

Ampere's Law

18.1

In electrostatics

Coulomb's Law • calculate \vec{E} field from point charges

- for highly symmetric charge distributions

- can solve more easily with Gauss's Law

For magnetism

Biot-Savart Law • calculate \vec{B} field from current segments

- Consider analogy w/ \vec{E} field

Is there a magnetic analogy to Gauss's Law?

Consider a straight wire

- lines of B form circles around wire

$$B \propto I$$

$$B \propto a^{-1}$$



We seek relationship between \vec{B} and I we can determine

- for certain symmetric current distributions

- above example \rightarrow circular symmetry around wire in plane (A)

$$\vec{B} \cdot d\vec{s} \text{ for wire} = B ds \quad (\vec{B} \parallel d\vec{s})$$

"Path integral" around wire:

$$\oint \vec{B} \cdot d\vec{s} = \oint B ds = B \oint ds = \frac{\mu_0 I (2\pi a)}{2\pi a}$$

\leftarrow constant on our chosen path

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 I} \quad \text{Ampere's Law}$$

\leftarrow generally applicable

Note,

$\vec{B} \cdot d\vec{s}$ around any closed path proportional to the current passing thru the surface bounded by the closed path

Similar to Gauss's Law case,

- we are only concerned with sources of field inside path
- these are the current-carrying wires enclosed

Magnetic Field from a Cylinder (18.4) of Current:



What is B for $r < R$?

Current, i , thru surface
bounded by path

- scales as area of surface

$$i_{\text{enclosed}} = \frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I$$

From Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$B(2\pi r) = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

18.5

\therefore $B \propto r$ when $r < R$

When $r > R$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$



Example

18.6

A thick wire with current 0.2 A registers a magnetic field of 1 mT at a radius half of its outer radius. What is the outer radius?

$$B = \frac{\mu_0 I (r/R)}{2\pi R}$$

$$10^{-3}\text{ T} = \frac{2 \times 10^{-7} \text{ Tm/A} (0.2\text{ A}) (0.5)}{2\pi R}$$

$$R = \frac{2 \times 10^{-4} \text{ Tm} (0.1)}{10^{-3} \text{ T}}$$

$$= 2 \times 10^{-5} \text{ m}$$

- or -

$$= \boxed{20\ \mu\text{m}}$$

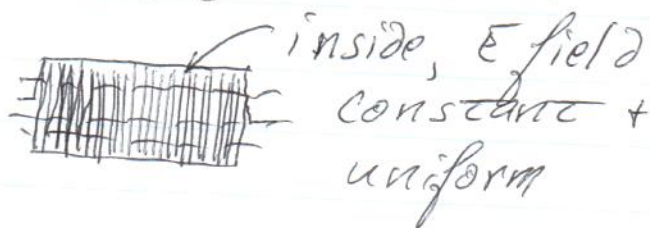
Solenoids:

18.7

- Definition: a long wire wound in a close-packed helix and carrying current I .



- when wires close
 - almost like a sheet of current



Ideal Case of Solenoid

12.8

Pick
paths \perp
or \parallel to
 \vec{B} field.



uniform field inside
- analogous to capacitor

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}$$

Since $h \gg$ width of wire, $i = i_0 n h$
where $n = \# \text{ turns / length}$

$$\int_a^b \vec{B} \cdot d\vec{s} = \int_a^b B ds = Bh = \mu_0 i$$

$$\underline{B} = \frac{\mu_0 (i_0 n h)}{h}$$

\vec{B} field for a
solenoid

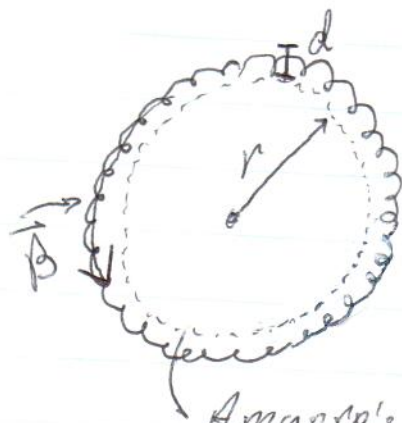
$$= \boxed{\mu_0 i_0 n}$$

- uniform \vec{B} field inside solenoid
- not a bad approximation for actual solenoids near axis

Toroids:

18.9

Like a solenoid bent closed to a donut shape



Ampere's path

We want to know \vec{B} field at r from center.

→ note circular symmetry

Using Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

$$B(2\pi r) = \mu_0 i N$$

turns total

$$B = \frac{\mu_0 i N}{2\pi r}$$

So B is NOT constant inside ring.

Example:

18.10

A toroid has 1000 turns over its circumference. The current is 7 mA in the wire. What is the magnetic field at a radius 1 m, which is less than the toroid radius.

$$\begin{aligned} B &= \frac{\mu_0 i_0 N}{2\pi r} \\ &= \frac{2 \times 10^{-7} \text{ T/A} (1000) (10^{-3})}{2\pi (1 \text{ m})} \\ &= 2 \times 10^{-7} \text{ T} (x 7) \\ &= 1.4 \times 10^{-6} \text{ T} \\ &\quad \text{-OR-} \\ &= \boxed{1.4 \mu\text{T}} \end{aligned}$$