(1)

Self Inductance

From Faraday's Law

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

and $\phi_B \propto B \propto I$ in a current loop.

$$\therefore \mathcal{E} \propto -\frac{dI}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

L → a constant called "inductance"
→ depends on geometry of coil

In general with coil of N turns

$$\mathcal{E} = \underbrace{-N\frac{d\phi_B}{dt} = -L\frac{dI}{dt}}$$

equating + integrating over time

$$N\phi_B = LI$$

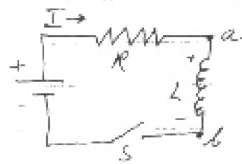
$$\therefore L = N\phi_B/I$$

units: 'henrys' $1\,H = 1\,V\cdot s/A$

(2)

RL Circuits

- an 'inductor' (symbol ⅏) has large self-inductance
  - so it opposes changes in current in a circuit



When we close the switch, I increases ($dI/dt > 0$)

$$\therefore \mathcal{E}_L = -L\,dI/dt < 0$$

$\mathcal{E}_L$ → back emf

→ the electric potential decreases when go from a + b

From loop rule

$$\frac{\mathcal{E}}{R} - \frac{IR}{R} - \frac{L}{R}\frac{dI}{dt} = 0$$

if take $x = \mathcal{E}/R - I$ then have

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\int_{x_0}^{x} dx/x = -(R/L)\int_0^t dt$$

$$\ln(x/x_0) = -R/L\; t$$

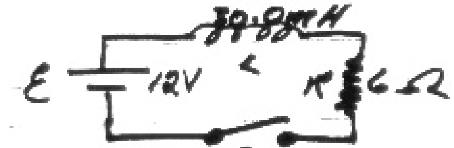
$$x = x_0 e^{-Rt/L}$$

$$\boxed{I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)}$$

$\tau = R/L$ time constant

$\frac{\mathcal{E}}{R}$ → equilibrium value of the current

Ex 32.3:



a) time constant?

$\tau = L/R = \frac{30\times10^{-3}H}{6\Omega} = \underline{5ms}$

b) switch, S closed @ $t = 0s$

$I = \frac{\mathcal{E}}{R}(1-e^{-t/\tau}) = \frac{12V}{6\Omega}(1-e^{-0.4})$

$= \underline{0.659A}$

~~1304/1404~~ L15, p10

c) ΔV across R + L

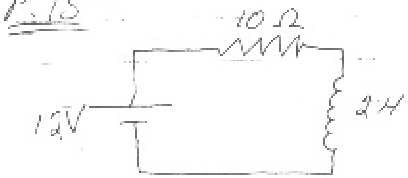
@ $t=0$, $I=0$; $\Delta V_R = 0$

$\mathcal{E}_L = -\mathcal{E}$

$\mathcal{E}_L \downarrow$ as $t \uparrow$ so $\Delta V_R \uparrow$

$\mathcal{E}_L + \Delta V_R = \mathcal{E}$

P.15



a) to reach 50 of final value

$0.5 = I/I_{max} = 1 - e^{-Rt/L}$

$0.5 = e^{-Rt/L}$

$\ln 0.5 = -Rt/L$

$\boxed{0.345} = Rt/L = t/\tau$

One can also use the loop rule Eq. to quantify the energy stored

$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$

$\frac{du}{dt} = LI\frac{dI}{dt}$ (rate energy stored)

$\int du = \int_0^I LI\,dI$

$\boxed{u = \frac{1}{2}LI^2}$ total E. stored

Ex 32.4

Does all E stored initially in inductor appear as internal E in R?

$$\frac{dU_R}{dt} = I^2R = (I_0e^{-Rt/L})^2R$$
$$= I_0^2Re^{-2Rt/L}$$

solve for dU + integrate from t=0 to t→∞

$$U = \int_0^\infty I_0^2Re^{-2Rt/L}dt$$
$$= I_0^2R\underbrace{\int_0^\infty e^{-2Rt/L}dt}_{\to L/2R}$$

$+\frac{L}{2R}(e^{0} - e^{-\infty})$

$$U = I_0^2R\left(\frac{L}{2R}\right)$$
$$= \underline{\underline{\frac{1}{2}LI_0^2}}$$

(same as stored in inductor)

Mutual Inductance

→ 2 adjacent coils

→ one has current, I_1 which sets up magnetic field, $\vec{B}_1$

some field lines pass thru coil 2 which has N_2 turns

→ call it ϕ_{12}

define Mutual Inductance

$$\boxed{M_{12} = \frac{N_2\phi_{12}}{I_1}}$$

→ from Faraday's Law

$$\mathcal{E}_2 = -N_2\frac{d\phi_{12}}{dt} = -N_2\frac{d}{dt}\left(\frac{M_{12}I_1}{N_2}\right)$$

$$= \underline{-M_{12}\frac{dI_1}{dt}}$$ proportional to rate other coil current changing

like self induced $= -L\frac{dI}{dt}$

LC Circuit

total initial energy

$$U = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$

Cap. Ind.

→ must remain constant in time $dU/dt = 0$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right)$$

$$= \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

but $I = \frac{dQ}{dt}$, so

$$\frac{Q}{C}\cancel{I} + L\cancel{I}\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

time-rate of charge & of current

similar to oscillating springs

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$x = A\cos(\omega t + \phi)$$

by analogy

$$Q = Q_{max}\cos(\omega t + \phi)$$

where $\omega = \frac{1}{\sqrt{LC}}$ (natural frequency)

Since charge, Q, varies sinusoidally, so does $I = dQ/dt$

$$I = -\omega Q_{max}\sin(\omega t + \phi)$$

→ ϕ dependent on initial conditions

→ current out of phase by 90° with charge

total energy:

$$U = \underbrace{\frac{Q_{max}^2}{2C}\cos^2\omega t}_{\text{E field}} + \underbrace{\frac{LI_{max}^2}{2}\sin^2\omega t}_{\text{B field}}$$

→ constant sum

1304/1404 L16, p4

RLC Circuit

→ in LC → current oscillates ad infinitem U const w/time
→ add R to circuit
→ U declines w/time to internal energy of resistor

$$\frac{dU}{dt} = -I^2R$$

→ so

$$LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R$$

$$LI\frac{d^2Q}{dt^2} + \frac{Q}{C}I + I^2R = 0$$

like damped harmonic oscill.

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

when R small, like damping

$$Q = Q_{max}e^{-Rt/2L}\cos\omega_d t$$

where $\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$