Energy Transport + the Poynting Vector

We have already encountered potential energy stored in electric + magnetic fields.

For EM wave, how much energy is delivered?

instantaneous value

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$

Poynting Vector

→ Energy-per area per time
→ in direction of travel of EM wave

or Power per area

Magnitude always

$$S = \frac{1}{\mu_0} E B$$

since $\vec{E} \perp \vec{B}$ for EM waves

- since $c = E/B$, we also have

$$S = \frac{E^2}{2\mu_0 c} = \frac{c B^2}{2\mu_0}$$

Intensity of EM Wave

Generally we do not care about the rapid variation in power for oscillations of fields in EM wave.

- Calculate average quantities

$$\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}} \; ; \; B_{rms} = \frac{B_{max}}{\sqrt{2}}$$

Substituting gives:

$$I (\equiv S_{avg}) = \frac{1}{\mu_0}\left(\frac{\mathcal{E}_{max}}{\sqrt{2}}\right)\left(\frac{B_{max}}{\sqrt{2}}\right)$$

$$\boxed{I = \frac{\mathcal{E}_{max} B_{max}}{2\mu_0}}$$

Since $c = \mathcal{E}/B$, we can also write

$$I = \frac{c B_{max}^2}{2\mu_0} = \frac{\mathcal{E}_{max}^2}{2\mu_0 c}$$

Energy Density

We would like to compare the energy per volume in the $\mathcal{E}$ and B field components.

$$\underline{u_E = \frac{1}{2}\epsilon_0 \mathcal{E}^2}$$ (from our study of capacitors)

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{(inductors)}$$

$$= \frac{(\mathcal{E}/c)^2}{2\mu_0} = \underline{\underline{\frac{1}{2}\epsilon_0 \mathcal{E}^2}}$$

So $|u_E| = |u_B|$ and the total is

$$u = u_E + u_B = \epsilon_0 \mathcal{E}^2 = B^2/\mu_0$$

The average is

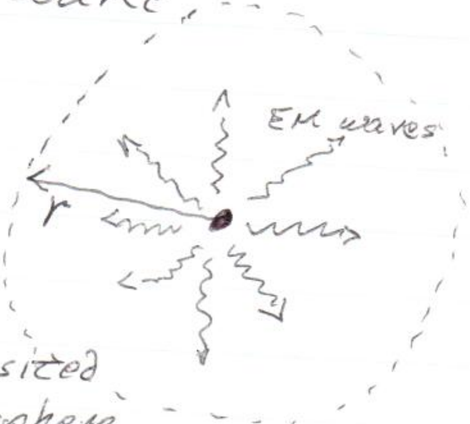
$$\boxed{\begin{aligned} u_{avg} &= \epsilon_0 \mathcal{E}_{max}^2/2 \\ &= B_{max}^2/2\mu_0 \end{aligned}}$$

$$\therefore \quad \underline{\underline{I = c\,u_{avg}}}$$

Variation of Intensity with Distance

How does power get distributed at a distance from source of EM waves?

Consider point source:



- have a sphere at radius, r
- all energy in wave @ source
  → must be deposited on surface of sphere
  - energy is 'spread out' as r gets larger

$$I = \frac{\text{power delivered}}{\text{area of sphere}} = \underline{\underline{\frac{P_s}{4\pi r^2}}}$$

this is why lights get dim the further away you are.

Example:

Calculate the energy density in sunlight. It's intensity is $1kW/m^2$.

$$u_{avg} = I/c$$

$$= \frac{10^3 W/m^2}{3\times10^8 m/s}$$

$$= \frac{1}{3}\times10^{-5} \frac{W/\cancel{m}^2}{\cancel{m}/s}$$

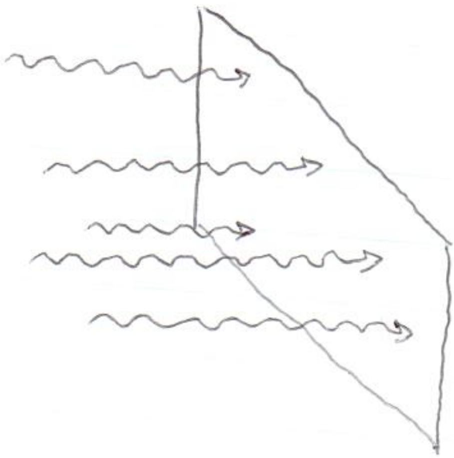
$$= \boxed{3.3\times10^{-6} J/m^3}$$

Radiation Pressure

Light has energy

Consider surface $\perp$ to direction of travel of wave

there is momentum associated with light



Total energy transferred to surface in time interval, Δt, is ΔU.

If complete absorption:

- momentum trasferred by radiation

$$\Delta U = c\Delta p$$

$$\therefore \boxed{\Delta p = \frac{\Delta U}{c}}$$

Radiation Pressure (cont.)

To calculate the force on the area, consider intensity

$$I = \frac{\text{energy/time}}{\text{area}}$$

$$\Delta U = I A \Delta t$$

Since force $= dp/dt$,

$$F = \frac{I A \cancel{\Delta t}}{c \cancel{\Delta t}} = \frac{IA}{c}$$

the pressure is then

$$\boxed{p_r} = F/A = \boxed{I/c}$$

For total reflection,

$$\boxed{\Delta p = 2\Delta U/c}$$

and

$$\boxed{p_r = 2I/c}$$

Example:

An intense light delivers $25\,W/m^2$ to a facing wall.

What pressure is exerted on the wall?

$$P_r = S_{avg}/c$$

$$= \frac{2.5\times 10}{3\times 10^{8}} \frac{W}{m^2\cdot m/s}$$

$$= \frac{2.5}{3}\times 10^{-7}\,\frac{N}{m^2}$$

$$\approx \boxed{8\times 10^{-8}\,N/m^2}$$