Gauss's Law

→ Coulomb's Law gives relation between electric force (field) + charge
→ calculations difficult in many charge distributions

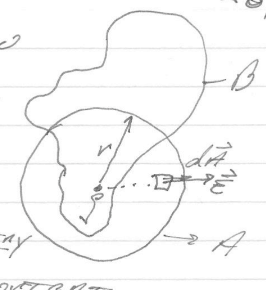
→ Gauss's Law another form of Coulomb's Law → simplifies some specific calculations
→ but it's now considered the more fundamental expression because of the insights it provides

- applies to any closed surface + any charge distribution

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

- where q_{in} is net charge enclosed by surface
→ if no charge ⇒ no flux (+ ∴ no field)
→ external charge makes no contribution to flux
→ used to evaluate $\vec{E}$ only if charge distribution very symmetric

Gauss's Law + Coulomb's Law



Consider point charge.
- spherical symmetry
- so use spherical surface
symmetry very important

→ $d\vec{A} \parallel d\vec{E}$

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = q/\epsilon_0$$

$$E \oint dA = E(4\pi r^2) = q/\epsilon_0$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}$$

in the presence of a second charge, we get

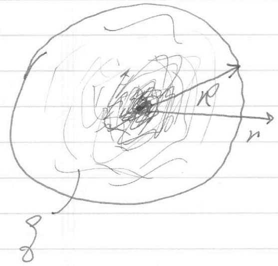
$$F = k\frac{q\,q_2}{r^2}$$

→ surface B → same # field lines (flux) since same q_{in}
→ but $\oint_B \vec{E}\cdot d\vec{A}$ very hard!!

net flux thru any closed surface surrounding a point charge $= q/\epsilon_0$
- independent of shape of surface

independent of any charge outside of surface

Some Applications: Spherically Symmetric Charge Distrib.



Some charge density, ρ, only function of r.

if $r > R$, then by Gauss's Law, only care about q

$$\phi_E = \oint \vec{E}\cdot d\vec{A} = E\oint dA = E\,4\pi r^2 = \frac{q}{\epsilon_0}$$

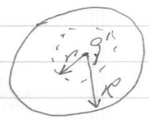
just like if a point charge.

$$\therefore\ \underline{E = k\frac{q}{r^2}}$$

Special case: ρ constant over volume (i.e. not dependent on r)

→ for a given radius, r

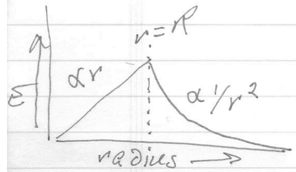
→ all charge outside r does not contribute to E.



$$\rightarrow\ q' = q\,\frac{V'}{V} = q\,\frac{(\frac{4}{3}\pi r^3)}{(\frac{4}{3}\pi R^3)} = q\,\frac{r^3}{R^3}$$

→ from q', we can again use above relation

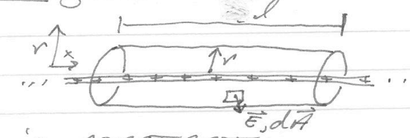
$$E = k\frac{q'}{r^2} = \boxed{k\,\frac{qr}{R^3}}$$



→ agree @ $r=R$

→ derivative discontinuous

Infinite Line of Charge



line of charge

→ λ is linear charge density + is constant

→ by symmetry, $\vec{E}$ must be directed radially (no 'x' component)

→ so a cylindrical symmetry is apparent

→ so closed surface is a cylinder of length l

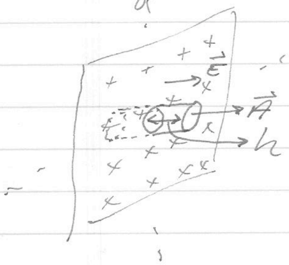
$$\phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}, \quad q = \lambda l$$

constant on cylinder

$$E\underbrace{\oint dA}_{2\pi r l} = \lambda l/\epsilon_0$$

$$\boxed{E = 2k\,\frac{\lambda}{r}}$$ → direction radially

Infinite Sheet of Charge



- uniform charge density
  $\sigma = q/A$
- $\vec{E} \perp$ to surface by symmetry

→ construct cylinder $\perp$ to plane
→ no flux thru cylinder, only thru ends

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = q/\epsilon_0$$

$$E\cancel{A} + E\cancel{A} = \sigma \cancel{A}/\epsilon_0$$

$$E = \frac{\sigma}{2\epsilon_0}$$

- no radial dependence
  ∴ E field same for all points on each side of sheet