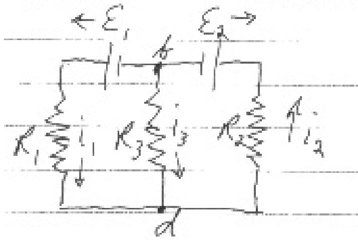
(1)

Junction Theorem



In multiloop circuits

- current in R_1 may not be same as in R_2 or R_3
- have two junctions @ b + d

At each junction, charge flows in on some conductors, + flows out on others

@ b: i_2 flows in while $i_3 + i_1$ flow out

$\therefore\ i_1 + i_3 = i_2$ (from conservation of charge)

– or –

$i_1 + i_3 - i_2 = \boxed{\sum i = 0}$

At any junction, the algebraic sum of the currents must be zero.

(2)

Consider previous diagram

For left loop in counterclockwise direction:

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$$

The right loop has

$$-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0$$

Plus we know junction rule

$$i_3 + i_1 - i_2 = 0$$

You can solve these 3 equations for the 3 unknown currents:

$$i_1 = \frac{\mathcal{E}_1 (R_2 + R_3) - \mathcal{E}_2 R}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2 (R_1 + R_3)}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

+

$$i_3 = - \frac{(\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1)}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

so we see i_3 goes up (i.e. opposite to our choice in Figure.)

(3)

Some notes about loop + junction rules:

\# of times can use junction rule equals one less than the # of junction pts.
(= 1 in example above)

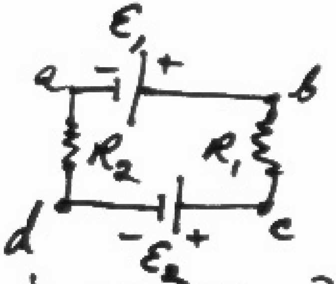
\# loops depends on finding new loops that have new unknown currents in each new equation
(= 2 for previous example since one loop has 2 new currents, + next loop adds the third remaining current)

The # of independent Eqs will equal the # of unknown currents.

1304/1404

L9, p3
~~L8, p13~~

Ex 28.8



a) what is current?

$\sum \Delta V = 0$ (loop rule)

so a→b→c→d

$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$

$-I(R_1+R_2) = \mathcal{E}_2 - \mathcal{E}_1$

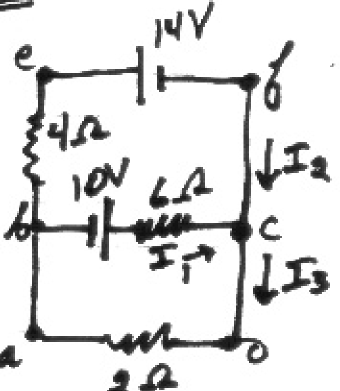
$$\boxed{I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1+R_2}}$$

$= \dfrac{6V - 12V}{8\Omega + 10\Omega} = \underline{\underline{-0.33A}}$

−ve sign means I goes opp. assumed, [a→d→c→b]

1304/1404 L8, p14 L9, p14

Ex 28.9



Can't reduce to R's in series + parallel

@c) (1) $I_1 + I_2 = I_3$

for abcda) (2) $10V - 6\Omega I_1 - 2\Omega I_3 = 0$

befcb) (3) $-14V + 6\Omega I_1 - 10V - 4\Omega I_2 = 0$

using (1) in (2),

$10V - 6\Omega I_1 - 2\Omega(I_1 + I_2) = 0$

(4) $10V = 8\Omega I_1 + 2\Omega I_2$

simplifying (3),

(5) $-12V = -3\Omega I_1 + 2\Omega I_2$

(4) - (5) gives $22V = 11\Omega I_1$

$\boxed{I_1 = 2A}$

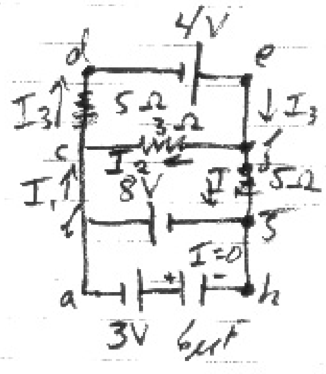
Use in (5) to get $\boxed{I_2 = -3A}$

Use in (1) to get $I_3 = -1A$

1304/1404 L9, p15

Ex 28.10

Multi-loop circuit



using $\Sigma I_{in} = \Sigma I_{out}$ at f, gives

$I_3 = I_2 + I_1$

Using $\Sigma \Delta V = 0$,

bcfgb: $8V + 3\Omega I_2 - 5\Omega I_1 = 0$

cdefc: $+4V - 3\Omega I_2 - 5\Omega I_3 = 0$

use $I_1 = I_3 - I_2$ into bcfgb,

$8V + 3\Omega I_2 - 5\Omega I_3 + 5\Omega I_2 = 0$

$8V + 8\Omega I_2 - 5\Omega I_3 = 0$

cdefc - bcfgb gives

$-4V - 11\Omega I_2 = 0 \Rightarrow I_2 = -\frac{4}{11}A$

using cdefc

$4V + \frac{12}{11}V - 5\Omega I_3 = 0$

$I_3 = +\frac{1}{5}\left(+\left(\frac{56}{11}\right)\right) = 1.02$

$I_3 = I_1 + I_2$

$I = 1.05 + \frac{4}{11} = 1.38A$

1304/1404 L9, p6

28.10 b)

What is Q on 6μF capacitor?

→ find ΔV across capacitor,

$\Sigma \Delta V = 0$

$-8V + \Delta V_{cap} - 3V = 0$

$\Delta V_{cap} = \underline{11V}$

$Q = C\,\Delta V_{cap}$ $C = 6\mu F$

$\boxed{Q = 66\mu C}$

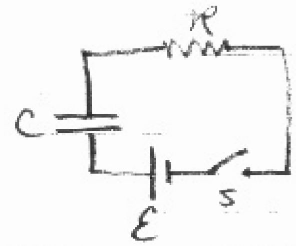
1304/1404 ~~L9, p7~~ L10, p1

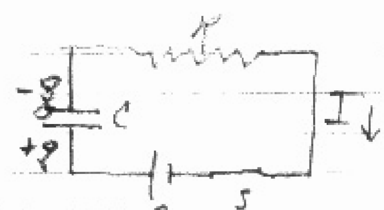
RC Circuits:

→ circuit containing Resistor + cap in series

→ Don't limit ourselves to steady state situation

→ consider time when cap is changing





→ ε sets up $\vec{E}$ in wires

→ causes charge ~~exchanged~~ between plates + connecting wires

→ until capacitor fully charged

→ max charge depends on voltage

→ I=0 when max Q reached

when switch closed

$$\mathcal{E} - \frac{q}{C} - IR = 0$$

q → instantaneous values

$\frac{q}{C}$ → potential diff. across cap

at $t=0$ $\quad I_0 = \frac{\mathcal{E}}{R}$ (I is maximum as no $\vec{E}$ in cap)

$\mathcal{E}$ entirely felt across resistor

when charged fully, $I=0$, so

$Q = C\mathcal{E}$ (max. charge)

To get analytical expression use $I = dq/dt$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$= -\frac{q - C\mathcal{E}}{RC}$$

$$\frac{dq}{q - C\mathcal{E}} = \frac{-1}{RC}\,dt$$

Integrating + using $q=0$ @ $t=0$

$$\int_0^q \frac{dq}{(q - C\mathcal{E})} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - \mathcal{E}C}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

$$\frac{q - \mathcal{E}C}{-\mathcal{E}C} = e^{-t/RC}$$

$$-q/\mathcal{E}C = -1 + e^{-t/RC}$$

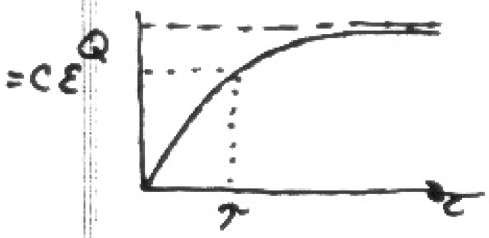
charge as fn. for charging cap

$$q(t) = \mathcal{E}C\,(1 - e^{-t/RC})$$

$$= Q\,(1 - e^{-t/RC})$$

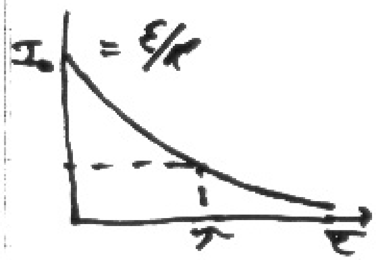
Differentiating: $\boxed{I = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}}$

So,



Capacitor charge vs time

no $Q \to$ max.



Circuit current

max $I \to 0$

time constant, $\tau = RC$

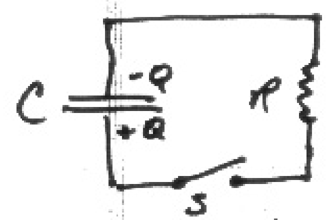
→ time by which I goes to $1/e$ of initial, I_0

↳ $= 0.368\, I_0$

→ in 2τ $I = e^{-2} I_0 = 0.135\, I_0$

Ex 28.13 Discharging a Capacitor

How do Q, I vary



loop eq:

$$-\frac{q}{C} - IR = 0$$

but $I = dq/dt$, so

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$\boxed{q(t) = Q e^{-t/RC}}$$

$$\boxed{I(t) = \frac{dq}{dt} = \frac{-Q}{RC} e^{-t/RC}}$$

$$I_0 = Q/RC$$

1304/1404 L10, p6

→ after how long is Q 1/4 of Q_0

$q(\tau) = Q e^{-\tau/RC}$

$\frac{Q}{4} = Q e^{-\tau/RC}$

$\ln\left(\frac{1}{4}\right) = \ln\left(e^{-\tau/RC}\right)$

$+\ln 4 = +\tau/RC$

$\tau = (\ln 4) RC = \boxed{1.39 \tau}$

→ how about half-life

$\frac{Q}{2} = Q e^{-\tau_{1/2}/RC}$

$\boxed{\tau_{1/2} = 0.693 \tau}$