Gauss's Law for Magnetic Fields

In electrical phenomena

$$\oint \vec{\Phi}\cdot d\vec{A} = q/\epsilon_0$$

- flux related to presence of charge

What 'charge' corresponds to magnetism?

N or S poles

- but we never see isolated "magnetic monopoles"

- a big mystery in physics

By same logic that gave us Gauss's Law, we have

$$\boxed{\oint \vec{\Phi}_B\cdot d\vec{A} = 0}$$

the flux thru any closed surface A is zero.

Induced Magnetic Fields

Faraday's law differs from Ampere's Law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \underline{\text{Faraday's}}$$

– vs –

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad \underline{\text{Ampere's}}$$

Why doesn't Faraday's look like Ampere's? No monopoles means no "magnetic current", so term is zero.

But why no induction term to Ampere's? eg. $\oint \vec{B}\cdot d\vec{l} \propto \frac{d\Phi_E}{dt}$

It's possible + was suggested by Maxwell!

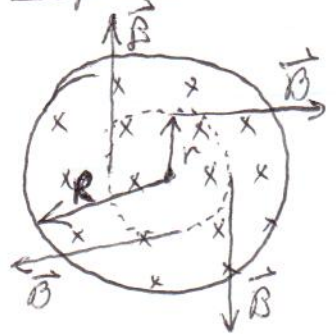
Ampere-Maxwell Law

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}}$$

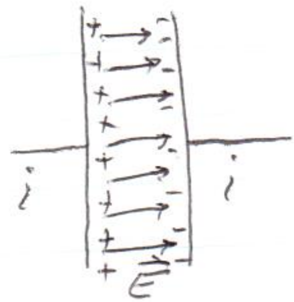
"a changing ~~magnetic~~ electric field can induce magnetic field."

Experiment: Charging a Capacitor

top view:



Side view:



Consider case where vary $|E|$ at a constant rate, $\Delta E/\Delta t$.

$r \le R$ case

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}(E(\pi r^2))$$

$$\boxed{B = \tfrac{1}{2}\mu_0 \epsilon_0 \, r \, dE/dt}$$

- - - - - -

$r > R$ case

$$B(2\pi r) = \mu_0 \epsilon_0 \frac{d}{dt}(E(\pi R^2))$$

$$\boxed{B = \mu_0 \epsilon_0 \frac{R^2}{2r} \, dE/dt}$$

If $r = R = 50\text{mm}$ and $\Delta E/\Delta t = 10^{12}\, V/m^2$

$$\boxed{B = \tfrac{1}{2}\mu_0 \epsilon_0 R \frac{\Delta E}{\Delta t} = 0.0028 \text{ gauss}}$$

↳ induced field very small

But measureable! Obeys induction Law.

Maxwells Equations

Fundamental electrical + magnetic relationships:

1) $\oint \vec{E} \cdot d\vec{A} = q_{in}/\epsilon_0$ — Gauss's Law
   - E field from charge distribution

2) $\oint \vec{B} \cdot d\vec{A} = 0$
   - no magnetic monopoles

3) $\oint \vec{E} \cdot d\vec{l} = -d\phi_B/dt$ — Faraday's Law
   - changing B-field gives E field

4) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{dE}{dt}$ — Ampere-Maxwell Law
   - moving charge or changing E field produces B-field

Describe all known electric + magnetic phenomena.

Wave Equations

Faraday's Law & the induction piece of Ampere's Law

- reveal connection between variation of E + B fields

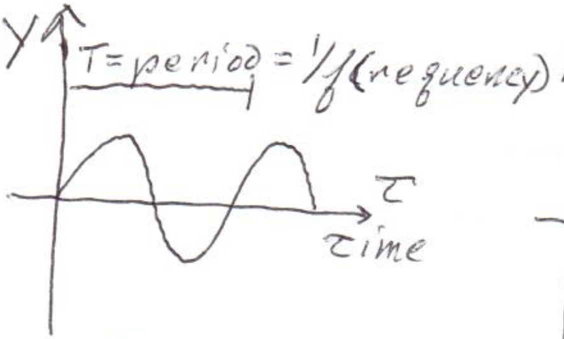
One can show that we can derive the following relation:

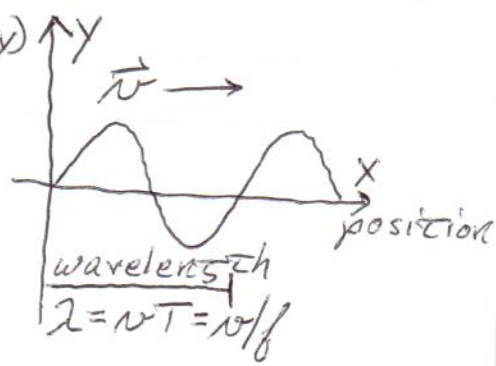
$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$

You've seen this structure before:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

wave velocity $\leftarrow\!\!\lrcorner = \sqrt{\omega/k} = \sqrt{\lambda f}$





$\lambda = \frac{2\pi}{k} \rightarrow$ wave #

$\omega = 2\pi f$

$\rightarrow$ angular frequency

Travelling Electromagnetic Waves

So in $\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$, we describe an electromagnetic wave

wave velocity $\frac{1}{v^2} = \mu_0 \epsilon_0$

$\therefore \boxed{v_{EM} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$

Speed of electromagnetic wave can be calculated from purely electric + magnetic constants

$$v_{EM} = \boxed{c = 3 \times 10^8 \, m/s}$$

this is the speed of light!!

- Light is a form of EM wave
- Electromagnetism 'unified'
- also subsume field of Optics

Note: Maxwell's Eqs consistent with Special Relativity. Newton's Laws are not.

Electromagnetic Spectrum 22.7

Maxwell's Eq: 'predict' speed of EM waves (light), but do not specify λ or ω.

Different names based on λ:

Names	λ range	notes
Radio	$>0.1\,m$	-radio, radar
Micro-wave	$10^{-4}\,m - 10^{-1}\,m$	-appliances
infrared	$7\times10^{-7}\,m - 10^{-4}\,m$	-room temperature -rotational, vibrational atoms
visible	$4\times10^{-7}\,m - 7\times10^{-7}\,m$	-we're most sensitive to yellow
ultra-violet	$6\times10^{-10}\,m - 4\times10^{-7}$	-Sun
X-rays	$10^{-12}\,m - 10^{-8}\,m$	
γ-rays	$<10^{-10}\,m$	-nuclear decay -gamma-ray bursts