Flux

flux: a property of all vector fields.
(fr. L. fluere = to flow)
→ related essentially to the amount of field passing thru an area, A ($\propto$ # field lines)

Consider water in pipe again:

- in all cases → # field lines thru surface is same if density, ρ, uniform

scalar

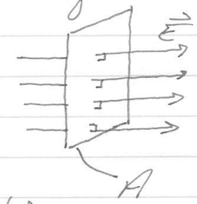
for 1st two: $\phi = \rho \vec{v} \cdot \vec{A}$ represents "mass flux" thru area
→ same flux, different orientation
→ works also for 3rd case: i.e. same # field lines (flux)

→ closed surface

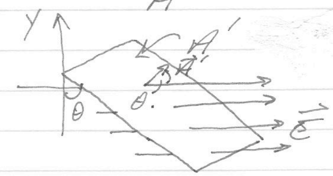
flux = 0
(#lines = #lines out)
→ if no source of flow inside surface

Flux of an Electric Field:

Electric field a vector field (like $\vec{v}$)



$\phi_E = \vec{E} \cdot \vec{A}$
$= EA$



$\phi_E' = \vec{E} \cdot \vec{A}'$
$= EA' \cos\theta$

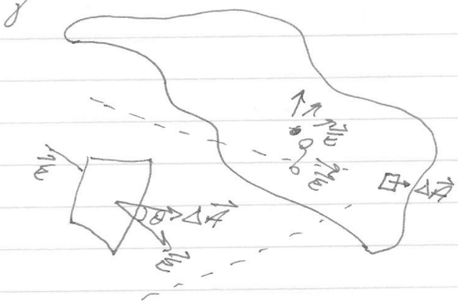
if $A' = A/\cos\theta$ (i.e. length in y direction is same)

→ $\phi_E = EA = \phi_E' = E(A/\cos\theta)\cos\theta$
$= EA$

$\vec{A}$ + $\vec{E}$ are ∥ → when θ = 0, flux is maximum
$\vec{A}$ + $\vec{E}$ are ⊥ → " " θ = 90°, flux = 0

a more complex surface:

$\Delta\vec{A}_i$ is $\perp$ to surface in region "i"



$\Delta\phi_{E_i} = \vec{E}_i \cdot \Delta\vec{A}_i$

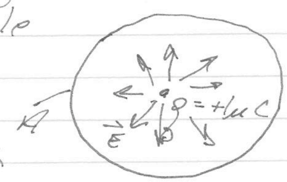
$= E_i \Delta A_i \cos\theta_i$

↳ sum over all ΔA_i

$$\Rightarrow \phi_E = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i = \int_{surface} \vec{E} \cdot d\vec{A}$$

A closed surface → use $\oint$

→ e.g. sphere Example



point source

$E = k\frac{q}{r^2}$; $A = 4\pi r^2$

$\vec{E} \parallel \vec{A}$

$\phi_E = \oint \vec{E} \cdot d\vec{A} = E\oint dA$

$= EA = 1.1 \times 10^5 \, Nm^2/c$

↳ units