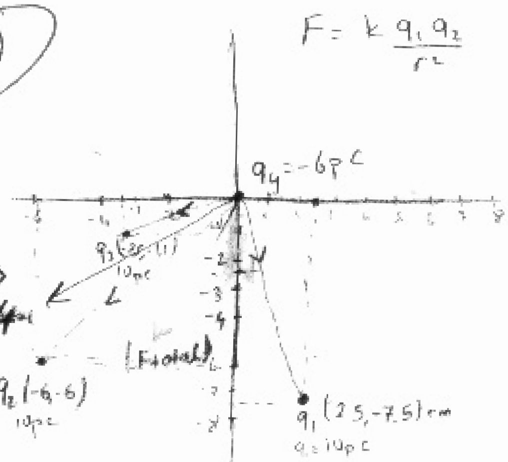
PHYS 1304 Spring 2007 Test # 1 2-13-07

1-) $C = \epsilon_0 \frac{A}{d}$ $A = \pi r^2 = \pi \cdot \left(\frac{0.01\,m}{2}\right)^2 = 7.85\times10^{-5}\,m^2$

$C = (8.85\times10^{-12})\left(\frac{7.85\times10^{-5}}{0.2\times10^{-3}}\right) \Rightarrow \boxed{C = 3.48\times10^{-12}\,F = 3.48\,pf}$

2-) False

3-)



$|r_{41}| = \sqrt{(2.5)^2+(-7.5)^2} = 7.9\,cm = 0.079\,m$

$|r_{42}| = \sqrt{(-6)^2+(-6)^2} = 8.5\,cm = 0.085\,m$

$|r_{43}| = \sqrt{(-3.5)^2+(-1.1)^2} = 3.67\,cm = 0.037\,m$

$F_{41} = \frac{(9\times10^9)(10\times10^{-12})\times(-6\times10^{-12})}{(0.079)^2} = -8.64\times10^{-11}$

$F_{42} = \frac{(9\times10^9)(10\times10^{-12})\times(-6\times10^{-12})}{(0.085)^2} = -7.5\times10^{-11}$ attrac

$F_{43} = \frac{(9\times10^9)(10\times10^{-12})(-6\times10^{-12})}{0.037^2} = -4.01\times10^{-10}\,N$

$\theta_1 = \tan^{-1}\left(\frac{-7.5}{2.5}\right) = 288.43°$

$\theta_2 = \tan^{-1}\left(\frac{-6}{-6}\right) = 225°$

$\theta_3 = \tan^{-1}\left(\frac{-1.1}{-3.5}\right) = 197.4°$

$\vec{F}_{41} = 8.63\times10^{-11}\cos\theta_1\,\hat{i} + 8.63\times10^{-11}\sin\theta_1\,\hat{j} = 2.73\times10^{-11}\,\hat{i} - 8.2\times10^{-11}\,\hat{j}$

$\vec{F}_{42} = 7.5\times10^{-11}\cos\theta_2\,\hat{i} + 7.5\times10^{-11}\sin\theta_2 = -5.3\times10^{-11}\,\hat{i} - 5.3\times10^{-11}\,\hat{j}$

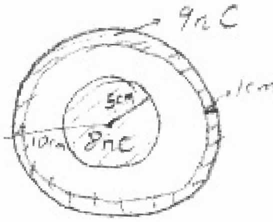
$\vec{F}_{43} = 4.01\times10^{-10}\cos\theta_3\,\hat{i} + 4.01\times10^{-10}\cos\theta_3 = -3.83\times10^{-10} - 1.2\times10^{-10}\,\hat{j}$

$\vec{F}_4 = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43} = -4.08\times10^{-10}\,\hat{i} - 2.55\times10^{-10}\,\hat{j}$

$|F_4| = \sqrt{(-4.08\times10^{-10})^2+(-2.55\times10^{-10})^2} \Rightarrow \boxed{|\vec{F}_4| = 4.81\times10^{-10}}$

$\theta = \tan^{-1}\left(\frac{-2.55\times10^{-10}}{-4.08\times10^{-10}}\right) \Rightarrow \boxed{\theta = 212°}$

or $\theta = 32°$ from negative x-axis

4-) 

a) $E_a = \dfrac{k\,Q_{sphere}}{R^3_{sphere}} = \dfrac{(9\times10^9)(8\times10^{-9})}{(0{,}05)^3}(0{,}01)$

$\boxed{E_a = 5{,}76\times10^3 N/C}$

b) $E_b = \dfrac{k\,Q_{sphere}\,r_{sphere}}{R^3_{sphere}} = \dfrac{(9\times10^9)(8\times10^{-9})}{(0{,}05)^3} = \boxed{23{,}04\times10^3 N/C}$

c) $E_c = \dfrac{k\,Q_{sphere}}{r^2} = \dfrac{(9.10^9)(8\times10^{-9})}{(0{,}08)^2} = \boxed{11{,}25\times10^3 N/C}$

d) $E_d = \dfrac{k\,(Q_{shell}+Q_{sphere})}{r^2} = \dfrac{(9\times10^9)(17\times10^{-9})}{(0{,}12)^2} = \boxed{10{,}63\times10^3 N/C}$

5-) $\dfrac{1}{C_{eff}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{1}{C} + \dfrac{1}{C} + \dfrac{1}{C} = \dfrac{3}{C}$

$C_{eff} = \dfrac{C}{3} = \dfrac{5pF}{3} \Rightarrow \boxed{C_{eff} = 1{,}67\,pF}$

6-) 

$E = 2\pi k \sigma \left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$E = 2\pi.(9\times10^9)(10^{-3})\left(1 - \dfrac{0{,}3}{\sqrt{0{,}3+0{,}2^2}}\right) =$

$\boxed{E = 9{,}5\times10^6 N/C}$

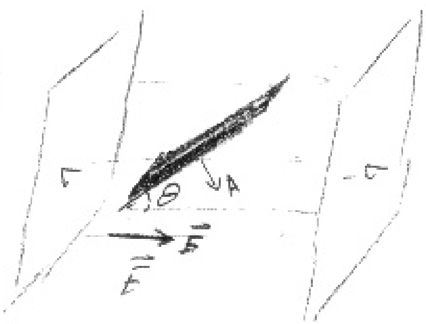
7-)

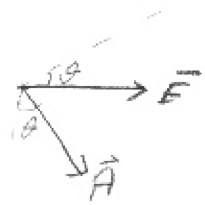
$$\Delta V = V_2 - V_1 = \frac{kq}{r_2} - \frac{kq}{r_1} = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= (9\times10^9)(-3\times10^{-3})\left(\frac{1}{0.25} - \frac{1}{0.05}\right) = 432\times10^6$$

$$\boxed{\Delta V = 4,32\times10^8\ V}$$

8-)





$$\phi = \vec{E}\cdot\vec{A}$$
$$= EA\cos(90-\theta)$$
$$= EA\sin\theta$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{(10\times10^{-3})}{(8.85\times10^{-12})} = 1,13\times10^9\ N/C$$

$$A = 4.4 = 16\,m^2$$

$$\phi = (1,13\times10^9)\times(16)\sin 43 = 12,3\times10^9$$

$$\boxed{\phi_E = 1,23\times10^{10}\ N\,m^2/C}$$