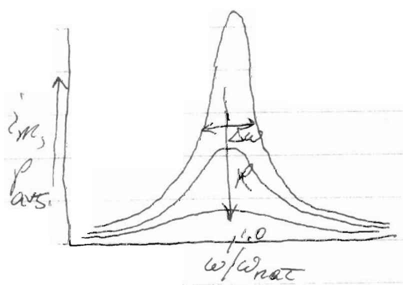
(1)

Resonance

The expression $i_m = V_m / \sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}$ gives the condition for maximum current (or $Power_{avg} = I^2 R$) as

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega_{max} = \frac{1}{\sqrt{LC}}$$

which is the same as the natural frequency of the LC components.



Peak narrows as R decreases
→ would 0 width if R = 0 (i.e. LC circuit case)

It's useful to define a parameter

$$Q = \frac{\omega_{nat}}{\Delta\omega} = \frac{\omega_{nat} L}{R}$$

↳ width when $P_{avg} = \frac{1}{2}$ maximum

(1.5)

P 33.22: RLC Circuit

$$\Delta V(t) = \underbrace{(40.0V)}_{V_{max}} \sin(\underbrace{100}_{\omega} t)$$

Also: L = 160 mH; C = 99 μF; R = 68 Ω

a) Impedance?

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$
$$= \sqrt{(68\Omega)^2 + (100 \cdot 0.16H - \frac{1}{100 \cdot 99\times10^{-6}F})^2}$$
$$= \sqrt{11849\Omega^2}$$
$$= \boxed{108.9\Omega}$$

b) maximum current?

$$I_{max} = V_{max}/Z$$
$$= 40V/108.9\Omega$$
$$= \boxed{0.37A}$$

c) phase?

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$
$$= \tan^{-1}((\omega L - 1/\omega C)/R)$$
$$= \tan^{-1}((16\Omega - 101\Omega)/68\Omega)$$
$$= \tan^{-1}(-1.25)$$
$$= \boxed{-0.896 \text{ rad}}$$

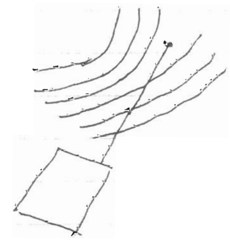
So, $i(t) = 0.37A \sin(100t + 0.896 \text{ rad})$

Since $X_C > X_L$ → $\phi < 0$ and current leads the voltage

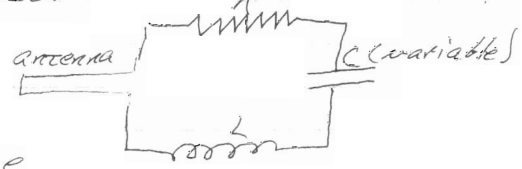
Radio:

radio waves (oscillating E + B fields)

- when impinge on antenna
- induce emf + current on antenna



Attach to RLC circuit with a variable capacitor



Adjust capacitance C so that ω_{nat} changes.

- when $\omega_{nat} = 2\pi f_{FM\ STATION}$ then induced emf gives maximum current in circuit.
- so tuning a radio makes it sensitive to different ω of incoming EM wave
- important to have a narrow "filter" to distinguish overlapping signals

choose R so that have desired Q in narrow range of ω's (like for FM) (88MHz - 108MHz)

P33.38: AM tuning

LC combination used

$L = 0.2\,mH$

C = variable

→ circuit can resonate for range of frequencies from 550kHz to 1650kHz

What is range needed for C?

$$\omega_{Low} = 2\pi f = 2\pi \cdot 550\,kHz = 3500\,rad/s$$

$$\omega_{high} = 2\pi f = 2\pi \cdot 1650\,kHz = 10\,krad/s$$

Since $\omega_{res} = 1/\sqrt{LC} \Rightarrow LC = \frac{1}{\omega_{res}^2}$

$$C = \frac{1}{L\,\omega_{res}^2}$$

$$C_{lo} = \left[0.2\times10^{-3}H \cdot (3500\,rad/s)^2\right]^{-1} = 408\,\mu F$$

$$C_{high} = \left[0.2\times10^{-3}H \cdot (10^4\,rad/s)^2\right]^{-1} = 5\times10^{-5}F \quad (= 50\,\mu F)$$

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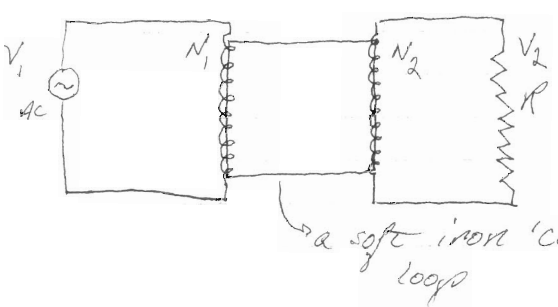
Transformers

- power distribution system
  - → want low voltage @ generation + end-user points
  - → want lowest possible current to ~~reduce~~ minimize $P = I^2R$ losses
  - → so need to have a voltage changing device: transformer



2 coils:

$$V_1 = -N_1 \frac{d\phi_B}{dt}$$

$$V_2 = -N_2 \frac{d\phi_B}{dt}$$

$$\Big\} \quad V_2 = \frac{N_2}{N_1} V_1$$

if $N_2 > N_1$: "step up" transformer

if $N_2 < N_2$: a "step down" transformer

current transforming: $i_1 V_1 = i_2 V_2$

$\therefore \quad i_2 = i_1 (N_1/N_2)$

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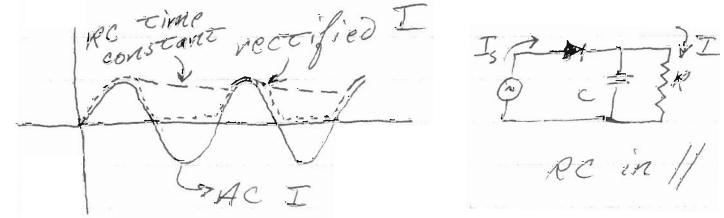
Rectifier

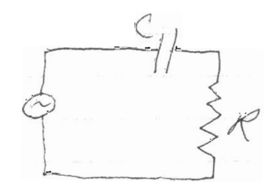
- sometimes we want DC
  - → Diodes (→|) allow current in one direction



→ when supplied I_s goes → 0, C discharges thru R + keeps I high
- residual "ripple" undesireable

Filters

- recall $X_c = 1/\omega C$ is large when ω low



V_R → nonzero for high frequencies

V_c → nonzero fo low frequencies