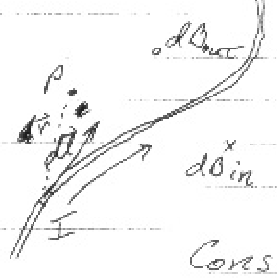
Calculating B: the Biot Savart Law



How calculate $\vec{B}$ @ P near a current-carrying conductor of arbitrary shape?

Consider Coulomb's Law for E field by analogy:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r} \quad (d\vec{E} \parallel \hat{r})$$

we would integrate to get

$$E = \int dE$$

For magnetism from a current element, $id\vec{l}\times\hat{r}$

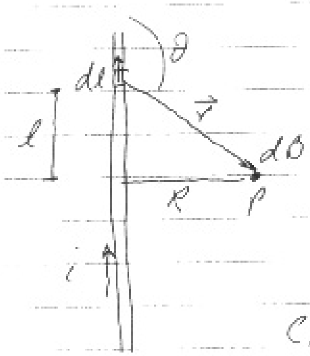
$id\vec{l}\times\hat{r}$ is analogous to $dq\,\hat{r}$ ~~for~~ E field.

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}}$$ Biot-Savart Law

μ_0 = permeability constant $(= 4\pi\times10^{-7}\ Tm/A)$

$$B = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\hat{r}}{r^2}$$

Example: Straight Wire



To find $\vec{B}$ @ P

$$dB = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

- $d\vec{l}\times\hat{r}$ has same direction (into page) for all $d\vec{l}$

Convert to scalar expression + integrate

$$B = \int dB = \frac{\mu_0 I}{4\pi}\int_{l=-\infty}^{l=\infty}\frac{\sin\theta\, dl}{r^2}$$

Since l, r + θ are dependent $(r = \sqrt{l^2+R^2};\ \sin\theta = \sin(\pi-\theta) = R/\sqrt{l^2+R^2})$, we substitute to get

$$B = \frac{\mu_0 i}{4\pi}\int_{-\infty}^{\infty}\frac{R\,dl}{(l^2+R^2)^{3/2}} = \frac{\mu_0 i}{4\pi}\left[\frac{l}{R(l^2+R^2)^{1/2}}\right]_{-\infty}^{\infty} = \boxed{\frac{\mu_0 i}{2\pi R}}$$

where 'R' is ⊥ distance from the wire

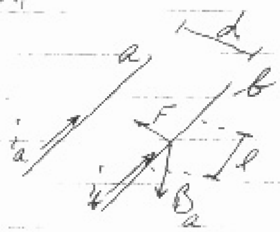
→ So we have that B forms a series of concentric circles around the wire with $1/R$ dependence.

Two Parallel Conductors

A wire feels force from external B if there is a current

$$\vec{F} = i\vec{l} \times \vec{B}_{external}$$

Consider wire a:



- get $\vec{B}_a$ nearby
- at wire b, the magnitude of B_a is

$$B_a = \frac{\mu_0 i_a}{2\pi d} \quad \text{(direction down)}$$

The force on b is thus

$$\underline{\underline{F_b}} = i_b \vec{l} \times \vec{B}_a = i_b l B_a = \boxed{\frac{\mu_0 l i_a i_b}{2\pi d}}$$

→ For anti-parallel wires: <u>attraction</u>
→ <u>same</u> sign currents: <u>repulsion</u>

<u>Definition</u>: When a conductor carries a <u>steady</u> <u>current</u> of <u>1A</u>, the quantity of <u>charge</u> that flows thru a cross section of <u>conductor</u> in 1s is 1C.