Thin lenses

At first surface
\[ \frac{n_e}{o} + \frac{n_e}{i} = \frac{n_e}{r} \]

The image from first surface is the source, or object, for the 2nd, O
\[ \frac{n_e}{o} + \frac{n_e}{r} = \frac{n_e}{r'} \]

If lens thin enough, we can ignore \( k \) in this relation, + set
\[ \frac{n_e}{o} + \frac{n_e}{r} = \frac{n_e}{r'} \]

Adding (1) + (2) gives
\[ \frac{n_e}{o} + \frac{n_e}{i} = (n - n_o) \left( \frac{i}{r} - \frac{i}{r'} \right) \]

which can be written for a thin lens in air or vacuum
\[ \frac{1}{o} + \frac{1}{i} = (n - 1) \left( \frac{i}{r} - \frac{i}{r'} \right) \]

where \( i \) is the image distance for the lens.
Problem 28

Contact lens has \( n = 1.5 \) and radii of curvature +1.0 cm and +2.5 cm. What is focal length?

Note both radii are \( > 0 \) →

\[
\frac{1}{f} = (n - 1) \left[ \frac{1}{r'} - \frac{1}{r} \right]
\]

\[
f = (1.5 - 1)\left[ \frac{0.01m}{0.01m} - \frac{0.025m}{0.01m} \right] = 0.05m
\]

\[
f = 20cm
\]

2a) Biconvex lens → left side \( r' = 12cm \) → right side \( r'' = 18cm \)

\( n = 1.94 \) → so \( r > 0 \) and \( r'' < 0 \)

2b) Focal Length:

\[
f = (n - 1) \left( \frac{1}{r''} - \frac{1}{r} \right)
\]

\[
f = 0.94 \left( \frac{0.018m}{0.018m} - \frac{0.012m}{0.012m} \right)
\]

\[
f = 14.4cm
\]

2c) Turn it around

\[
f = (n - 1) \left( \frac{1}{r'} - \frac{1}{r} \right)
\]

But now \( r' < 0 \), \( r'' > 0 \)

\[
f = 0.04 \left( \frac{0.018m}{0.018m} - \frac{0.012m}{0.012m} \right)
\]

\[
f = 14.4cm \quad \text{again}
\]

Problem 31

Thin lens \( w/f = 25cm \)

c) Describe image when \( o = 26cm \)

\[
\frac{1}{i} = \frac{1}{o} + \frac{1}{f} = \frac{1}{0.25m} + \frac{1}{25m} = \frac{1}{0.25m} - \frac{1}{0.35m}
\]

\[
\frac{i}{f} = \frac{1}{25m} = \frac{0.25m}{0.35m}
\]

\[
i = +46.5m
\]

\[
M = -\frac{i}{o} = \frac{46.5m}{26m} = \frac{36}{26} < 0
\]

Image: Real, inverted and enlarged

b) \( o = 21cm \)

\[
\frac{1}{i} = \frac{1}{0.25m} - \frac{1}{21m} \Rightarrow \frac{i}{12m} = \frac{1}{21m} \Rightarrow \frac{i}{12m}
\]

\[
M = -\frac{i}{o} = \frac{12m}{21m} = -\frac{12}{21} = -\frac{4}{7} < 0
\]

Image: Virtual, upright and enlarged