in a Box

- Interaction of a particle with its environment is represented by potentials and
    - One or more "boundary conditions"
    - If a particle is restricted to a finite region of space

- Particle moving back & forth in classical physics with impenetrable walls
    - by 'impenetrable walls' we can also say infinite 'binding energy' meaning that the particle held within the box by a force that is infinitely strong
    - Can have any momentum or energy
- In quantum physics
    - wave-function for particle in a box
        - somewhat analogous to plucked string held fixed at both ends
        - probability to find particle at position, x, within box
            - not flat in x
            - have 'nodes' where the particle never occurs, and 'antinodes' where the particle often occurs.
    - zero probability to be at or outside of walls.
    - this constraints what wavelengths are possible for particles in this box.
        - can never have a particle of zero energy within such a box, since this would correspond to infinite wavelength, which is not possible in a box of finite size.
        - probability of particle to be at or outside of 'walls' is zero
        - only wavelengths which fit an integer number times within the box are permitted (again, somewhat in analogy with a vibrating string held fixed at both ends)

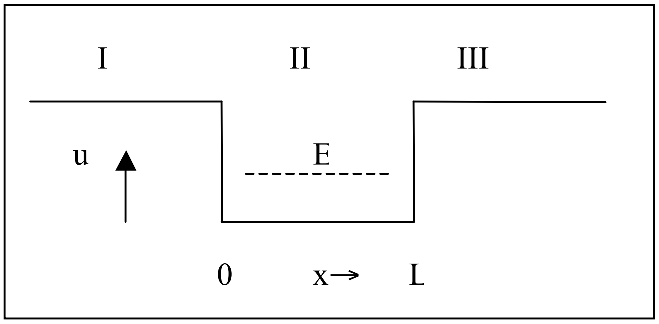


Remember that each energy for a particle corresponds to a specific frequency or wavelength. Since only certain wavelengths 'fit', the energy levels of the trapped particles are discrete.

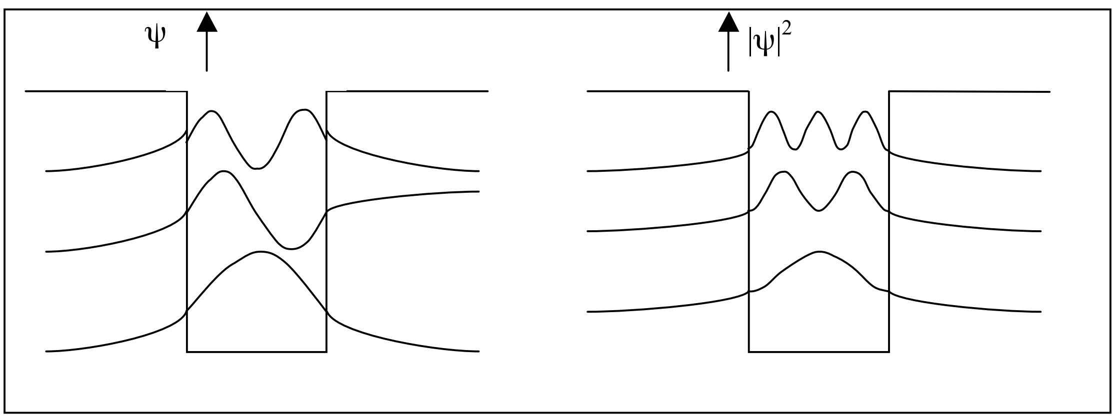
Therefore, **energy of particle in this box is quantized!**

Boundary Conditions & Finite Potential Well

- If walls not quite impenetrable
    - a more realistic case
    - Ie. binding energy, $U < \infty$
    - Classically, if the kinetic energy (energy of motion) of a particle is less than the binding energy ($KE < U$), the particle will always be in the well

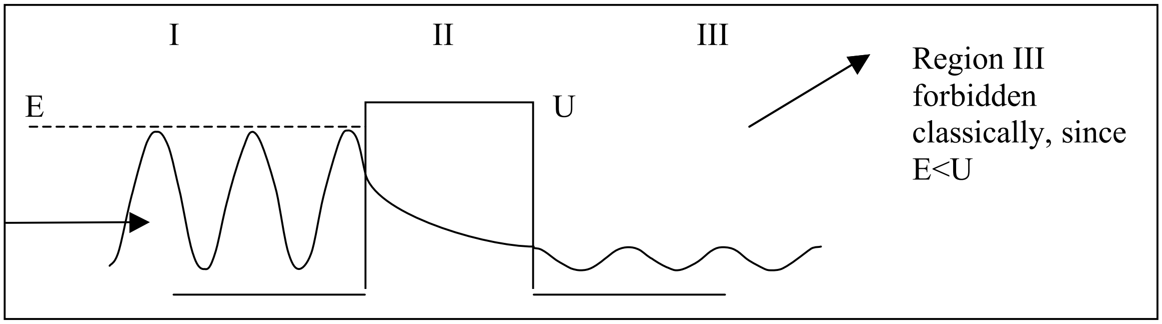


- Quantum mechanics
    - Probability to find particle at given position, x, must be smooth at boundaries
    - Probability density $\neq 0$ in walls (i.e. regions I & III)
    - Because energy of system uncertain can occasionally have $E > U$
    - In II, sinusoidal → but boundary conditions require continuity



Tunneling L11p4

-
- Consider moving particles with energy E
    - Heading toward a Potential barrier of height U>E
    - Finite width. (region II is some finite thickness)



- probability Sinusoidal in Region I
- probability Exponential in Region II
    - But there is a non-zero probability of finding the particle to the right side of the barrier.
- probability Sinusoidal in Region III with reduced amplitude (also non-zero)

∴ The particle can be anywhere!

Thus we get tunneling from I□III, which is completely contrary to classical physics.

Questions

1) A blackbody is observed to emit light over a broad range of wavelengths. (T or F) [2 pts]

2) A blackbody's peak wavelength of emission goes to higher values when, [2 pts]

a) temperature increases?
b) intensity increases?
c) frequency increases?
d) none of the above

3) In the photoelectric effect, the electric current observed to come from a metal plate with light incident on it has the following property, [2 pts]

a) the current increases with intensity of incident light.
b) the current depends ont he frequency of the incident light.
c) above some 'cutoff' frequency, no current is observed.
d) a and b
e) a, b and c

4) How did Einstein use the quantum idea to resolve the photoelectric effect? [10 pts]

5) Electrons have wave-like properties. (T or F) [2 pts]

6) State the Heisenberg Uncertainty Principle. [2 pts] Discuss how this follows from the wave-like nature of particles. [8 pts]

7) Describe how the idea of "the quantum" resolved the theoretical problems in understanding black-body radiation. [10 pts]

8) Describe how the idea of "the quantum" resolved the classical problems with the photoelectric effect. [10 pts]