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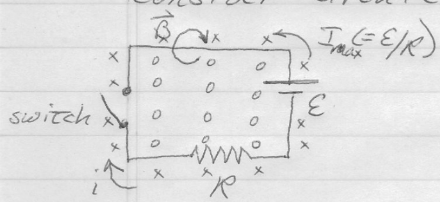
Self Inductance:

Consider circuit with current and emf



$\vec{B}$ is out of page inside of loop

when throw switch

$I = 0 \Longrightarrow I_{max} = \mathcal{E}/R$

↳ $\therefore B = 0 \Longrightarrow B = B_{max}$ (Biot-Savart Law)

$\therefore \underline{\underline{\Delta\phi_B > 0}}$

So, we will have induced emf + current

$$\mathcal{E}_L = -\,d\phi_B/dt$$

• Lenz's Law means $\mathcal{E}_L$ will oppose $\Delta\phi_B$ and induce "back emf" + current, i

Since $\phi_B \propto B \propto I$: $\quad \mathcal{E}_L = -L\, dI/dt$

proportionality constant termed "inductance"

<u>inductance</u>: a measure of how much emf induced by loop (or coil - see later)

$\boxed{\text{units: 'henrys' } \ 1H = 1\,V{\cdot}s/A}$

Get a slowdown in change in current: $I = I_{max} + i$

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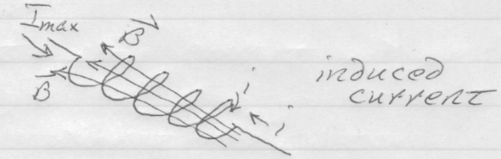
Inductors



We can increase inductance by multiplying # loops, as shown for coil above.

$$\mathcal{E}_L = -N\frac{d\phi_B}{dt} = -L\frac{dI}{dt}$$

Equating both sides + integrating over time:

$$N\phi_B = LI$$

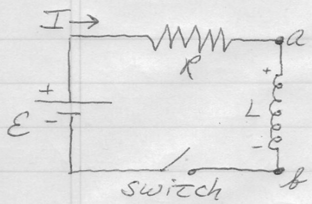
$$\therefore \boxed{L = N\phi_B/I}$$

An 'inductor' has large self-inductance

symbol: ⌇⌇⌇

- opposes changes in current in a circuit

RL Circuits



When close switch, I increases $(dI/dt > 0)$

$\therefore \mathcal{E}_L = -L\frac{dI}{dt} < 0$

Electric potential decreases when go from $a \to b$

From Kirckoff's loop rule:
(+ divide by R)

$$\frac{\mathcal{E}}{R} - \frac{IR}{R} - \frac{L}{R}\frac{dI}{dt} = 0$$

if take $x = \mathcal{E}/R - I$, then

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\int_{x_0}^{x} dx/x = -(R/L)\int_0^t dt$$

$$\Rightarrow \ln(x/x_0) = -R/L\, t$$

exponentiate: $\therefore$

$$x = x_0 e^{-Rt/L}$$

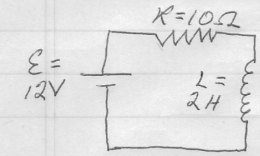
$\boxed{\tau = L/R}$ time constant

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

↳ equilibrium value of the current

Prob. 15

RL circuit



How long until current reaches 50% of its maximum value?

$$\frac{I}{I_{max}} = 0.5 = 1 - e^{-Rt/L}$$

$$0.5 = e^{-Rt/L}$$

$$\ln(0.5) = -Rt/L$$

$$+0.69 = +Rt/L \Rightarrow \underline{t = 0.69\tau}$$

In seconds: $\underline{t} = 0.69(2H/10\Omega) = \underline{0.14s}$

Energy Stored:

Use the Loop rule to quantify energy stored in an inductor in an RL circuit

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

$$I\mathcal{E} = I^2R + IL\frac{dI}{dt}$$

$$P_L = \underset{\underset{I\mathcal{E}}{\parallel}}{P_{TOT}} - \underset{\underset{I^2R}{\parallel}}{P_R} = \frac{du}{dt}$$

$$\frac{du}{dt} = LI\frac{dI}{dt}$$ rate energy stored

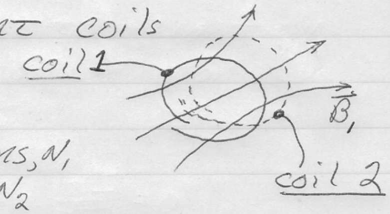
Integrating gives

$$\int du = \int_0^{I} LI\,dI$$

$$\boxed{u = \frac{1}{2}LI^2}$$ Total energy stored

Mutual Inductance:

Take two adjacent coils



- coil 1 has current I_1 and number of turns, N_1
- coil 2 has I_2, N_2

Coil 1 has current which sets up magnetic field, B_1

- some of these field lines pass thru coil 2 $\Rightarrow$ ϕ_{12} is flux thru 2 from 1

From Faraday's Law ($\phi_{12} \propto I_1$)

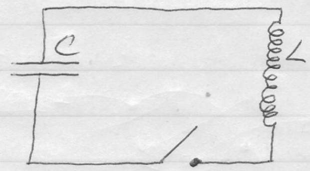
$$\mathcal{E}_2 = -N_2\frac{d\phi_{12}}{dt} = -M_{12}\frac{dI_1}{dt}$$

"mutual inductance" $= N_2\,\phi_{12}/I$

So $\mathcal{E}_2$ is proportional to the rate other coil's current is changing.

LC Circuit

Charge up a capacitor with charge Q, and place in circuit with inductor, L



switch

Consider total energy (note: no losses due to resistors)

$$U = \underbrace{\frac{Q^2}{2C}}_{\text{capac.}} + \underbrace{\frac{1}{2}LI^2}_{\text{induct.}}$$

- must remain constant in time ($dU/dt = 0$)

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{1}{2}LI^2\right)$$

$$= \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

Since $I = dQ/dt$,

$$\frac{Q}{C}\cancel{I} + L\cancel{I}\frac{d^2Q}{dt} = 0$$

$Q =$

time-rate of change of current

$$\therefore \frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

NOTE: similarity to oscillating spring $\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x = -\omega^2 x$

Solution $\Rightarrow x = A\cos(\omega t + \phi)$

So solution would be

$$Q = Q_{max}\cos(\omega t + \phi)$$

where $\omega = 1/\sqrt{LC}$ is the natural frequency.

We can calculate the current

$$I = dQ/dt = -\omega Q_{max}\sin(\omega t + \phi)$$

so I is 90° out of phase with charge.

Total Energy

$$U = \underbrace{\frac{Q_{max}^2}{2C}\cos^2\omega t}_{\text{E-field}} + \underbrace{\frac{LI_{max}^2}{2}\sin^2\omega t}_{\text{B-field}}$$

$= \text{constant}$