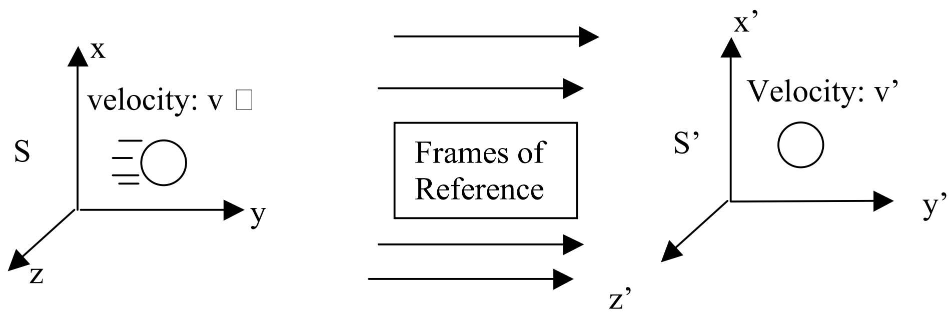
Reference Frames

To describe a physical event, we need to establish a 3-dimensional coordinate system associated with measurement.

Let's consider two reference frames. One, called S, is shown at left. The other, called S', is shown at right. Let's imagine a meatball which is moving with velocity 'v' within reference frame S. Reference frame S' may be moving with respect to frame S, so the perceived velocity of the meatball in S' may be different than an observer in S would measure. In fact, v' could be zero. So the value of the y' coordinate may also differ in the two reference frames. However, as drawn, the x (x') and z (z') coordinates would be the same in the two reference frames.



Inertial Reference Frames

Inertia: An object moves at constant velocity unless acted upon by an external force.

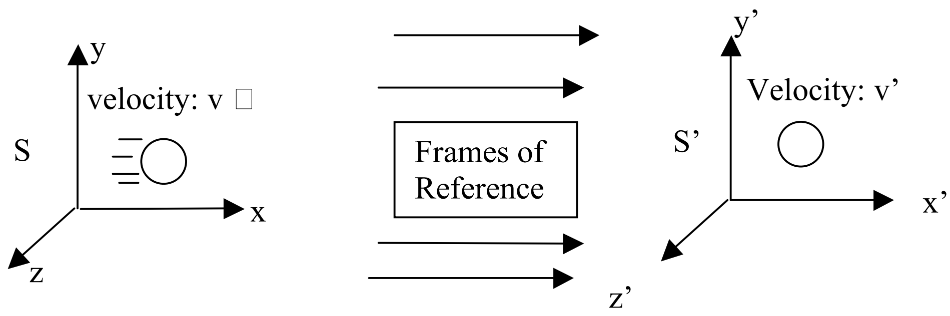
Given the concept of inertia, we find it useful to talk about 'inertial reference frames' which are three-dimensional coordinate systems which travel at constant velocity. In such a frame, an object is observed to have no acceleration when no forces are acting on it. If a reference frame moves with constant velocity relative to an inertial reference frame, it also is an inertial reference frame. There is no absolute inertial reference frame, meaning that there is no state of velocity which is special in the universe. All inertial reference frames are equivalent. One can only detect the relative motion of one inertial reference frame to another.

Principle of Galilean Relativity

"Laws of mechanics must be same in all inertial frames of reference"

Galilean Transformations

Consider a meatball in frame S moving with velocity, v, within that frame, and S' is moving with velocity V' relative to frame S. This is shown in the following Figure.



We want to know how to determine the coordinates in S' when we know them in frame S. In the picture above, in the S frame the meatball is moving and the {x,y,z} axes are fixed. When we transform to the S' coordinate system (so that {x',y',z'} are at rest), it now looks like the meatball has velocity v' & that the old axes {x,y,z} are moving with velocity v in the NEGATIVE x' direction. "v" then appears as the relative velocity of the PRIMED coordinate system {x',y',z'}, S', compared to the UNPRIMED coordinate system {x,y,z} or S.

To determine the coordinates of the meatball in one frame, S', when we know it's coordinates in another frame, S, we employ the Galilean space and time transformations.

Galilean space-time transformations:

If S' has a velocity relative to S so that v' = 0, then we have

$x' = x+vt$
$y'=y$
$z'=z$
$t'=t$

(Note: remember, v or v' are vectors, so they have a sign.)
The time interval between any two events is the same in any frame of reference.

Galilean velocity transformation

We also want to know the velocity of the meatball in frame S'. From the above situation, frame S' is moving with V' relative to S, and the meatball has velocity v in frame S. So the velocity within frame S' would be $v' = V' + v$. This is the Galilean velocity transformation.

Energy & Momentum

Classically, several forms of energy are recognized.

- energy of motion (‘kinetic energy’) KE
- energy from forces (‘potential energy’) PE or binding energy
- heat

It was recognized that energy can change form from one to the another. Kinetic energy can be converted into heat, for instance. If one considers a swing, then when the swing is at the bottom of its trajectory, it has a maximum velocity (and so maximum KE), and a minimum potential energy. At the highest points of its path, its velocity goes to zero (no KE), but its potential energy is a maximum. However, it was always observed that the total amount of energy (=KE+PE+heat) in a closed system was constant. This is known as ‘conservation of energy’: Energy may not be created nor destroyed in a closed system.

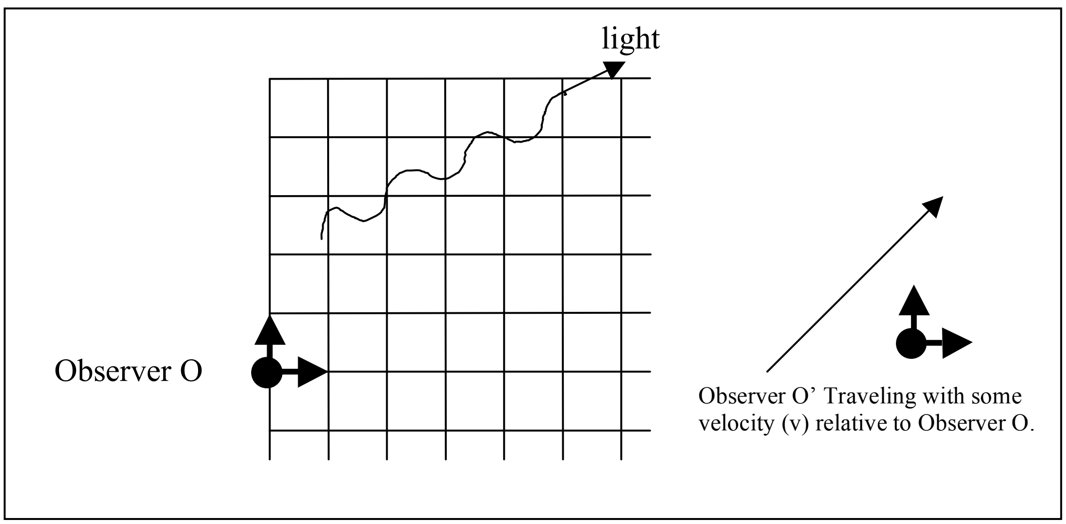
A related concept concerns the momentum of a particle, or series of particles in a closed system. In general, momentum (p) = mv and it preserves the direction of the velocity (unlike energy). As is indicated by the Law of inertia, the magnitude and direction of an object’s velocity does not change unless it is acted upon by an external force. This is what $F = ma$ means. Since acceleration is a change in velocity per time, it equals $\Delta v/\Delta t$. This means that force can also be expressed as a change in momentum per time, $F = \Delta p/\Delta t$. The net momentum of a closed system cannot change, so when two particles collide in the same way, the sum of their outgoing momenta will equal the sum of their incoming momenta.

Speed of Light

Recall that the speed of light was predicted by Maxwell's equations to be the enormous value of $c=3x10^8$ m/s. Questions immediately arise about what this velocity is to be measured with respect to. Also, recall that Galilean relativity only encompassed mechanical phenomena. Does Galilean relativity apply to electricy, magnetism, & optics?

First, we need to establish what light is. From Maxwell's equations, we know that it is an electromagnetic wave. But what is waving? It turns out something called electric and magnetic fields are waving. These fields are related to the electrical and magnetic forces associated with particular charged objects. But in general, there is always a medium that a wave phenomenon is waving 'in'. So what is light waving 'in'? Maxwell postulated that there is an omnipresent 'ether' which is the medium of propagation. The ether would represent a special, absolute reference frame. We on Earth, in our laboratory, can have a net velocity relative to this ether.

Observers with different motion should observe different apparent values of c.



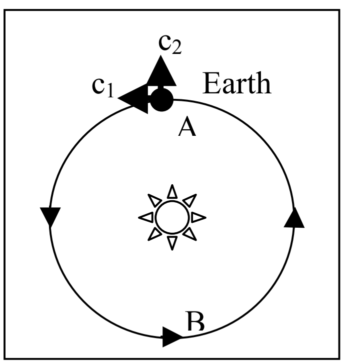
If reference frame O corresponds to the ether reference frame, then the velocity of light within this frame is 'c'. The Galilean velocity tranformations should hold for observers of light in any frame moving at a speed v relative to absolute ether. If an observer O' moves at a velocity v (as pictured), then she should measure v_{light} in her rreference frame to be c-v.

Michelson-Morley Experiment

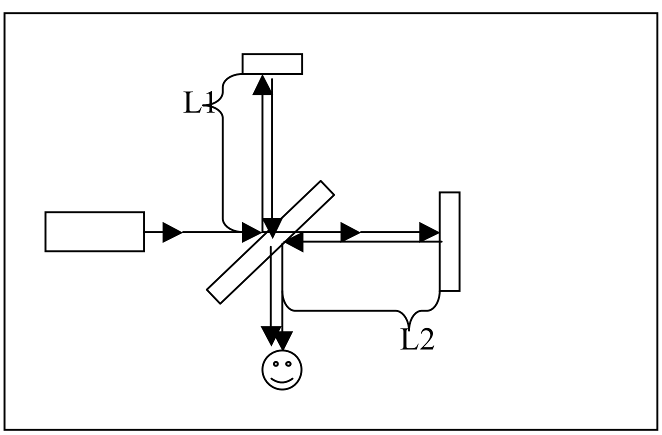
So how do we detect the unique ether reference frame? We can't know, for instance, what the velocity of any particular inertial reference frame is relative to the ether. Taking our cue from the above, we should measure the speed of light in several different states of motion and comparing our measured speeds for light will show differences due to the motion of the ether. However, we run into the problem that the velocities that we are usually dealing with on Earth (10s of mph) are incredibly small compared to the speed of light.

One way to attain somewhat higher velocities is to utilize the motion of a point on the surface of the Earth. The Earth rotates on its axis and it revolves around the Sun. Let's consider a particular location on the Earth, say Cleveland, Ohio ;) Let's also assume, for the sake of argument, that the motion of a lab in Cleveland in June is such that the ether reference frame has zero relative velocity with respect to our lab. Call this point A in Earth's orbit. In this case, our measurement of the velocity of light in this situation would give us $v_{measured} = c$. Now consider the scenario 6 months later (call this point B). Earth is now moving in the opposite direction with a velocity v relative to the ether rest frame. In this situation, we should measure that $v_{observed} \neq c$. It turns out that, although the orbital velocity of the Earth is still small compared to c, an experiment could be devised in the late 1800s to detect the expected shift in light's velocity.

The Michelson-Morley experiment sought to measure the speed of light in two different reference frames and thereby determine the motion of the observer thru the absolute ether reference frame. As mentioned, it used the Earth's orbital motion to produce a measurable effect. Instead of waiting 6 months for Earth to reverse it's motion, the scientists chose to compare the measured speed of light parallel to the Earth's orbital motion (c1) and perpendicular to Earth's orbital motion (c2) at a given point. This orientation maximizes the velocity difference for a particular point on Earth's orbit. If at point A, Earth's orbital motion is not causing it to be at rest (stationary, or zero velocity) in the ether reference frame, then we should observe c1 not to equal c2.



The specific apparatus looked like:



where $L_1=L_2=L$. A velocity difference in light's travel thru L_1 & L_2 cause beams to arrive at the observer at different times. (Note: in practice, the two lengths are not precisely equal and it is necessary to rotate the apparatus by 90 degrees and make measurements at other times of day or year to make sure this difference is corrected for.) The difference in arrival times is given by

$$\frac{\Delta t1 - \Delta t2}{\Delta t1} = \frac{v^2}{2c^2}$$

If you plug in the value for Earth's orbital velocity, you get a relative difference in time of $5x10^{-9}$. This is too small to measure directly.

The experiment therefore used interference of two beams of light which were split into L1 and L2 and then recombined. If the velocity of light in one of the directions was greater with respect to the Earth's motion because of the relative motion of the ether reference frame, then this would result in a different number of wavelengths being traveled by that light. The intensity of the recombined light at the observer would be subject to interference of the light traveling the two paths. If there was an absolute ether reference frame, then one would expect to see a change in the observed light intensity with the orientation of the apparatus with respect to the relative motion of the ether reference frame. The precision of the experiment is aided by the tiny wavelength of visible light, which is less than a millionth of a meter.

No variation in light intensity was observed, meaning that the light traveling L1 was arriving at the observer with the same time relative to light going in path L2, irrespective of the motion of the laboratory. This means $c_1=c_2$! In general, no change in the velocity of light was observed for the different reference frames tested. A lot of effort was expended to explain this, but no luck. The velocity of light is always c, regardless of your motion relative to the source of the light!

Lecture 5: Special Relativity

Maxwell's Eq's

- imply speed of light (electromagnetic waves) is a constant, 'c'
    - contradicts Galilean transformation
        - rests on idea of absolute space & absolute time

Michelson-Morley Experiment

- refutes Galilean transformation as applies to light
- impossible to measure absolute velocities of light with respect to "ether" frame
    - idea of ether looses any relevance

light (EM waves) convey electromagnetic forces

- for these to be same in all inertial reference frames, c=constant

Conclusions drawn from electromagnetic theory (Maxwell's Equations):

Einsteinian Relativity

<u>Einstein Principle of Relativity</u>

1) The laws of physics (=mechanics AND electromagnetism) must be the same in all inertial reference frames (a generalization of Galilean relativity.)

2) The speed of light in a vacuum ($c=3x10^8$ m/s) has same value in all inertial reference frames, regardless of velocity of the observer or velocity of the light emitting source

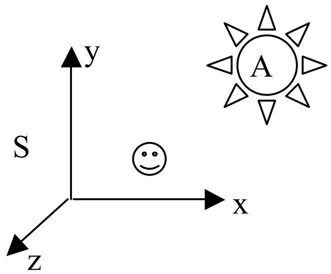
2) is required by 1)

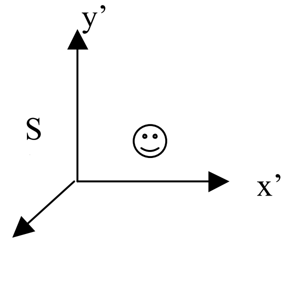
1) means: no preferred inertial reference frames

- impossible to detect "absolute motion"
- experiments (like measuring 'c') must get same result in all inertial frames
- explains Michelson-Morley result (Einstein may not have known experiment).

Classical Time & Simultaneity

Classically, consider an event ‘A’





Here event A appears at:
x = 1 km
y = 2 km, t = noon

Here event A appears at:
x’ = –10 km
y’ =2 km, t’ = noon

- same event, different coordinates
- length in xyz same as in S & S'
- length in t same in S & S'

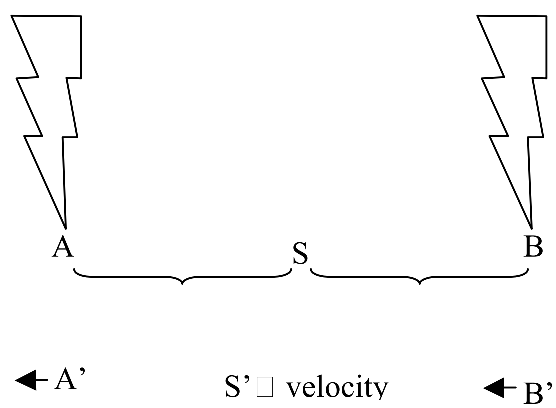
  independent of motion of observer

- a universal time scale exists that is the same for all observers

  simultaneous events are simultaneous for everybody

Relativistic Time

Let's say S sees lightning strike at A & B simultaneously

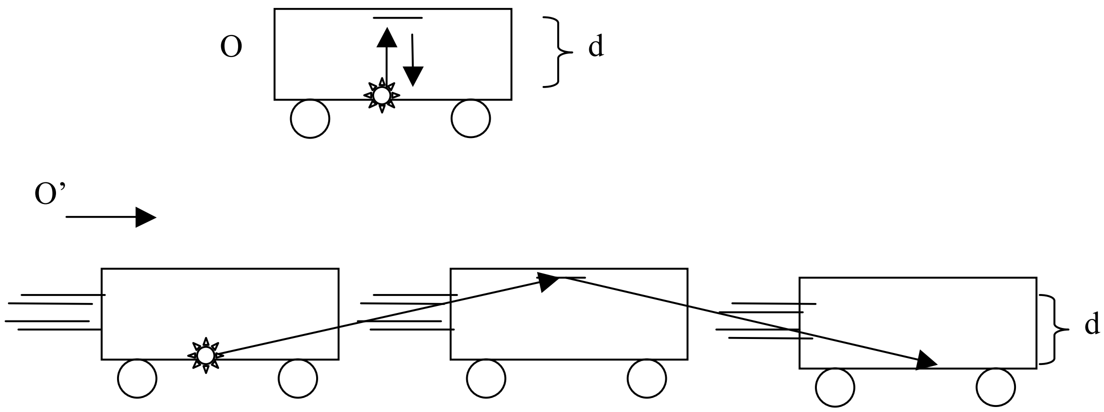


S is midway between A & B so the distances to A & B are equal (so S sees simultaneous strikes)

- when S sees strikes, S' has already seen B', but has not seen A'
- therefore S' does not see simultaneous events
  - because c is not dependent on motion of observer

even if correct for transit time, S' sees non-simultaneous events.

Time Dilation



For observer O, light travels distance , D=2d, so Δt_p=2d/c (This is the proper time, the time measured when at rest with respect to the clock).

For observer O', light travels distance D>2d since the mirror moves during transit of light

since c=constant
Δt=D/C > 2d/c It takes longer!

Specifically $\Delta t = \gamma\ \Delta t_p$

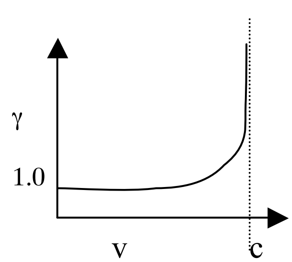
$$\gamma \equiv 1/\sqrt{(1 - v^2/c^2)} \quad \neq 1 \text{ only when v is similar to c.}$$

The time measured by an observer moving with respect to a clock is > time measured if at rest relative to a clock

Time Dilation Calc.

if v=25mph = 0.01 km/s
$\gamma = 1/\sqrt{(1 - (.01\text{ km/s})^2/(3\text{x}10^5\text{ km/s})^2)}$ = 1.000 000 000 000 001

if v=30 km/s	γ = 1.000 000 005
if v=10,000 km/s	γ=1.0006
if v=100,000 km/s	γ=1.06
if v=290,000 km/s	γ=3.9

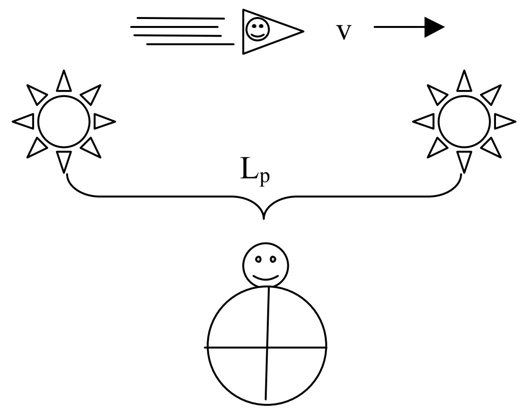


Length Contraction

- measured distance depends on frame of reference
- let's define "proper length"
    - proper length is length of an object measured by someone at rest relative to that object
        - i.e. observer for whom endponts of object remain fixed in position

<u>Example</u>

- observer on Earth
- i.e. at rest relative to 2 stars measure distance between stars=L_p
- Δt is the time ship takes to travel between the stars (according to O on Earth)

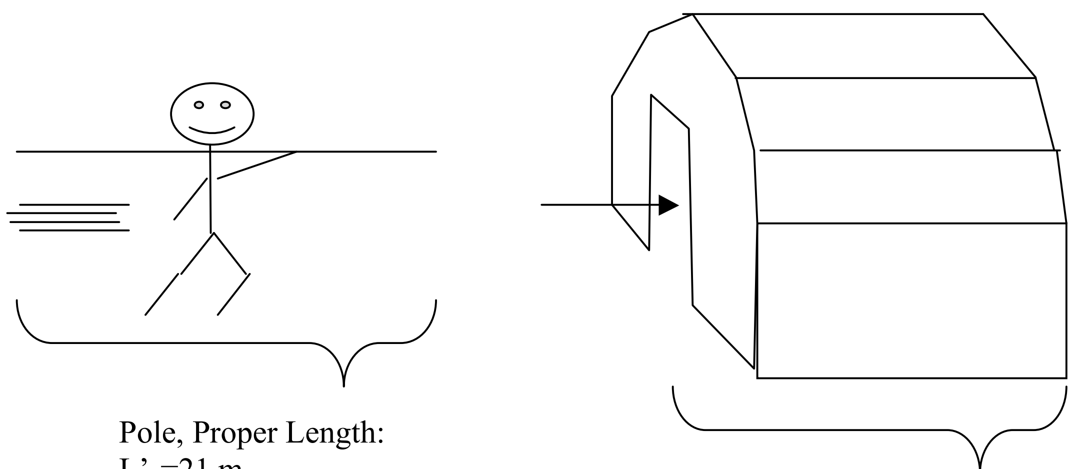


<u>space traveller</u>
stars cross at same position to her
see $\Delta t_p=\Delta t/\gamma$

therefore the space traveler must measure distance between stars, $L=L_p/\gamma$

since the space traveler sees a shorter time, she measures a shorter distance.

Pole in the Barn



Pole, Proper Length:
L'_p=21 m

Barn, Proper Length:
L_p = 20 m

- Pole-vaulter runs into barn
    - is there a moment when front & back gates can close & open simultaneously and not hit the pole?
    - in frame of barn, doors close simultaneously
        - see length L (=L_p/γ) since $\gamma \geq 1.0$, then $L < L_p$
        - if v=c/3, γ = 1.06 and
        - L will be 19.7m!

Okay, what about in frame of runner?
- she doesn't see pole shrink, but the barn, which was already to small, shrinks by 6%!
    - the doors don't close simultaneously
    Leading end of pole exits rear door before the trailing end comes to front door

Lorentz Transformations

- Newtonian mechanics predicated on Galilean transformations

- Galilean transformations need to be replaced with new ones:

$x'=\gamma(x-vt)$
$y'=y$ Lorentz
$z'=z$ Transformations
$t'=g(t-v\ x/c2)$

$vx'=(vx-v)/(1-vx\ v/c2)$

Recall Galilean transformation
$x'=x-vt$
$y'=y$
$z'=z$
$t'=t$
$v_x'=v_x-v$

Relativistic Momentum & Energy

Momentum:

(recall Newtons' F=ma) □ not correct in Relativity.

Force = change in momentum per unit of time

- at low velocity: twice the change in velocity=2x the momentum
- at high velocity: "γ" factor means small change in velocity produces larger changes in momentum
  - as v approaches c then $\gamma \rightarrow \infty$

  takes infinite force to accelerate to v=c
  Therefore, a massive particle can never attain v=c

Energy

- classically: different forms of energy (heat, "kinetic energy" of motion)
- relativistically: $E=\gamma mc^2$ (Total Energy: This what's conserved)
  - when v=0; $E=mc^2$ = rest energy of particle

mass is a form of energy

Total Energy - This what's conserved

Special Relativity (summary):

Given the predictions of Maxwell's equations that the speed of light was a constant, and it's inconsistency with the Galilean statement of the Principle of Relativity, Einstein postulated a new concept of relativity to bring mechanical and electromagnetic phenomena under one roof. In this statement, he hypothesized that all physical phenomena, including electromagnetic, should proceed regardless of the intertial reference frame they in which they occur. This modification had the result that it stipulated the speed of light to be constant irrespective of inertial reference frame, and so 'explained' the Michelson-Morley experimental result.

The results of this hypothesis were a number of predications of how space and time would behave in cases where velocities approached the speed of light. The first impact was the removal of the concepts of an absolute space and an absolute time. For time, this may be most clearly seen in a discussion of simultaneity. Classically, if two events are observed to occur simultaneously by one observer, then all other observers will see them as simultaneous. Relativistically, however, the speed of light is constant. As a result,

Another manifestation of the relativity of time concerns the phenomenon of 'time dilation'. If an object is moving in a particular inertial reference frame, O, with a velocity approaching the speed of light in frame O, then a clock attached to that object will appear to run slower to an object stationary in frame O. We discussed an illustration of this using a flashlight and a mirror mounted on a train. In this illustration, the light takes longer to return to the flashlight as observed by a stationary observer in O when compared to the time measured by an observer on the train. This is because it must travel a longer distance in the O reference frame than just twice the distance from the flashlight to the mirror.

Space also becomes relative because as objects move faster, their apparent lengths appear to shrink. We discussed this issue with a description of the Pole in the Barn 'paradox'. In this situation, a pole-vaulter runs toward a barn with a pole which is longer than the size of the barn. Doors are set such that they momentarily close and open simultaneously (in the frame of the barn) when the pole-vaulter is at the center of the barn. Since the length of the pole shrinks with increased velocity, there is a velocity for which the pole will fit within the barn in the barn's reference frame and will clear the doors when they close. The 'paradox' arises when we consider that the pole-vaulter measures the proper length of her pole and sees the barn length contracted to be even smaller than its proper length. Since it's proper length is smaller than the pole's proper length, the pole cannot possible fit within the barn. However, we are saved by the fact that the doors which close simultaneously in the barn's reference frame, do not close simultaneously in the runner's reference frame. The back door closes and opens first, then the front door, thus allowing the runner thru without breaker her pole.

Finally, we described some of the implications of relativity on our concepts of energy. In classical physics, energy can come in several forms. For a moving particle, one form of energy is that of motion, i.e. its kinetic energy. An unheated particle at rest (velocity = 0) has no energy in the classical scheme. In relativity, it was realized that total energy of a particle is the sum of a kinetic term and a 'rest energy'. The rest energy is usually expressed as $E = mc^2$ and expresses the idea that energy and mass are equivalent.

These ideas of relativistic physics have been observed in some terrestrial situations (eg. time dilation with GPS satellite measurements).

Lecture 7: Relativity & Gravitation

Special relativity has a limitation

- we can only discuss physics which is happening in inertial reference frames. This is a fairly limiting criterion. What happens if we want to understand physics in an accelerating frame, or a frame in a gravitational field? Einstein wanted to understand this question when he worked out the theory of 'general' relativity.

Classical Mechanics

Newton's force law: $F = m_i a$

Here we are dealing with an 'inertial mass', which is the property of a body which indicates how hard it is to accelerate that body for a given force. If a body is twice as massive as another, it will require twice the force to acclerate it to a specific acceleration.

Newton's Law of Gravity: $F = -G\, m_1\, m_2/r^2 = m_2 g$

Here, m1 and m2 are 'gravitational masses'. They are like gravitational 'charges' in that they indicate how strongly a body generates a gravitational pull. We can simplify the above equation for a body on the surface of the earth (m1 is for Earth) to be in terms of a gravitational mass, m2, and a constant gravitational acceleration, $g = 9.8\ m/s^2$. There is no reason to suppose that this gravitational mass 'charge' has any relationship to the inertial mass, and Newton was aware of this. He found, however, that an appropriate choice of constant, G, would permit one to use the same value in both of the above two equations. What is surprising is that this single constant, G, works for all masses. This has been tested over 12 orders of magnitude in mass. So the gravitational mass property of a body is always exactly proportional to the inertial mass property. Why is this possible?

- Depends completely on the notion of universal time.
    - The Forces that the sun & planets exert on each other are determined by respective distances from each other <u>at the same time</u>.

    - "Action at a distance" is at the heart of Newtonian Gravity.

But in Special Relativity, there is a relative space & time which depends on the observer. As a result, ambiguities arise

Generalizing Relativity L7p3

In the Special Theory of relativity: Go to a frame where there are no accelerations.

- $\therefore$ far from other masses (i.e. Gravity)

Acceleration seems special because we can feel it. Is this true?

First, can we detect gravity. Consider:

1. you are in elevator in otherwise empty space accelerated upward by "a"=9.8 m/s^2.
2. You are standing on the surface of Earth and so being accelerated towards its center by 'g' = 9.8 m/s^2

You can't distinguish 1) from 2). Therefore they are equivalent.

Free Falling Reference Frame L7p4

Now consider you are in an elevator which is in free-fall in the gravity of the Earth. There is an apple sitting on the floor next to you. During your free-fall, you will not feel the floor pushing up on you since you have the same velocity and acceleration as the elevator. You will also see the apple is stationary with respect to you. In other words, it has no acceleration within the local reference frame associated with your motion. In other words, your local reference frame (the elevator) shows no evidence of forces acting. So gravity is undetectable by observing relative motion of these two bodies. Acceleration due do gravity is undetectable by a frame that is accelerated in time with physical objects subject to gravity & no other forces.

"free fall reference frame": cannot, however, be extended arbitrarily far thru space & time since the strength of gravity will change.

- "With respect to a free-falling frame of reference, material bodies will be unaccelerated if they are free from non-gravitational forces."
- This is the same as inertial frames of reference when there is no gravity present
  - When there is NO gravity present, a frame of reference is extensible (can be extended in space)
- But when gravity is present, we only can construct a local frame of reference. The reference frame cannot be extended arbitrarily far thru space and time since the strength of gravity will change with position.
  - $\therefore$ the presence of gravity is tantamount to the non-extensibility of local free falling frames of reference
  - The study of gravity is replaced with the study of inhomogeneities of motion in space.

Postulates of General Relativity L7p5

All local frames of reference are equally valid – there is no way to choose a class of preferred frames of reference with which to formulate the laws of nature.

<u>Principle of Covariance</u>
All laws of nature have the same form for observers in any frame of reference, whether that frame is accelerated or not.

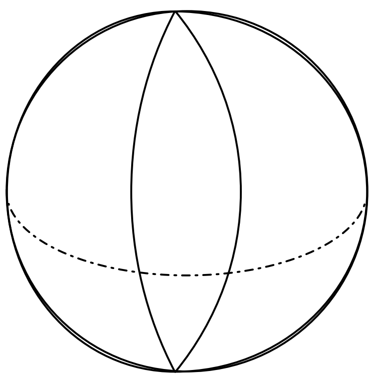
<u>Principle of Equivalence</u>
"In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects."

Spacetime L8p1

4 dimensional geometry : x, y, z, <u>time</u>,
- Frame of reference is a 4D coordinate system

Consider a more easily visualized lower number of dimensions. We are familiar with 3 spatial dimensions, x,y,z. Now, think of your perspective if you were confined to two dimensions. A flat 2 dimensional geometry would correspond to a tabletop, for instance. A curved 2 dimensional geometry would correspond, for instance, to the surface of a sphere, say a ball. A person confined to the surface of the ball can only move in two possible ways: left-right, or forward-backward. They cannot move up or down. The interesting thing is, this geometry can be ‘curved’ whereas the tabletop ‘flat’ geometry is not curved. This shape of the geometry will affect motion observed.

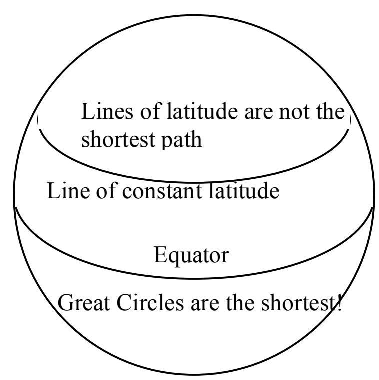
Consider two people walking parallel to each other in a flat geometry. They will never approach or recede from each other. In other words, their relative motion will be constant and there will be no accelerations between them. So there is no apparent force being exerted on either of them from each other’s vantage point.



Now consider the same two people on the surface of a sphere. Let’s say that they are on the equator and they both start walking north (i.e. parallel to each other). Their paths will converge at the pole. Their relative velocity will change from the time they start at the equator to the time they get to the pole. This is interpreted as a force being exerted of attraction between the two people.

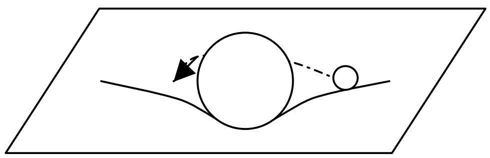
Now we want to consider the motion of light in spacetime. The shortest distance between any two points in spacetime is a ‘geodesic’. This is a ‘straight line’ on a curved surface or in a curved space. Light, by virtue of it’s having the maximum velocity in nature, travels along a geodesic in spacetime.

We can consider these geodesics in our sphereical geometry case. For a sphere, geodesics are segments of ‘great circles’ which have a diamater which is the same as the diameter of the sphere. Lines of constant latitude are not great circules – their segments are not the shortest paths. This is why airplanes do not travel along lines of constant latitute, but tend to bow northward (in the northern hemisphere) to take the shortest path.



Relativistic Gravity L8p2

It turns out the geometry of curved spacetime can have many possibilities
How can we figure out which possibility is true?



Geometry affects motion
- Acceleration in this case

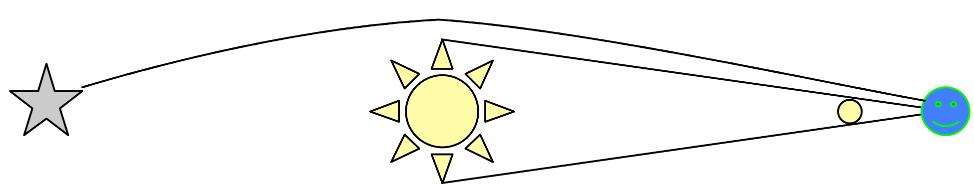
- Gravity is interpreted as a curvature of a 4D spacetime geometry

- We need to find geometry which presents accelerations we see
  - The combination of the Principle of Covariance & Newton's Gravitational Law gives a unique Theory of General Relativity. The curvature is such that it causes masses to attract one another, as shown above, rather than repel each other. The amount of attraction is set by the kind and degree of curvature.

Bending of Light L7p6

To extend an inertial frame through space & time
- Must compare distant clocks & straight rulers
  - Requires light to carry information

If we are in the presence of gravity, the shape of space-time is changed from flat to curved. Light has energy, & by Special Relativity and Principle of Equivalence, it is subject to gravity.
The speed of light cannot change, but it's direction can, according to the curvature of spacetime.



Solar Eclipse 1919 – Starlight observed bent by gravity of the sun!

1) Consider two inertial reference frames, S and S'. S' is moving with velocity, v, relative to S. When an object is moved with velocity V' in frame S', state the Galilean velocity transformation which will allow you to aclculate the object's velocity in frame S. [4 pts]

2) Diagram the Michelson-Morley experiment and explain the measurement obtained. [8 pts]

3) Why did the Michelson-Morley experiment call classical mechanics into question? [10 pts]

4) What is the difference between Galileo's and Einstein's statement of the Principle of Relativity? [3 pts] In what way does Einstein's formulation reconcile mechanics and electromagnetism, or account for the result of the Michelson-Morley experiment? [7 pts]

5) Describe the way Special Relativity changes our conception of time by referring to how it effects the observation of simultaneous events in different inertial reference frames. [10 pts]

6) An object moving at velocities approaching the speed of light will appear longer than if that object were stationary. (T or F) [2 pts]

7) Clocks moving at velocities approaching the speed of light will appear to register time fore slowly than if they were stationary. (T or F) [2 pts]

8) Describe the 'Pole-in-the-Barn' paradox. [10 pts]

9) In special relativity, the total energy of a particle includes a term which is based on the particle mass, $E = mc^2$. (T or F) [2 pts]

---------------- General Relativity Questions

1) State the Principle of Covariance. How does this relate to the Principle of Relativity as expressed in Einstein's Special Theory of Relativity? [10 pts]

2) State the Principle of Equivalence. Discuss how this follows from thinking about free-falling reference frames. [10 pts]

3) Motion of bodies is directly tied to the curvature of the geometry of space and time in General Relativity. (T or F) [2 pts]

4) Explain the concepts of inertial mass and gravitational mass. Why are they related? [10 pts]

5) Describe, possibly with a 2-dimensional spatial geometry analogy, how geometry can influence motion. [10 pts]

6) A free falling reference frame is not extensible to large distances. (T or F) [2 pts]