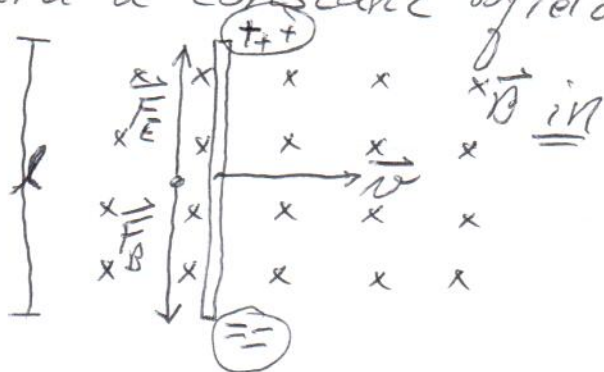


Motional Emf

20.1

\vec{E} induced in a conductor moving thru a constant \vec{B} field

straight conductor
 $\vec{v} \perp \vec{B}$



Electrons ($q < 0$) feel force
$$\vec{F}_B = q \vec{v} \times \vec{B}$$

\therefore electrons pushed to bottom

Since rod becomes 'polarized'

- and + at opposite ends

- electrons feel force

$$\vec{F}_e = q \vec{E} \text{ upwards}$$

Equilibrium condition:

$$\vec{F}_B = q \vec{v} \times \vec{B} = q v B = \vec{F}_e = q \vec{E}$$

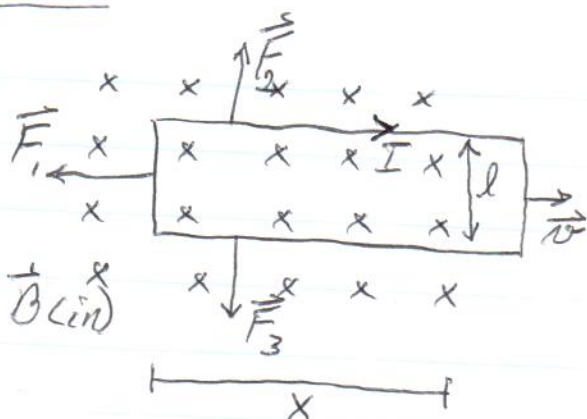
$v B = E$

Loop in a \vec{B} Field

20.2

Rectangular
loop pulled out
of B field

w/ velocity, \vec{v}



Flux, Φ_B thru loop changes
with time
($x = x(t)$)

$$\Phi_B = BA = Blx$$

Use Faraday's Law,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -Bl\frac{dx}{dt} = \underline{-Blv}$$

- Negative sign means
 - induced I clockwise
 - sets up field \parallel existing

$F_2 + F_3$ equal & opposite: \therefore cancel

F_1 counters our effort to
move loop

$$\begin{aligned}\vec{F}_1 &= I\vec{l} \times \vec{B} = IlB\sin 90^\circ (-\hat{i}) \\ &= \boxed{-BlI\hat{i}}\end{aligned}$$

Magnetic Brake

Generators & Motors

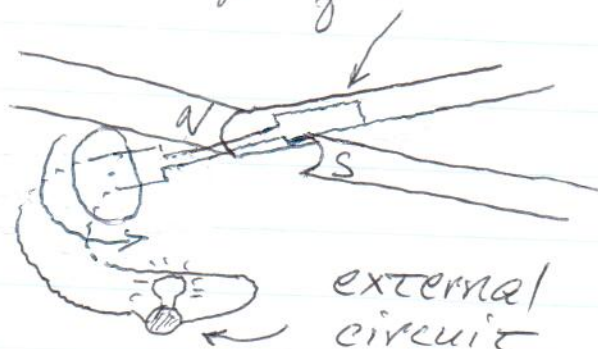
20.3

Electric Generators

- energy created by work
- transferred by electrical transmission

AC Generator

- loop of wire rotated in B field



Φ_B thru loop changes with time $\rightarrow \mathcal{E}$

$$\Phi_B = BA \cos \omega t$$

where ' ω ' is rotation frequency

We can calculate induced \mathcal{E}

$$\begin{aligned} \underline{\underline{\mathcal{E}}} &= -N \frac{d\Phi_B}{dt} = -NAB \frac{d(\cos \omega t)}{dt} \\ &= \underline{\underline{NAB\omega \sin(\omega t)}} \end{aligned}$$

where N is number of turns in loop

Self Inductance

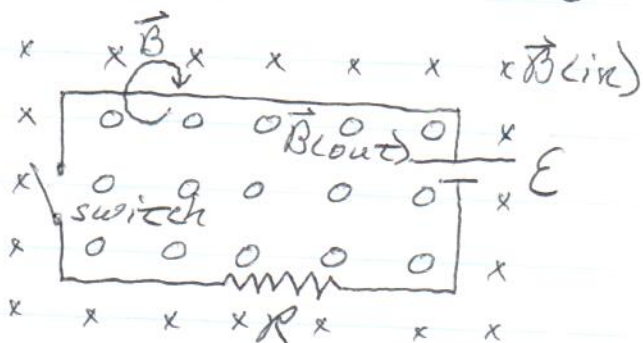
20.4

Consider a circuit with current and emf,

$$I_{max} = \mathcal{E}/R$$

after throw switch at $t=0$

$$(I(t=0) = 0)$$



When @ $I_{max} \rightarrow$ have $B = B_{max}$
(fr. Biot-Savart)

$$\therefore \Delta \Phi_B > 0$$

So, we'll have induced emf + current

$$\mathcal{E}_L = - \frac{d\Phi_B}{dt}$$

happens on same wire where \mathcal{E} $\left\{ \begin{array}{l} \text{Lenz's law means } \mathcal{E}_L \text{ will} \\ \text{oppose } \Delta \Phi_B \text{ and induce} \\ \text{'back-current' + emf.} \end{array} \right.$

$$\Phi_B \propto B \propto I; \quad \boxed{\mathcal{E}_L = -L \frac{dI}{dt}}$$

"inductance"

Inductance

20.5

- constant of proportionality relating induced emf and change in current
- a measure of how much emf induced by loop (or coil - see later)

units: 'henrys' $\boxed{1\text{H} = 1\text{V}\cdot\text{s}/\text{A}}$

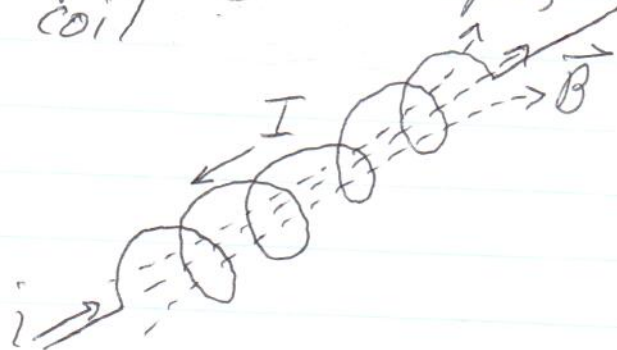
Causes a slow down in change in current, I

$$\underline{I = I_e(\tau) + i_l(\tau)}$$

Inductors

20.6

We can increase inductance by multiplying # loops, as for coil



Giving us

$$\underline{\underline{\mathcal{E}_L = -N \frac{d\phi_B}{dt} = -L \frac{dI}{dt}}}$$

Equating both sides +
integrating over time

$$N\phi_B = LI$$

$$\therefore \boxed{L = N\phi_B/I}$$

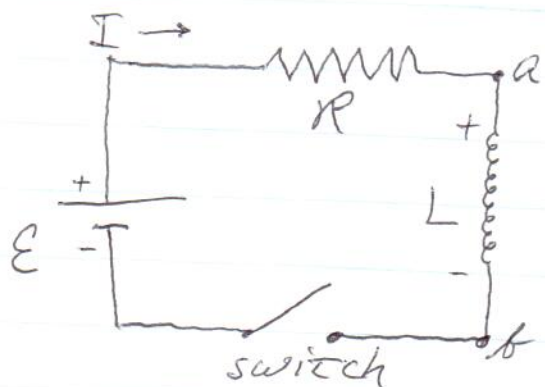
An 'inductor' has large self-inductance, L

- opposes changes in current in a circuit

Symbol: 

RL Circuits

20.7



When close switch
 I increases, $\frac{dI}{dt} > 0$

$$\therefore \mathcal{E}_L = -L \frac{dI}{dt} < 0$$

Electric potential
decreases over $a \rightarrow b$

From Kirchoff's loop rule:

$$\frac{\mathcal{E}}{R} - \frac{IR}{R} - \frac{L dI/dt}{R} = 0$$

Using $X = \mathcal{E}/R - I$, we have

$$X = -\frac{L}{R} \frac{dX}{dt}$$

$$\frac{R}{L} dt = -\frac{dX}{X}$$

Integrating

$$\int_0^t \frac{R}{L} dt = -\int_{X_0}^X \frac{dX}{X}$$

We then have

$$-\frac{R}{L} \tau = \ln\left(\frac{x}{x_0}\right)$$

Exponentiating gives

$$x = x_0 e^{-R\tau/L} \quad (\text{where } \tau = L/R)$$

$$\frac{\mathcal{E}}{R} - I = \left(\frac{\mathcal{E}}{R} - I_0\right) e^{-R\tau/L}$$

$$-I = -\frac{\mathcal{E}}{R} + \frac{\mathcal{E}}{R} e^{-R\tau/L}$$

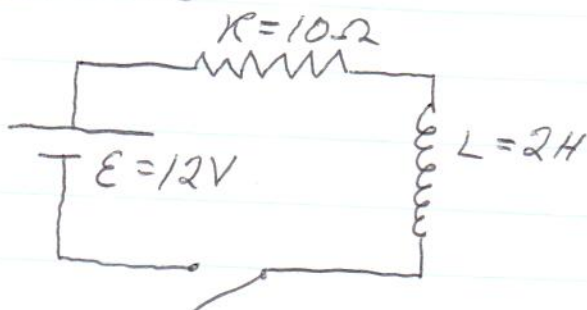
$$I = \left(\frac{\mathcal{E}}{R}\right) (1 - e^{-R\tau/L})$$

└ equilibrium value
of the current

Example

20.9

RL circuit



How long until current reaches 50% of its maximum value?

$$\frac{I}{I_{\max}} = 0.5 = 1 - e^{-Rt/L}$$

$$0.5 = e^{-Rt/L}$$

$$\ln(0.5) = -Rt/L$$

$$0.69 = Rt/L$$

$$\underline{\underline{T}} = 0.69\tau = 0.69\left(\frac{2H}{10\Omega}\right) \\ = \underline{\underline{0.14s}}$$

Energy Stored

20.10

Use loop rule on RL circuit
- quantify energy stored
in inductor

$$\mathcal{E} - IR - L \frac{dI}{dt} = 0$$

$$I\mathcal{E} - I^2 R = IL \frac{dI}{dt}$$

In terms of Power

$$P_L = P_{\text{tot}} (= I\mathcal{E}) - P_R (= I^2 R) = \frac{dU}{dt}$$

So

$$\frac{dU}{dt} = LI \frac{dI}{dt} \quad \text{- rate energy stored}$$

Integrating gives

$$\int dU = \int_0^I LI dI$$

Total
Energy
Stored

$$U = \frac{1}{2} LI^2$$