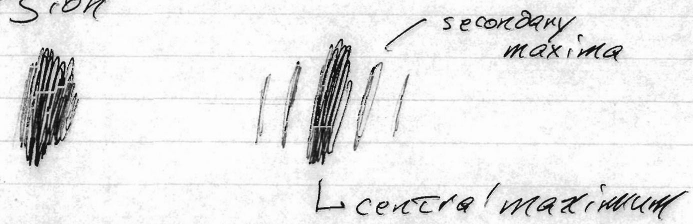
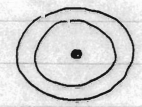
Diffraction Patterns

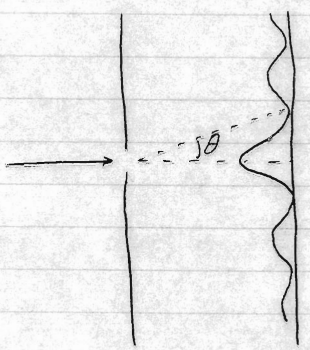
→ diffraction → refers to fact that wave spreads beyond shadow of aperture

→ we don't just see wide illuminated region



→ illuminate penny
→ get bright spot



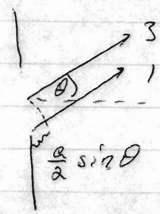


assume rays reaching screen approx. parallel

Fraunhofer diffraction pattern

→ a slit has a non-zero width
→ each point in slit ~~is~~ a source of light waves
→ as if multiple slits, all interfering

- Consider a slit with two halves



- a path difference between 1 & 3

- if $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$: destr. interference
waves from upper half of slit destructively with lower half

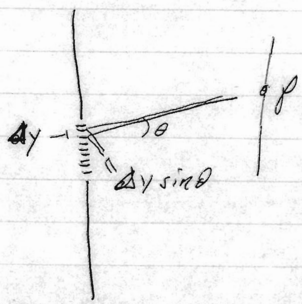
first min. — i.e. $\boxed{\sin\theta = \pm\frac{\lambda}{a}}$

→ if divide into 4ths, 6ths..

$\sin\theta_{dark} = m\frac{\lambda}{a} \quad (m = \pm 1, 2, 3 \ldots)$

↳ where have dark regions

Intensity - Single Slit



each subsection of slit, Δy is source of coherent radiation
→ gives some ΔE @ point, P

- adjacent Δy's → phase different by Δy sinθ

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$

→ total phase difference across slit

$$\beta = N\Delta\beta = \frac{2\pi}{\lambda}N\Delta y\sin\theta = \frac{2\pi}{\lambda}a\sin\theta$$

$a = N\Delta y$ (width of slit)

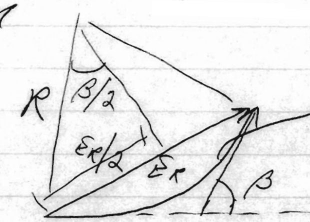
→ 1st max → $\beta = 0$ → $\varepsilon_R = N\varepsilon_0$
→ 1st min → $\beta = 2\pi$ → $\varepsilon_R = 0$ ($\beta = N\Delta\beta = 2\pi$)
→ 2nd max → $\beta = 3\pi$ → $\varepsilon_R = ?$
- 2nd min → $\beta = 4\pi$ → $\varepsilon_R = 0$

$$2\pi = \frac{2\pi}{\lambda}a\sin\theta_{dark}$$

$$\boxed{\sin\theta_{dark} = \lambda/a}$$

Consider limit of $\Delta y \to 0$ (dy)
→ all $\Delta\varepsilon$'s delineate arc in phasor diagram

$$\sin\frac{\beta}{2} = \frac{\varepsilon_R/2}{R}$$



ε_0 is arc length $= R\beta$

- Geometrically ε_R is a "chord" thru circle partly containing arc

$$\frac{\varepsilon_R}{2R} = \sin\frac{\beta}{2} \Rightarrow \varepsilon_R = 2\left(\frac{\varepsilon_0}{\beta}\right)\sin\frac{\beta}{2} = \varepsilon_0\frac{\sin(\beta/2)}{\beta/2}$$

$$I = I_{max}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

↳ $\beta/2 = \pi a\sin\theta/\lambda$

when $\beta/2 = m\pi$ → minima
↳ $m = \pm 1, 2, 3 \ldots$

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Two slit diffraction

- → with two slits → diffraction from each slit
  - interference between 2 slits

$I \propto I_{single-slit} \times I_{2-slit}$

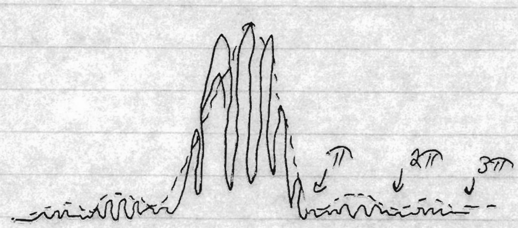
(diffr.) ↓ (interf.) ↓

$$I = I_{max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2 \times \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

↳ slit width (center peak highest)

↳ 2-slit seperation

peak I's roughly constant

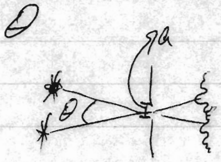


Resolution

→ 2 sources of light

→ ea. produce seperate image/dot + diffraction pattern

→ angular sep., θ



Rayleigh's criterion

→ when central image of ① falls on first min. of ②

→ termed "just resolved"

$\sin\theta = \boxed{\lambda/a \approx \theta_{min}}$ (slit of width 'a')

→ circular aperture

$$\boxed{\theta_{min} = 1.22\, \lambda/D}$$

↳ Diam. of aperture