Calculating Magnetic Field Due to a Current 17.1

The Lorentz force law $\vec{F}=q\vec{v}\times\vec{B}$ suggests a connection between moving charges and magnetic fields. Unravelling this further reveals a law analogous to Coulomb's Law for electric fields

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

but it does not involve magnetic charges.

The unit of B-field generating elements is not a charge but an electric current segment:

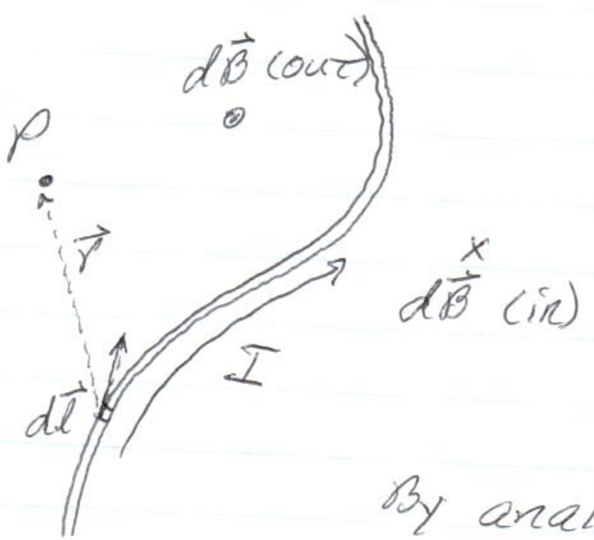
Biot-Savart Law $$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\hat{r}}{r^2}}$$

where $I d\vec{l}$ is current segment and μ_0 is the "permeability constant"

$$\underline{\mu_0 = 4\pi\times10^{-7}\ \mathrm{Tm/A}}$$

Using Biot-Savart

How do we calculate $\vec{B}$ at some point P near a current-carrying conductor of arbitrary shape?



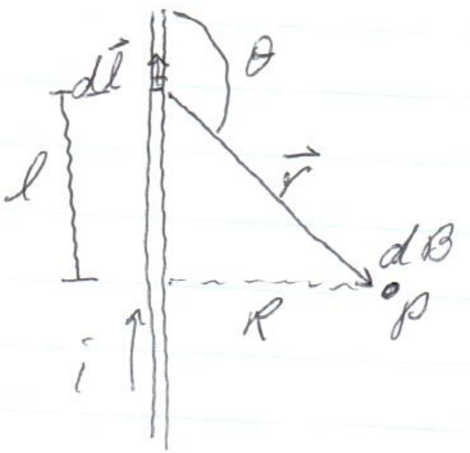
By analogy to
$\vec{E} = \int d\vec{E}$

we have

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}}$$

Magnetic Field from a Straight Wire



Point P a $\perp$ distance R from wire.

$d\vec{l} \times \hat{r}$ has same direction for all dl
- into page

Convert to scalar expression and integrate

$$B = \frac{\mu_0}{4\pi} i \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 i}{4\pi} \int_{l=-\infty}^{l=+\infty} \frac{\sin\theta \, dl}{r^2}$$

But we know

$$r = \sqrt{l^2 + R^2}$$

$$\sin\theta = \sin(\pi - \theta) = R / \sqrt{l^2 + R^2}$$

Straight Wire (cont.)

By substitution

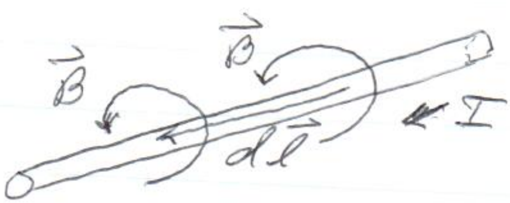
$$B = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{R\,dl}{(l^2+R^2)^{3/2}}$$

Integrating

$$B = \frac{\mu_0 i}{4\pi} \left[\frac{l}{(l^2+R^2)^{1/2}} \right]_{-\infty}^{+\infty} = \frac{\mu_0 i}{4\pi} \{1-(-1)\}$$

$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$

So, B forms a series of concentric circles around the wire with $1/r$ dependence.



Example

17.45

A straight wire has uniform current over its length of 50mA. If the magnetic field is 1 μT, what is the distance from the wire?

$B = 10^{-6}\,T$

$i = 5\times10^{-2}\,A$

$$R\ \cancel{B} = \frac{\mu_0\, i}{2\pi R} = \frac{\cancel{4\pi}^{2}\times10^{-7}\,\cancel{T}m/\cancel{A}\ (5\times10^{-2}\,\cancel{A})}{\cancel{2\pi}\ (\cancel{R})(10^{-6}\,\cancel{T})}$$

$R = 2\times10^{-1}\,m\,(5\times10^{-2})$

$\underline{\underline{R}} = 0.01\,m = \boxed{1\,cm}$

What direction of $\vec{B}$

$d\vec{l}\,||$ I arrow

$d\vec{l}\times\hat{r}$ means

$\vec{B}$ out here

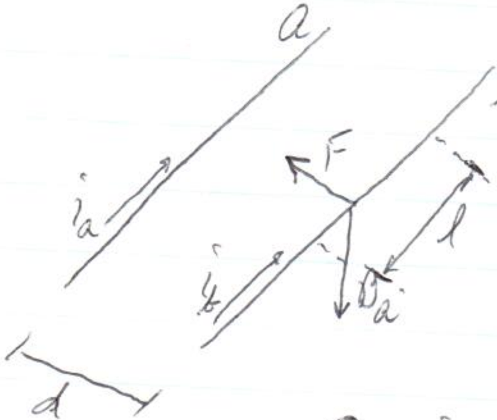
I

$\vec{B}$ in here

Two Parallel Conductors

A wire feels a force from external B if there is a current.

$$\vec{F} = i\vec{l} \times \vec{B}_{external}$$



What is B_a?

@ wire 'b' the magnitude of B_a is

$$B_a = \frac{\mu_0 i_a}{2\pi d} \text{ (down)}$$

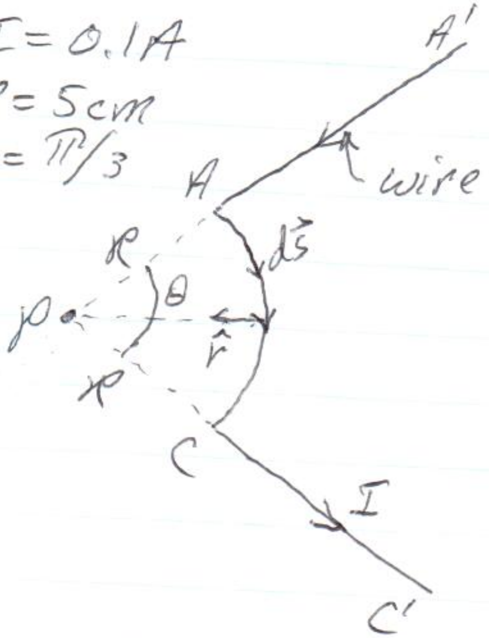
Force on 'b' is then

$$F_b = i_b \vec{l} \times \vec{B}_a = i_b l B_a = \boxed{\frac{\mu_0}{2\pi} \frac{l\, i_a i_b}{d}}$$

parallel currents: attraction
antiparallel currents: repulsion

Example:

$I = 0.1A$
$R = 5cm$
$\theta = \pi/3$



For segments AA' and CC', $d\vec{s} \times \hat{r} = 0$ since $d\vec{s}$ & $\hat{r}$ $\parallel$ or $\uparrow\downarrow$.

Just care about AC

On AC, each $d\vec{s}$ provides $d\vec{B}$@P
- all same since all $d\vec{s}$ here at same R
- $d\vec{s} \perp \hat{r}$, so $d\vec{s} \times \hat{r} = ds$

$$\therefore dB = \frac{\mu_0 I}{4\pi} \frac{ds}{R^2}$$

Example (cont.)

Assuming I and R are constants

$$\underline{\underline{B}} = \int dB = \frac{\mu_0}{4\pi}\int I \frac{ds}{R^2}$$

$$= \frac{\mu_0 I}{4\pi R^2}\int ds = \frac{\mu_0 I s}{4\pi R^2}$$

Direction into page

$$= \boxed{\frac{\mu_0 I}{4\pi R}\theta} \quad (\text{since } s = R\theta)$$

θ in radians

For our wire

$$\underline{\underline{B}} = \frac{\cancel{4\pi}\times 10^{-7}\,\frac{T\cancel{m}}{\cancel{A}}}{\cancel{4\pi}}\left(\frac{0.1\cancel{A}}{5\times 10^{-2}\cancel{m}}\right)\left(\frac{\pi}{3}\right)$$

$$= \frac{2\pi}{3}\times 10^{-7}\,T = \boxed{2.1\times 10^{-7}\,T}$$

Consider a full loop of current

- B @ center, radius R
- increase angle $\theta \to 2\pi$

$$\underline{\underline{B}} = \frac{\mu_0 I}{4\pi R}(2\pi) = \boxed{\frac{\mu_0 I}{2R}}$$