Maxwell's Equations

Fundamental electrical + magnetic relationships:

Gauss's Law

$$\oint \vec{E}\cdot d\vec{A} = q/\epsilon_0 \qquad (1$$

→ E field from a charge distribution

$$\left(\oint \vec{B}\cdot d\vec{A} = 0 \right) \qquad (2$$

→ no magnetic charge distribution since no monopoles

Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -d\phi_B/dt \qquad (3$$

→ changing B-field gives E-field

Ampére's Law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \qquad (4$$

→ moving charge gives B field

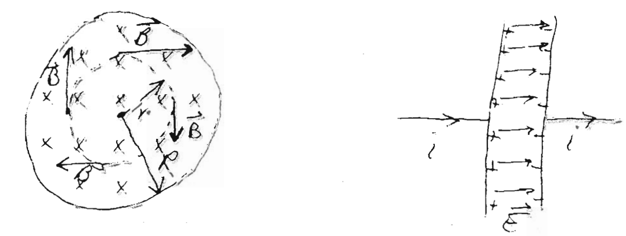
Maxwell saw potential symmetry of (3 + (4.

$$\oint \vec{B}\, d\vec{l} = \mu_0 I + \underline{\epsilon_0 \mu_0 \frac{d\phi_E}{dt}}$$

→ a changing electric field can generate a B-field

Induced Magnetic Fields

Consider circular parallel plate capacitor



(from side-view)

Here, we consider case when $|E|$ is varying at a constant rate, $\Delta E/\Delta t$. Observation reveal a magnetic field $\vec{B}$ induced by changing $\vec{E}$,
- obeys "induction" piece of the generalized Ampére's Law.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

r ≤ R case

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}\left(E(\pi r^2)\right)$$

$$\boxed{B = \tfrac{1}{2}\mu_0 \epsilon_0\, r\, dE/dt}$$

r > R case

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}\left(E(\pi R^2)\right)$$

$$\boxed{B = \mu_0 \epsilon_0 \frac{R^2}{2r}\, dE/dt}$$

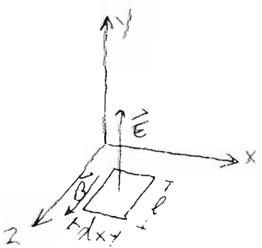
Consider $r = R = 50\text{mm}$ and $\Delta E/\Delta t = 10^{12}\, V/m s$.

$$\underline{B} = \tfrac{1}{2}\mu_0 \epsilon_0 R \frac{\Delta E}{\Delta t} = \boxed{0.0028 \text{ gauss}}$$

Induced B-fields very small.

What is relation of $E + B$?

If $q = I = 0$, can write



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\Downarrow \qquad \Downarrow$$

$$B_x \ell - B_{x+dx}\,\ell = \mu_0 \epsilon_0 \frac{\partial}{\partial t}(EA)$$

$$-\partial B\,\ell = \mu_0 \epsilon_0\, \ell\, dx \frac{\partial E}{\partial t}$$

$$-\cancel{\ell}\frac{\partial B}{\partial x}\cancel{dx} = \mu_0 \epsilon_0 \cancel{\ell\, dx}\, \frac{\partial E}{\partial t}$$

$$\boxed{\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}}$$

One can do same for Faraday's Law and get

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}}$$

If we take $\frac{\partial}{\partial x}$ of both sides, get

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right)$$

$$= -\frac{\partial}{\partial t}\left(-\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Electromagnetic Waves

The expression $\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$ is structurally similar to

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \left(\text{wave eq.}\right)$$

$\hookrightarrow (\omega/k = \lambda f) \rightarrow$ speed of wave

What is waving? Good Question

$\rightarrow$ we say E or B fields

Speed of electromagnetic wave:

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3\times10^8\,\text{m/s}\,!!!$$

speed of light

Light is a form of EM wave.

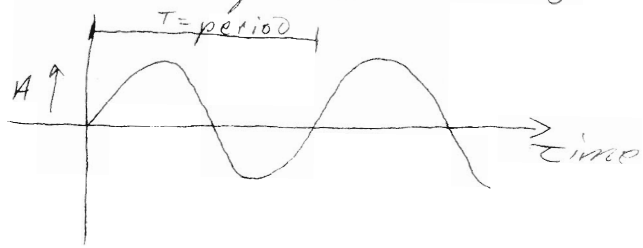
Electromagnetism not only 'unified', but it subsumes now the field of Optics.

By the way, Maxwell's Eq's consistent with Special Relativity

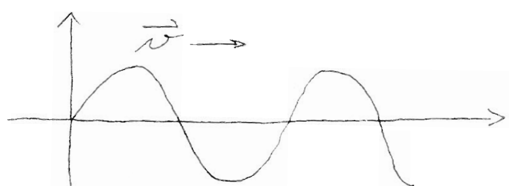
(Newton's Laws isn't).

Waves 101

If you disturb medium in one place
- motion imparted to neighbors



A is maximum displacement
$T = \frac{1}{f} = \frac{2\pi}{\omega}$



$\underbrace{\lambda = vT}_{\text{wavelength}} = v/f$ (distance travelled by maximum during one period)

transverse wave: $\vec{A} \perp \vec{x}$ (water, string)
longitudinal wave: $\vec{A} \parallel \vec{x}$ (sound in air)

polarization: when orientation of transverse waves favors particular direction in transverse plane