E-field of a Continuous Charge Distribution

If charge distribution is continuous (has vast number of charges)

- each small piece in $dxdydz$ with charge $d\rho$ can be summed
  \[ \Delta E = k \frac{d\rho}{r^2} \]

  so that
  \[ E = k \sum \frac{d\rho}{r^2} \]

  as $d\rho \to 0$
  \[ E = k \int \frac{d\rho}{r^2} \]

If charge arranged in a:

- line, $l$
  \[ \lambda = \frac{\rho}{l} \]

- surface, $A$
  \[ \sigma = \frac{\rho}{A} \]

- volume, $V$
  \[ \rho = \frac{\rho}{V} \]

if $\rho$ is uniformly distributed

$Q = \text{total charge}$

Example: E-field of Charge

- E-field at $r$ on axis of point charge $Q$ at distance $x$ from its center

For element $d\rho$, we have $dE = \frac{kd\rho}{r^2}$

Need to sum $E$ vectorially for all elements!

Components $1 \to x$ cancel
- Components along axis are
  \[ dE = \cos \theta = k \frac{d\rho}{r^2} \cos \theta \approx \frac{d\rho}{r^2} \]

  Since $\theta, a$ are the same for all $d\rho$s, we have the integral
  \[ E = \int \frac{dE \cos \theta}{r^2} \]

  \[ = \frac{k}{2} \int \frac{x}{x^2 + a^2} \frac{d\rho}{r^2} \]

  when $x \gg a$, $E \approx \frac{kQ}{x^2}$ - acts like a point charge.
Motion of Charged Particles in E Field

Consider a charged particle in an E field. What is force acting on it?

\[ \vec{F} = q \vec{E} \Rightarrow F = q \vec{E} \]

What is the particle’s motion?

\[ F = ma \]

\[ \Rightarrow q \vec{E} = ma \]

\[ \Rightarrow a = \frac{q \vec{E}}{m} \]

when \( q > 0 \) \( \vec{a} \parallel \vec{E} \)

when \( q < 0 \) \( \vec{a} \parallel \vec{E} \)

Example:

E Field between 2 Plates

1) Consider a with mass, \( m \), placed in an E field uniform between two plates.

2) Take charge \( q \) stationary + \( q \) on 2 oppositely charged metal plates.

\[ a = \frac{q \vec{E}}{m} \rightarrow a_y = 0, \quad a_x = \frac{q \vec{E}}{m} \]

Uniformly accelerated motion

\[ v = at = \frac{q \vec{E} t}{m} \]

\[ y = \frac{1}{2} a t^2 = \frac{q \vec{E} t^2}{2m} \]

2) Deflecting an electron

- Travel with velocity, \( v_0 \) in \( x \)-direction: \( x(t) = vt_0 \)

Given expression for \( y \) above, this gives

\[ y = \frac{q \vec{E}}{m} \left( \frac{x}{v_0} \right)^2 \]

→ a parabolic path until exit, then tangent.