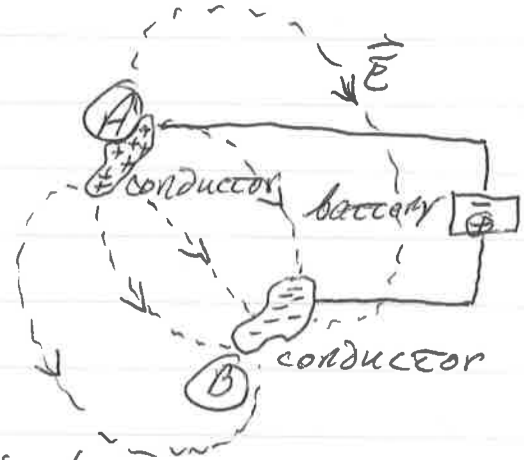
Capacitance

Consider 2 conductors:

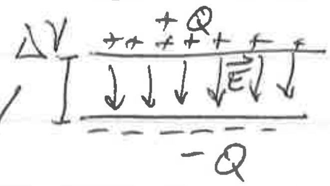


- separated by insulator (e.g. air)
- attach to battery terminals
  - → A + B become charged, Q
  - all $\vec{E}$ field lines from A terminate on B
  - a potential difference ΔV develops between A + B

2 charged // plates:
- uniform $\vec{E}$

$$Q \propto \Delta V$$
$$= \text{const.}\,\Delta V$$
$$\boxed{= C\,\Delta V}$$

Observe



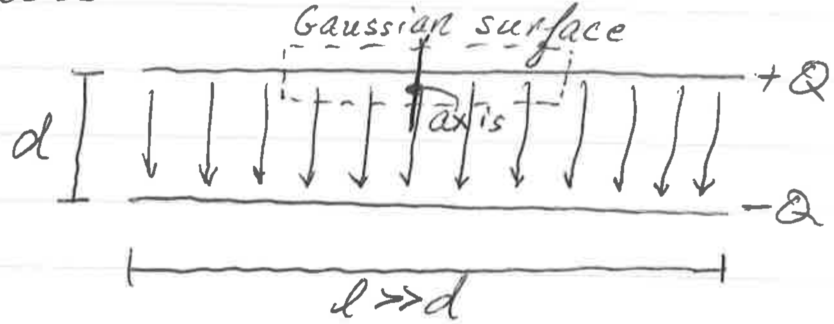
$\boxed{C \equiv Q/\Delta V} \Rightarrow$ capacitance

"amount of charge a 'capacitor' can store per unit potential difference."

units 'farad' = coulomb/volt

Calculating the Capacitance 9.3

Use as example 2 // conducting plates



We already applied Gauss's Law to this configuration.

- cylindrical symmetry + surface: axis ⊥ plates
- top surface inside conductor
  → no field flux
- side cylinder $\vec{E} \cdot A = 0$
- bottom surface $\vec{E} // \vec{A}$

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

Calculating the Potential Difference 9.4

For a uniform $\vec{E}$ field, we also know

$$\Delta V = Ed$$

Considering our expression for E, we have

$$\Delta V = \left(\frac{\sigma}{\epsilon_0}\right)d = \left(\frac{Q/A}{\epsilon_0}\right)d$$

$$\boxed{\Delta V = \frac{Qd}{\epsilon_0 A}}$$

Q = total charge on plate
A = area of plate (not just Gaussian area)

We can calculate capacitance for || plates:

$$C = Q/\Delta V = Q/\left(\frac{Qd}{\epsilon_0 A}\right)$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

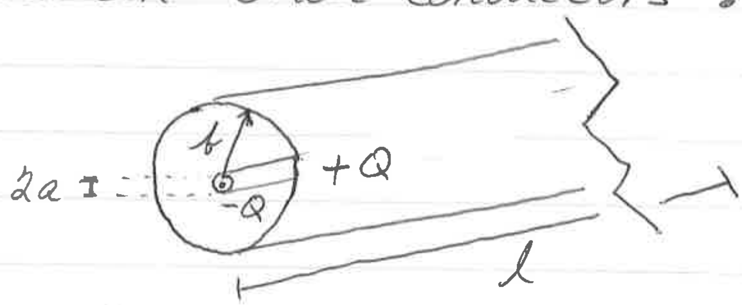
Note: only geometry of plates enter

- bigger plates: more Q & C

- larger separation: less C

Example 9.4b

A cylindrical capacitor 1cm long has a wire of radius 100μm and outer conductor radius 1mm. When 2nC of charge are deposited on this capacitor, what is the potential difference between these conductors?



$C = \frac{\ell}{2k \ln(b/a)}$

$a = 100\mu m$
$b = 1mm$
$\ell = 1cm$
$Q = 2nC$

$= \frac{10^{-2}\cancel{m}}{2\times 9\times 10^{9} \frac{Nm^2}{C^2} \ln\left(\frac{10^{-3}\cancel{m}}{10^{-4}\cancel{m}}\right)}$

$= \frac{10^{-11} C^2}{18\ Nm \ln(10)}$

$= 0.024 \times 10^{-11}\ C^2/Nm$

$= 2.4\times 10^{-13}$ farad

$\Delta V = Q/C = 2\times 10^{-9} C / 2.4\times 10^{-13}$ farad

$= 0.8\times 10^{4}$ Volt

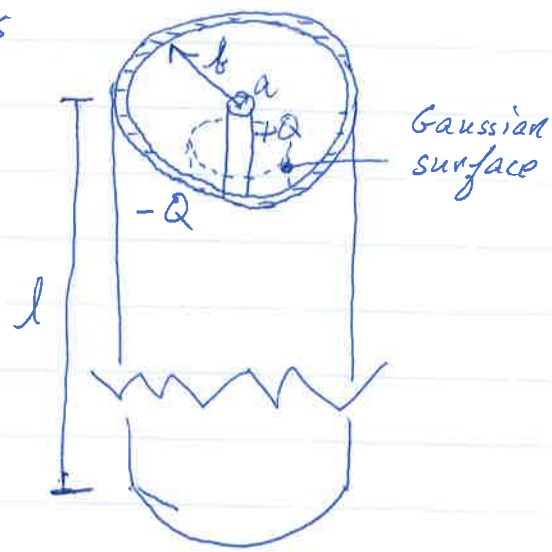
$= \boxed{8{,}000 \text{ Volt}}$

Cylindrical Capacitor

Let's consider following geometry:

2 coaxial cylinders
- radii a & b
  - $b > a$
- length l
  - $l \gg b$

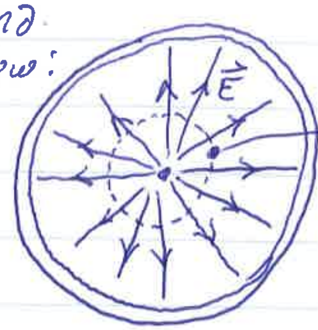
We stay far from ends of cylinder



What is $\vec{E}$ field internal to outer cylinder?

Gauss's Law: cylindrical symmetry
- coaxial with inner conductor
- no flux from outer cylinder
  - doesn't contribute to E-field

$\vec{E}$ is $\perp$ to axis and $\parallel$ to $\vec{r}$



From the inner conductor, we have seen

$$\underline{E_r = 2k\lambda/r}$$

To calculate the potential difference between ⓐ and ⓑ, we have:

$$V_b - V_a = -\int_a^b E_r dr = -2k\lambda \int_a^b \frac{dr}{r}$$

$$\underline{\Delta V = -2k\lambda \ln(b/a)}$$

By substitution in $C = |Q|/|\Delta V|$

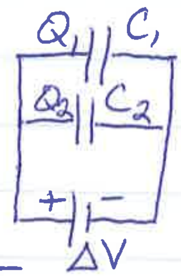
$$C = \frac{Q}{+2k\lambda \ln(b/a)} = \frac{Q}{2k(Q/l)\ln(b/a)}$$

$$\boxed{C = l/(2k \ln(b/a))}$$

Again, no λ or $Q \rightarrow$ only geometry l, b and a

Capacitors in Parallel

Consider our first electrical circuit:



- ⊣⊢ = capacitor
- ⊣⊢ = DC battery
- lines are conducting wire

These capacitors said to be in a "parallel" configuration

ΔV_1 across C_1 same as ΔV_2

Q_{TOTAL} is total charge stored
$= Q + Q_2$

So we can think of C_1 + C_2 together as one capacitor C_{eff}

$$\underline{Q_{TOTAL} = C_{eff} \Delta V}$$

To determine C_{eff} from known circuit elements, we recall

$$Q_1 = C_1 \Delta V$$
$$Q_2 = C_2 \Delta V$$

Thus,

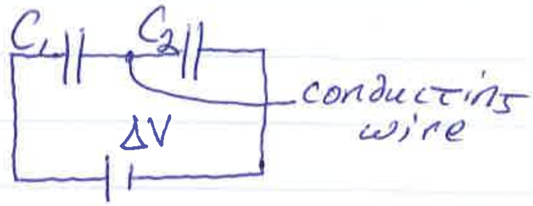
$$C_{eff} \cancel{\Delta V} = C_1 \cancel{\Delta V} + C_2 \cancel{\Delta V}$$

$$\boxed{C_{eff} = C_1 + C_2}$$

Capacitances sum when in parallel.

Capacitors in Series

Another circuit with 2 capacitors in "series"



Here, $Q_1 = Q_2$

- since charge conservation will not allow creation of charge in intervening conductor

But $\Delta V = \Delta V_1 + \Delta V_2$ since each step changes the potential

The equivalent capacitor to C_1 & C_2 is

$$\Delta V = Q/C_{eff} = \Delta V_1 + \Delta V_2$$

$$\frac{\cancel{Q}}{C_{eff}} = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2}$$

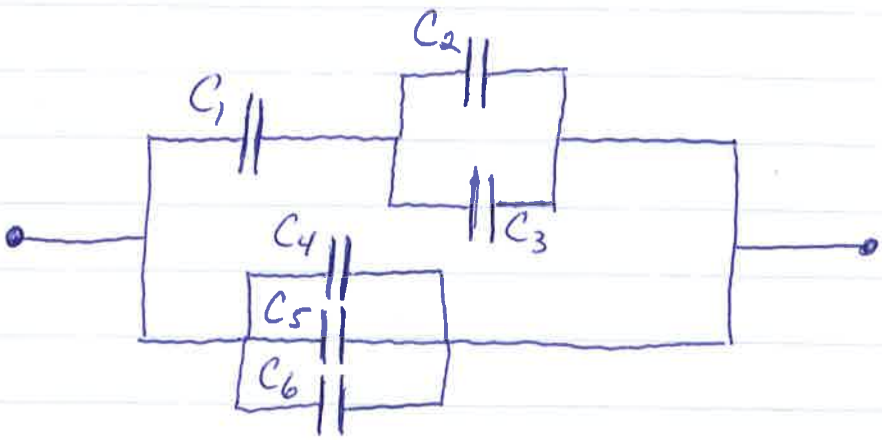
$$\boxed{\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Sum inverses of capacitances in series.

NOTE: $C_{eff} < C_1, C_2$ always

Example

9.10



What is the Effective capacitance?

$$C_{23} = C_2 + C_3$$

$$C_{456} = C_4 + C_5 + C_6$$

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$C_{eff} = C_{123} + C_{456}$$

$$= \boxed{\left[\frac{1}{C_1} + \frac{1}{C_2 + C_3}\right]^{-1} + C_4 + C_5 + C_6}$$