Ampere's Law

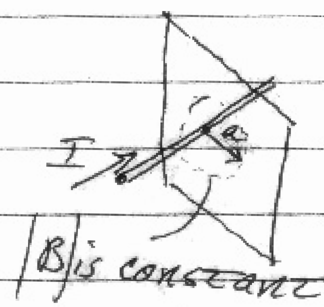
→ in electrostatics

Coulomb's Law → calculate $\vec{E}$ field from point charges

→ for highly symmetric cases, can solve with Gauss' Law more easily

→ by analogy: Coulomb's :: Biot-Savart
as Gauss :: ?

- Consider a wire



- lines of B form circles around a wire

$B \propto I, \; B \propto a^{-1}$

- we want a relationship between $\vec{B}$ & I easy to determine for certain easy shapes

$\vec{B} \cdot d\vec{s}$ for wire $= Bds$ since $\vec{B} \parallel d\vec{s}$

path integral around wire $\oint \vec{B} \cdot d\vec{s} = \oint B ds = B \oint ds = \frac{\mu_0 I}{\cancel{2\pi a}} (\cancel{2\pi a})$

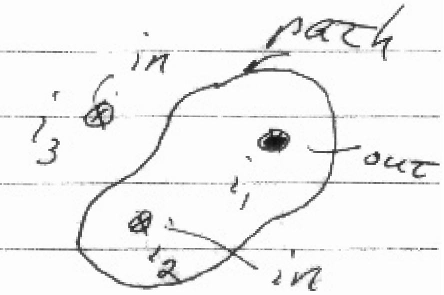
constant on our chosen path

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I}$$

(generally applicable)

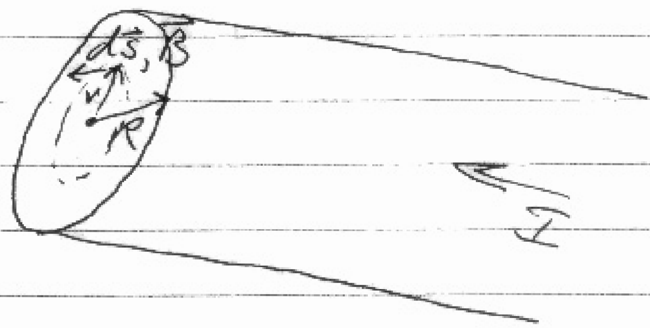
So, $\vec{B}\cdot d\vec{s}$ around any closed path is proportional to the current passing thru the surface bounded by the closed path.

- Like w/ Gauss's Law, we only are concerned with sources of field (current-carrying wires) inside path



$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, i_{in}$$
$$= \mu_0 \underbrace{(i_1 - i_2)}_{\text{net current}}$$

- of course, this path + scenario is not symmetric enough to proceed further



$d\vec{s} \parallel \vec{B}$ (symmetry)

What is B for $r < R$?

- current, i, thru surface scales as area of surface:

$$i = \frac{\cancel{\pi} r^2}{\cancel{\pi} R^2} I$$

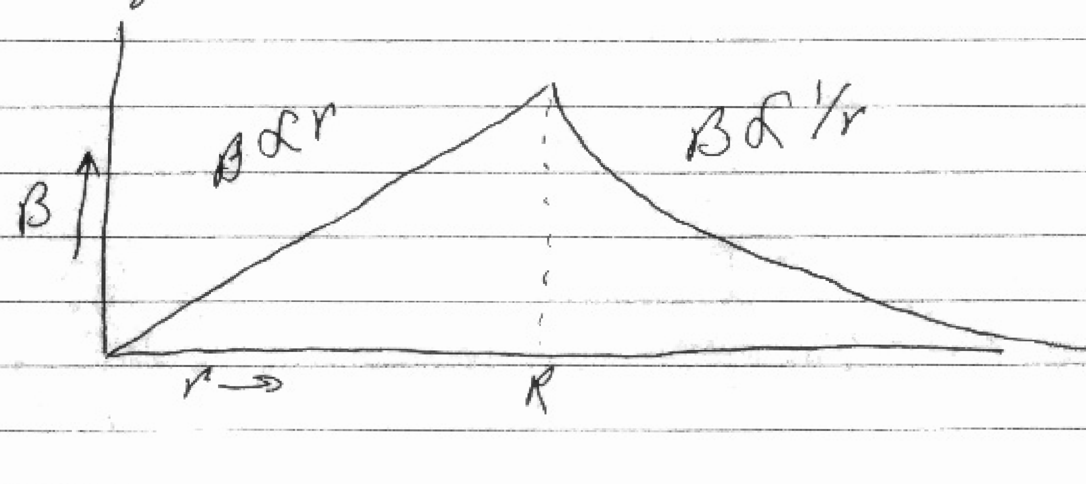
From Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

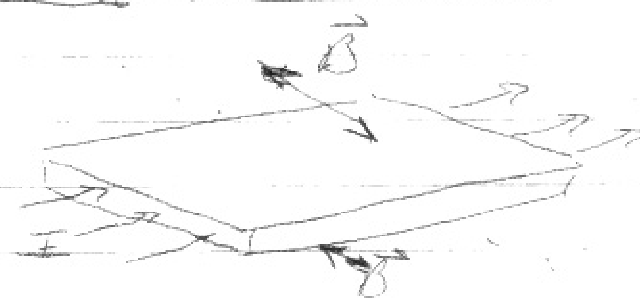
$$B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2} I\right)$$

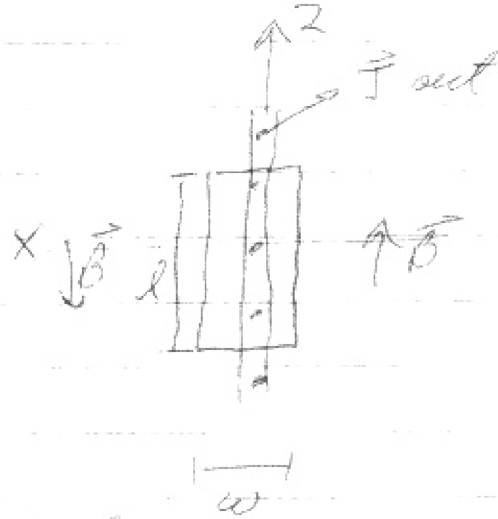
$$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

so B-field scales as r in this region



Infinite Current Sheet





→ rectangular path
→ 2 horiz. sides of length w
→ no contrib. since $\vec{B} \cdot d\vec{s} = \phi$
→ direction of field needs to be ∥ to sheet
→ can't penetrate sheet or violate Biot-Savart

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I = \mu_0 J \ell$

current in plane of rectangle

$2B\ell = \mu_0 J \ell$

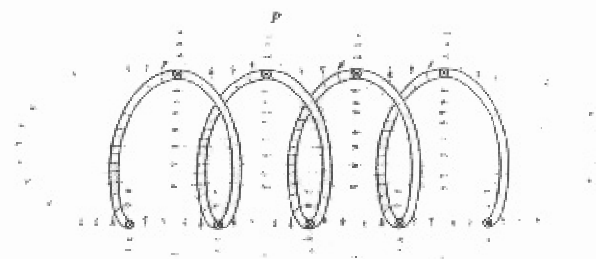
$\boxed{B = \mu_0 J/2}$

→ so does not depend on distance
(analogous to $E = \sigma/2\epsilon_0$)

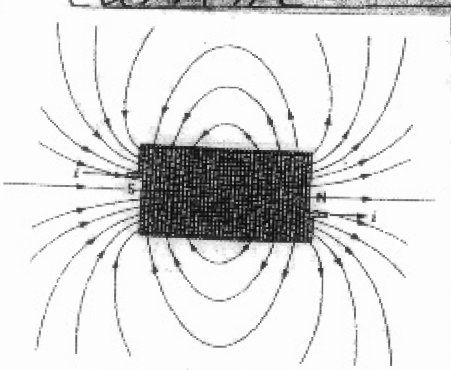
Solenoids + Toroids

solenoid: a long wire wound in a close-packed helix and carrying current I.

→

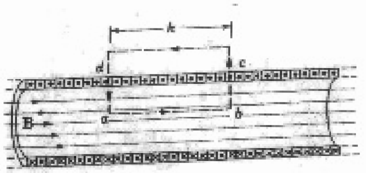


→ when wires close, acts almost like a sheet of current



→ ideal case: length >> diameter

pick paths ∥ or ⊥ to $\vec{B}$



$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \cancel{\int_b^c \vec{B}\cdot d\vec{s}} + \cancel{\int_c^d \vec{B}\cdot d\vec{s}} + \cancel{\int_d^a \vec{B}\cdot d\vec{s}}$$

ϕ ϕ ϕ

Since $h \gg l_{wire}$ → $i = i_0 n h$

n = # turns/length

As aresult

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} = Bh = \mu_0 i$$

$$B = \frac{\mu_0 (i_0 n \cancel{h})}{\cancel{h}}$$

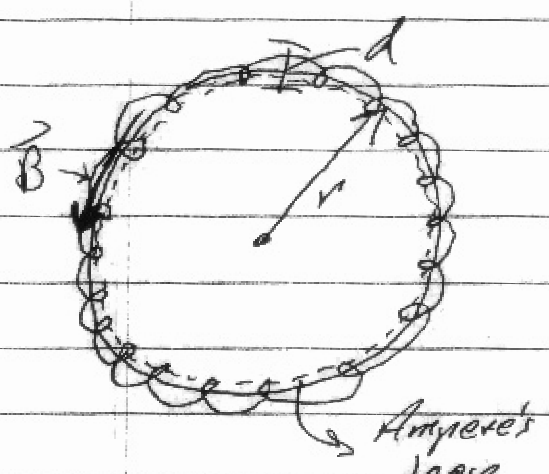
$\vec{B}$ for a solenoid

$$\boxed{B = \mu_0 i_0 n}$$

So we have a uniform B-field inside the solenoid.

→ not a bad approximation for actual solenoids near axis

Toroid: like a solenoid bent closed to a donut shape.



Using Ampere's Law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i$$

$$B(2\pi r) = \mu_0 i_0 N \rightarrow \text{\# turns total}$$

$$\boxed{B = \frac{\mu_0 i_0 N}{2\pi r}}$$

So B is not constant

(note: tokamak)

Magnetic moment of atoms

e^- in a current loop around atoms. The current

$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

T: time interval

magnetic moment, $\mu = IA$

$$\mu = \frac{ev}{2\pi r}(\pi r^2) = \frac{1}{2}evr$$

$$= \frac{e}{2m_e}L$$ where L is orbital ang. momentum of e^- $(L = m_e v r)$

→ permanent magnetic moment, para + ferromagnetic materials

→ not diamagnetic materials