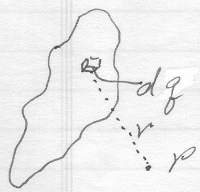
Electric Potential in Continuous Charge Distribution

If q distribution is continuous



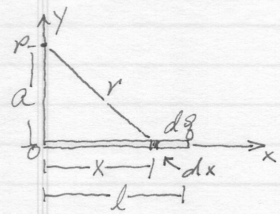
$$V = \int dV \qquad \left(dV = k\frac{dq}{r}\right)$$

$$= k\int \frac{dq}{r}$$ over whole distribution

NOTE: $V = 0$ when $r \to \infty$

Ex 25.7

Consider a rod of length, l, uniformly charged w/ $Q \Rightarrow \lambda = Q/l$



What is V @ P?

Element dq @ dx

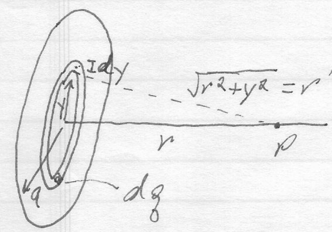
$$r = \sqrt{x^2 + a^2}$$

$$\therefore dV = k\frac{dq}{r} = k\lambda\, dx/\sqrt{x^2+a^2}$$

$$\underline{\underline{V}} = k\lambda\int_0^l \frac{dx}{\sqrt{x^2+a^2}} \quad \text{or} \quad k\frac{Q}{l}\int_0^l \frac{dx}{\sqrt{x^2+a^2}}$$

$$= \boxed{k\frac{Q}{l}\ln\left(\frac{l+\sqrt{l^2+a^2}}{a}\right)}$$

Example: A Charged Disk



What is V@P?

uniform Surface charge density

$$\sigma = Q/\pi a^2$$

Consider annulus of radius 'y'

$$dq = \sigma\underbrace{\left[(2\pi y)\, dy\right]}_{\text{area of annulus}}$$

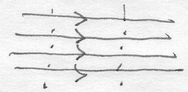
all dq on this annulus give same potential

$$dV = k\frac{dq}{r'} = k\frac{\sigma\, 2\pi y\, dy}{\sqrt{y^2+r^2}}$$

Integrating for all radii y

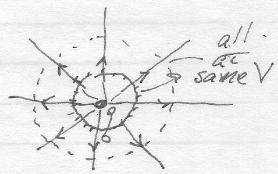
$$\underline{\underline{V}} = \underbrace{k(2\pi\sigma)}_{\left(\frac{\sigma}{2\epsilon_0}\right)}\int_0^a \frac{y\, dy}{\sqrt{y^2+r^2}}$$

$$= \boxed{\frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+r^2} - r\right]}$$

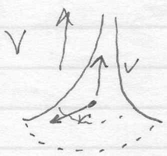


Equipotentials

If have point charge
→ radial $\vec{E}$
→ spherical surfaces have constant V



equipotential surfaces ⊥ to E-field lines passing thru them



"shape" of V vs. 'r' for pt. charge

Every change in r causes a change in potential

$$dV = -\vec{E}\cdot d\vec{s} \quad \text{(General)}$$

(potential difference for 2 points, $d\vec{s}$ apart)

$$dV = -E_r\,dr \quad \text{(pt. charge)}$$

$$\boxed{E_r = -\frac{dV}{dr}}$$

∴ Electric field is a measure of rate of change w/position of electric potential

In General $\left\{ E_x = -\dfrac{\partial V}{\partial x};\ E_y = -\dfrac{\partial V}{\partial y};\ E_z = -\dfrac{\partial V}{\partial z} \right\}$

(partial derivatives)

Potential for Charged Conductor

Take a solid conductor in electrostatic equilibrium → charge on outer surface of the conductor
→ $\vec{E}$ field outside ⊥ to surface
→ $\vec{E}$ field inside = 0

Consider path between A+B on the surface.

$$\vec{E} \perp d\vec{s} \Rightarrow dV = -\vec{E}\cdot d\vec{s} = 0$$

∴ surface is equipotential

Since $E = 0$ inside, $V = \int \vec{E}\cdot d\vec{s} =$ constant

spherical conductor (again)