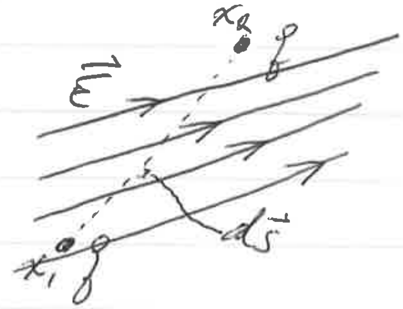
Electric Potential Energy 7.1

It requires work to move charge q in $\vec{E}$ field

Work to move $d\vec{s}$



$$dW = \vec{F} \cdot d\vec{s}$$

which equals the change in potential energy $dW = dW$

the difference in potential energy at x_1 and x_2 is

$$\Delta U = -q\int_{x_1}^{x_2} \vec{E} \cdot d\vec{s} = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{s}$$

- independent of path taken
- only x_1, x_2 matters

Electric Potential

As with electric field + force, we want expression independent of charges, q_0, placed in field

Define 'electric potential'

$$\Delta V = V_{x_1} - V_{x_2} \equiv \Delta U / q_0$$

this gives us an expression based on electric field

$$\Delta V = V_{x_1} - V_{x_2} = \boxed{-\int_{x_1}^{x_2} \vec{E} \cdot d\vec{s}}$$

Units: work/charge => joule/coulomb

$$1 \text{ Volt} = 1 V = 1 \frac{Nm}{C}$$

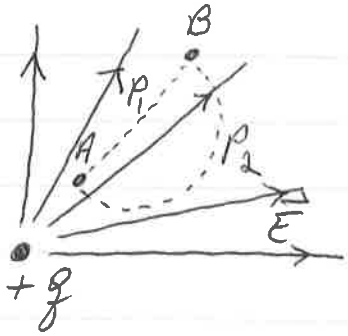
$\vec{E}$-field units therefore volts/m

Implications of Conservation of Energy 7.3

$\vec{F}_E$ is a 'conservative' force:

"a force is conservative if the kinetic energy of a particle on which it acts returns to its initial value after any round trip."

Examples:
- gravity
- ideal spring
- E-field



reminder: friction is nonconserving

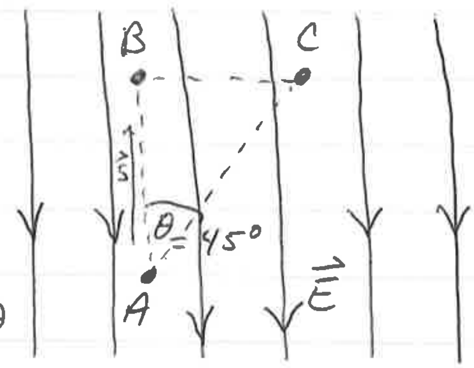
$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s}$$

only radial variation in path matters

Paths A + B have same radial extent so $\int$ along P_1 gives same result as P_2

Potential Difference in a Uniform $\vec{E}$ field

Consider area where $\vec{E}$ constant for all points:



A & B are separated by $|\vec{s}| = d$

$$\vec{S}_{AB} \parallel \vec{E}$$

When going from $A \rightarrow B$, go to higher potential

$$\underline{\Delta V_{BA}} = -\int_A^B \vec{E} \cdot d\vec{s} = -\int_A^B E \cos 180° \, ds$$

$$= +\int_A^B E\,ds = E\int_A^B ds$$

$$= \boxed{Ed} \quad \text{(a scalar)}$$

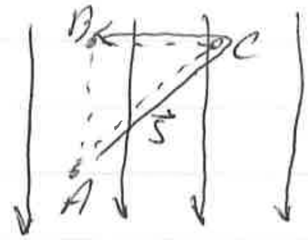
$$V_A > V_B$$

<u>Electric field lines point in direction of decreasing potential</u>

7.5

Consider also path ACB

$\Delta V_{ACB} = \Delta V_{AC} + \Delta V_{CB}$



$$\Delta V_{CA} = -\int_A^C \mathcal{E} \cos 135^\circ\, ds$$

$$= +\mathcal{E}\,\frac{\sqrt{2}}{2}\int_A^C ds \quad = \sqrt{2}\, d$$

$$= +\mathcal{E}\,\frac{2}{2}\, d$$

$$= \underline{\underline{+\mathcal{E} d}}$$ ~~same as~~

$$\Delta V_{BC} = -\int_C^B \mathcal{E} \cos 90^\circ\, ds = \underline{\underline{0}}$$

$$\underline{\underline{\Delta V_{ACB}}} = \mathcal{E} d + 0$$

$$= \boxed{\mathcal{E} d}$$ $\underline{\text{same as } \Delta V_{BA}}$

Example

A probe is placed in a uniform electric field of 10^{-3} N/C. It starts at the origin and is moved 2cm to the right, then 5cm upward. It is moved 3cm forward and 1cm down. What is the potential difference over this path?

- we just care about the net distance, not the path length.

$$\vec{d} = +2\text{cm}\,\hat{i} + (5\text{cm} - 1\text{cm})\hat{j} + 3\text{cm}\,\hat{k}$$

$$= 2\text{cm}\,\hat{i} + 4\text{cm}\,\hat{j} + 3\text{cm}\,\hat{k}$$

$$|d| = \sqrt{(2\text{cm})^2 + (4\text{cm})^2 + (3\text{cm})^2}$$

$$= 5.4\text{cm} = \underline{0.054\text{m}}$$

$$\boxed{\Delta V = Ed = 5.4 \times 10^{-5}\,\text{V}}$$

E-field does work:

1) when positive charge moves in direction of $\vec{E}$:

→ q feels force $q\vec{E}$ downward

→ accelerate, gain K.E.

-potential energy drops

→ It requires positive work for us to "go against" by moving q from $A \to B$

2) when negative charges move in direction of $\vec{E}$

→ opposite behavior

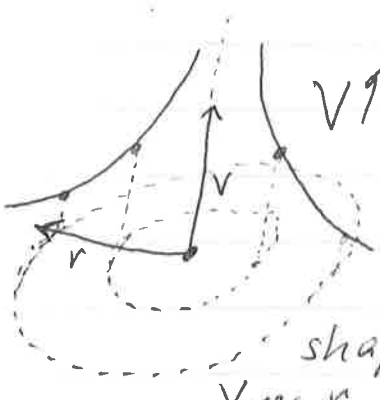
→ gain electric potential energy

Equipotentials

We're used to drawing field lines, but another line tells about potentials.

If have point charge
→ radial $\vec{E}$
→ spherical surfaces have constant V

equipotential surfaces ⊥ to E field lines passing thru them



shape of V vs. r for point charge

Potential difference for 2 points $d\vec{s}$ apart

$$dV = -\vec{E}\cdot d\vec{s} \quad \text{(general)}$$
$$= -E\,dr \quad \text{(point charge)}$$
$$\boxed{E_r = -\frac{dV}{dr}}$$

$\therefore$ Electric field is a measure of the rate of change with position of the electric potential