$\vec{E}$ field of a Continuous Charge Distribution

If charge distribution is continuous (i.e. has huge # of charges)

- each small piece in $\Delta x \Delta y \Delta z$ with charge Δq can be summed

$$\Delta \vec{E} = k \frac{\Delta q}{r^2} \hat{r}$$

so that

$$\vec{E} \simeq k \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

as $\Delta q_i \to 0$,

$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

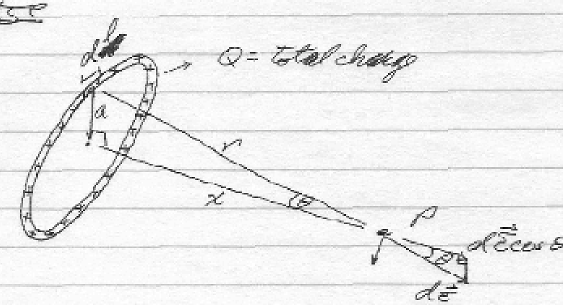
If charge arranged in a:

	charge density is:*	dq is
line, l	$\lambda = Q/l$	$\lambda\, dl$
surface, A	$\sigma = Q/A$	$\sigma\, dA$
volume, V	$\rho = Q/V$	$\rho\, dV$

* if Q is uniformly distributed.
Q = total charge

Example: Ring of Charge

E field at P on axis of ring a distance 'x' from it's center



For element, dl, we have $dq = Q\,dl/2\pi a$

Need to sum $\vec{E}$ vectorially for all elements!

- components $\perp$ to x cancel
- components along axis are

$$dE \cos\theta = k \frac{dq}{r^2} \cos\theta = k \frac{dq}{(a^2+x^2)} \frac{x}{\sqrt{a^2+x^2}}$$
$$= k \frac{x\, dq}{(a^2+x^2)^{3/2}}$$

Since x, a are the same for all dq's, we have the integral

$$E = \int dE \cos\theta$$
$$= k \frac{x}{(a^2+x^2)^{3/2}} \int dq$$
$$= \boxed{k \frac{x\, Q}{(a^2+x^2)^{3/2}}}$$

when $x >> a$, $E \to k \dfrac{Q}{x^2}$ + ring acts like a point charge.

Motion of Charged Particles in E field

Consider a charged particle in an $\vec{E}$ field. What is force acting on it?

$$\vec{E} = \vec{F}/q \implies \underline{\vec{F} = q\vec{E}}$$

What is the particles motion?

F also $= ma$

$$\therefore q\vec{E} = m\vec{a} \implies \boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

when $q > 0$ $\quad \vec{a} \parallel \vec{E}$

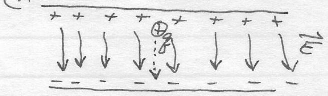
$q < 0$ $\quad \vec{a} \downarrow\uparrow \vec{E}$

Example:

$\vec{E}$ field between 2 Plates

A) Consider q with mass, m, placed in $\vec{E}$ field uniform between two plates:

{ATLAS Calorimeter!!}



2 oppositely charged metal plates

→ take charge q stationary & let go.

$$\vec{a} = q\vec{E}/m \rightarrow a_x = 0, \; a_y = qE/m$$

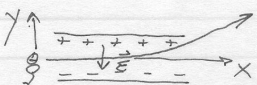
- uniformly accelerated motion

→ for uniform acceleration

$$v = at = qEt/m$$

$$\therefore \underline{\underline{y}} = \tfrac{1}{2}at^2 = \underline{\underline{\frac{qEt^2}{2m}}}$$

B) Deflecting an Electron



→ travel with velocity, v_0, in x-direction: $\underline{x(t) = v_0 t}$ {TV!}

Given expression for y above, this gives

$$y = \frac{qE}{2m}\left(\frac{x}{v_0}\right)^2$$

→ a parabolic path until exit, then tangent.