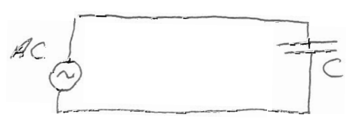
## Capacitive Circuit

$V_c = V_{c,m} \sin \omega t$




What is relationship between voltage + current? Recall $q_c = C V_c$

$$q_c = C V_{c,m} \sin \omega t$$

Take derivative to get the current

$$i_c = \frac{dq_c}{dt}$$

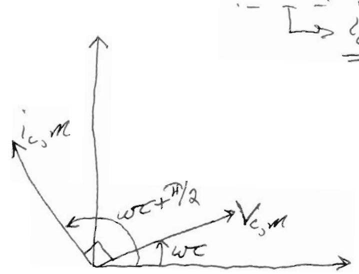
$$i_c = \omega C V_{c,m} \cos \omega t$$

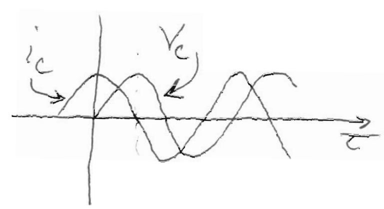
$\omega C \to \frac{1}{X_c} \to$ capacitive reactance

$$= \frac{V_{c,m}}{X_c} \sin(\omega t + \pi/2)$$

$\frac{V_{c,m}}{X_c} \to \underline{\underline{i_{c,m}}}$

90° out-of-phase with V






## Inductive Circuit:

$V_L = V_{L,m} \sin \omega t$


We know $V_L = -L \frac{di}{dt}$, so we have

$$i_L = \int \frac{di}{dt} dt = -\frac{V_{L,m}}{\omega L} \int \sin \omega t \, d(\omega t)$$

$$= -\frac{V_{L,m}}{\omega L} \cos \omega t$$

$\omega L \to X_L$ = inductive reactance

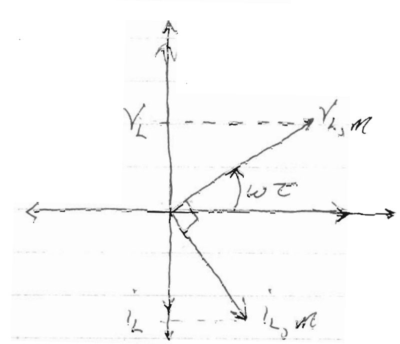
Noting that $-\cos \omega t = \sin(\omega t - \pi/2)$,

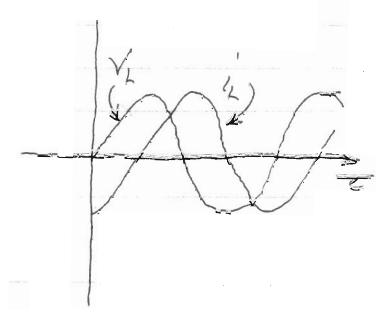
$$i_L = \frac{V_{L,m}}{X_L} \sin(\omega t - \pi/2)$$

$\frac{V_{L,m}}{X_L} \to \underline{\underline{i_{L,m}}}$

$-90°$ out-of-phase w/ voltage

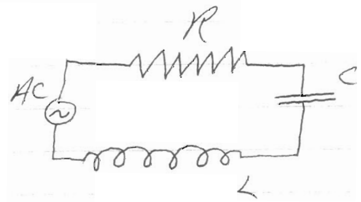
Phasor diagram:



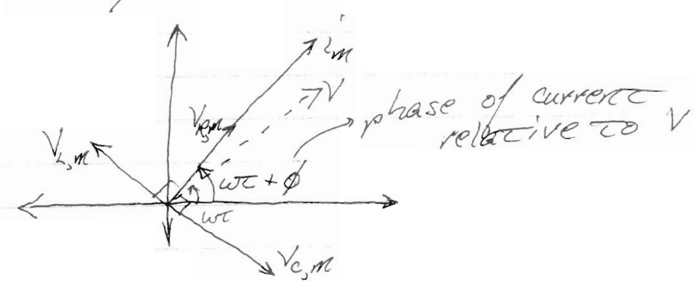
## The RLC Circuit Revisited:

$V = V_m \sin \omega t$




Again, we would like to understand the relationship between current, $i$, and $V$.

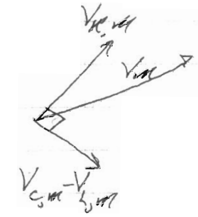
Since each element has voltage drop, at any instant $V = V_R + V_L + V_C$. The current thru all three elements must be the same at any given time, so relative phases exhibited by voltages






## RLC Impedance

Consider components // & ⊥ to $V_{R,m}$




$$V_m^2 = V_{R,m}^2 + (V_{C,m} - V_{L,m})^2$$
$$= (i_m R)^2 + (i_m X_C - i_m X_L)^2$$
$$= i_m^2 (R^2 + (X_C - X_L)^2)$$

Thus $i_m = V_m \Big/ \sqrt{R^2 + (X_C - X_L)^2} = V_m / Z$

and we define denominator as impedance,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$= \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \quad \text{(a function of } \omega\text{)}$$

Note, $\omega$ here is driving frequency of impressed voltage. This can be different than the natural frequency of the LC components $\left(\omega_{nat} = \frac{1}{\sqrt{LC}}\right)$.

The final phase angle $\phi$ is

$$\underline{\tan \phi} = \frac{V_{C,m} - V_{L,m}}{V_{R,m}} = \frac{i_m X_C - i_m X_L}{i_m R} = \boxed{\frac{X_C - X_L}{R}}$$