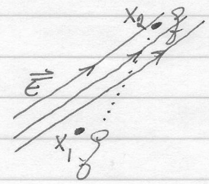
Electric Potential



- takes work to move charge q in $\vec{E}$ field

  → force felt @ x_1 ≠ force @ x_2
  → difference in potential energy at x_1 + x_2, ΔU

$$\text{work} = \Delta U = -q\int_{x_1}^{x_2} \vec{E}\cdot d\vec{s} \quad \left(= \int_{x_1}^{x_2} \vec{F}\cdot d\vec{s}\right.$$

Done by $\vec{E}$ field

- integral independent of path taken → only x_1 & x_2 matter

- Define "electric potential" difference

$$\Delta V = V_1 - V_2 \equiv \frac{\Delta U}{q_0} = -\int_{x_1}^{x_2} \vec{E}\cdot d\vec{s}$$

independent of test charge, q_0

→ if x_1 @ ∞, $\vec{E} = 0$ (& $V_1 = 0$)

$$\therefore \quad V = \frac{\Delta U}{q_0} = \frac{W}{q_0}$$

: Units of Electric Potential :

→ work / charge ⇒ joule / coulomb

$$\equiv \boxed{1 \text{ volt}}$$

$1\,V = 1\,\frac{Nm}{C}$ so $\vec{E}$ is Volts/m

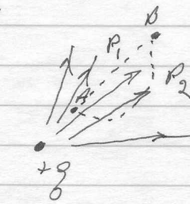
: path independence :

- a result of F_E being a "conservative force"

"A force is conservative if the kinetic energy of a particle on which it acts returns to its initial value after any round trip."

Examples: gravity, ideal spring, $\vec{E}$-field
non-conserving: friction

Since $\Delta V = -\int_{A}^{B} \vec{E}\cdot d\vec{s}$



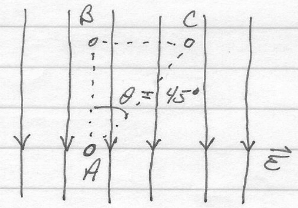
→ only radial changes in position result in dV for the integral → so Path 1 (P_1) same ΔV as P_2.

Potential Diff. in Uniform $\vec{E}$ field

→ $\vec{E}$ constant for all points

A & B are separated by distance $|\vec{s}_{AB}| = d$

→ $\vec{s}_{AB} \parallel \vec{E}$



$$\Delta V_{BA} = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_A^B E\cos 180^\circ ds = +\int_A^B E ds$$

$$= +E\int_A^B ds = \underline{\underline{+Ed}} \quad (\text{i.e. } V_A > V_B)$$

go from A → B (to higher potential)

This means that electric field lines point in the direction of decreasing potential.

If we consider path ACB:

$$\Delta V_{CA} = -\int_A^C E\cos 135^\circ ds = +E\frac{\sqrt{2}}{2}\int_A^C ds \rightarrow \sqrt{2}\,d$$

$$= +E\,\frac{2}{2}d =$$

$$= \underline{+Ed} \quad (\text{same as above})$$

$$\left(\Delta_{BC} = -\int_C^B E\cos 90^\circ ds = \underline{\underline{0}}\right)$$

E-field does work when positive moves in direction of $\vec{E}$:

→ q feels force $q\vec{E}$ downward

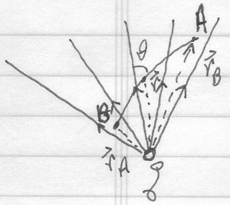
- as accelerate & gain K.E., P.E. drops

(also, it takes positive work for us to "go against" by moving q from A → B.)

For negative charges, it's the opposite:

Gain electric potential energy when move in field direction

Point Charge + Electric Potential



$\vec{E}$ directed radially outward and $\parallel \vec{r}$

θ is angle between $\hat{r}$ + $\vec{s}$

What is V @ r?

Consider $V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$

$\vec{E} = k\frac{q}{r^2}\hat{r}$

$$\vec{E}\cdot d\vec{s} = k\frac{q}{r^2}\underbrace{\hat{r}\cdot d\vec{s}}_{ds\cos\theta = dr} = k\frac{q}{r^2}dr$$

by substitution

$$V_B - V_A = -\int_A^B k\frac{q}{r^2}dr = -kq\int_A^B \frac{dr}{r^2}$$

$$= kq\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

So, again, independent of path.

By convention, $r_A \to \infty$ and $r_B = r$

$$\boxed{V = kq/r} \quad \text{(a scalar)}$$

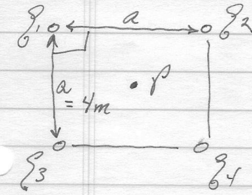
Potential for Multiple Charges

- for a group of charges, sum potential for each charge

$$V = k\sum_i \frac{q_i}{r_i}$$

[NOTE: no vectors!]

Example:



What is V @ pt. P?

$q_1 = +1\times10^{-8}\,C$; $q_2 = -20nC$

$q_3 = +20nC$; $q_4 = +30nC$

$$\underline{V} = k\sum_i q_i/r_i = k\frac{[q_1+q_2+q_3+q_4]}{r}$$

$$= 9\times10^9\,Nm^2/C^2\left(\frac{+40nC}{(\sqrt{2}\,2\,m)}\right)$$

$$= \boxed{127\ Nm/C}$$