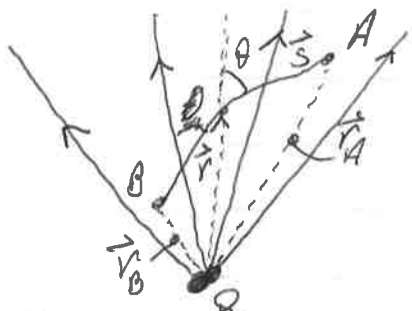
Potential Due to a charged Particle



θ is angle between $\hat{r}$ and $\vec{s}$

Consider potential difference calculated from A to B.

$\vec{E}$ directed radially outward $\parallel \hat{r}$

What is $V(r)$?

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} \qquad \vec{E} = k\frac{q}{r^2}\hat{r}$$

$$= -\int_A^B k\frac{q}{r^2}\hat{r} \cdot d\vec{s}$$

$$= -\int_A^B k\frac{q}{r^2}\,dr = -kq\int_A^B \frac{dr}{r^2}$$

$$= kq\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

so, again, independent of path

We often take the convention $r_A \to \infty$ and $r_B = r$

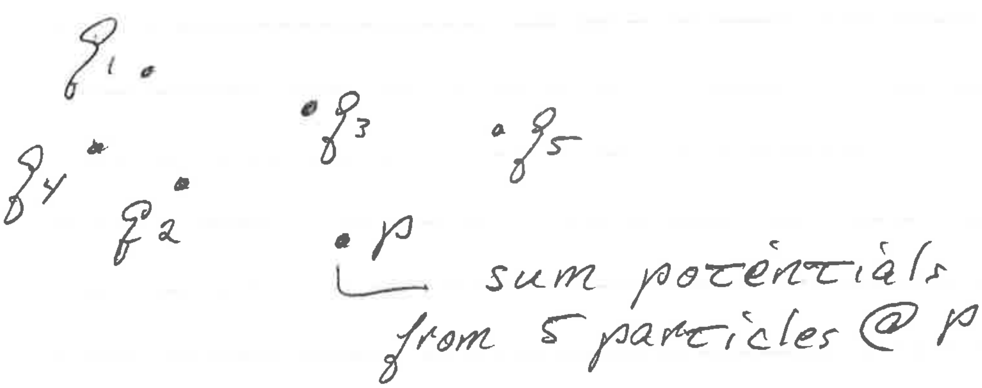
$$\boxed{V = kq/r}$$

Potential Due to a Group of Charges

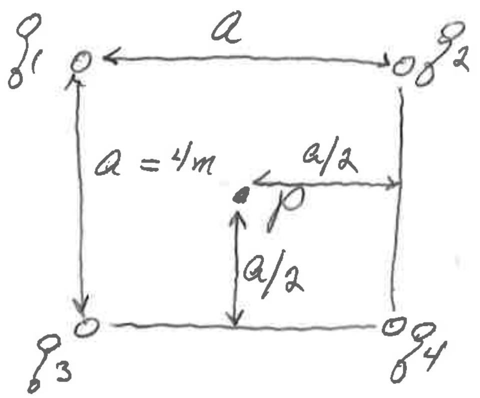
For a group of charges, treat each single charge, & sum potentials.

V is a scalar, so no need to keep track of vector components.

$$V = k \sum_i \frac{q_i}{r_i}$$



Example



$q_1 = +1 \times 10^{-8} C$
$q_2 = -20 nC$
$q_3 = +20 nC$
$q_4 = +30 nC$

What is V @ point P?

$\underline{r_1} = r_2 = r_3 = r_4 = \sqrt{(a/2)^2 + (a/2)^2} = \sqrt{\frac{2a^2}{4}}$

$= \underline{a/\sqrt{2}}$

$\underline{\underline{V}} = k \sum_i \frac{q_i}{r_i} = \frac{k}{r}[q_1 + q_2 + q_3 + q_4]$

$= 9 \times 10^9 \frac{Nm^2}{C^2} \left(\frac{1}{a/\sqrt{2}}\right) [40 nC]$

$= \boxed{127\ Nm/C}$

Electric Potential in 8.4
Continuous Charge Distribution

If charge distribution continuous

$$V = \int dV = \int k\frac{dq}{r}$$

$$= k\int\frac{dq}{r}$$

Integrate over whole charge distribution.

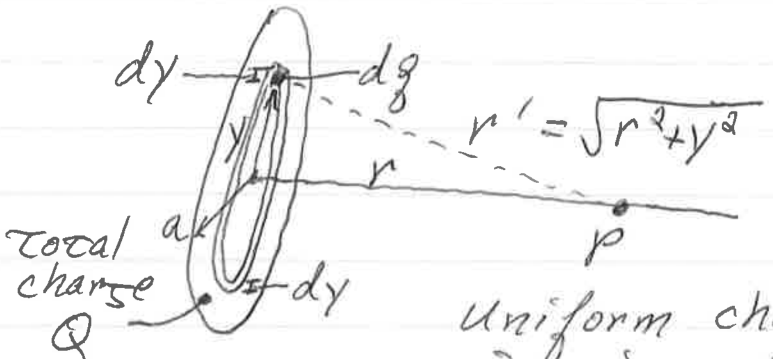
May involve uniform charge densities!

linear $dq = \lambda dx$

areal $dq = \sigma dA$

volume $dq = \rho dV$

Potential from a Uniformly Charged Disk



Uniform charge density $\sigma = Q/\pi a^2$

Consider annulus of radius 'y'

$$dq = \sigma\left[(2\pi y)\,dy\right]$$

(Area of annulus)

All dq on annulus give same potential

$$dV = k\frac{dq}{r'} = k\frac{\sigma(2\pi y)\,dy}{\sqrt{y^2+r^2}}$$

Integrating over all radii y

$$V = k(2\pi\sigma)\int_0^a \frac{y\,dy}{\sqrt{y^2+r^2}}$$

($k(2\pi\sigma) = \sigma/2\epsilon_0$)

$$= \boxed{\frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+r^2} - r\right]}$$

Example

An insulating disk has a uniform charge density throughout. The disk's outer radius is 12cm, and the total charge is +30nC. What is the electric potential at 30cm along the disk axis?

$$\sigma = \frac{Q}{A} = 30nC/(\pi(0.12m)^2)$$

$$\sigma = 6.7\times10^{-6}\ C/m^2$$

$$V = \frac{\sigma}{2\epsilon_0}\left[\sqrt{a^2+r^2} - r\right]$$

$$= \frac{6.7\times10^{-6}\ C/m^2}{2(8.85\times10^{-12}\ C^2/Nm^2)}\left[\sqrt{0.0144m^2+0.09m^2} - 0.3m\right]$$

$$V = 9.2\ \frac{Nm}{C}$$