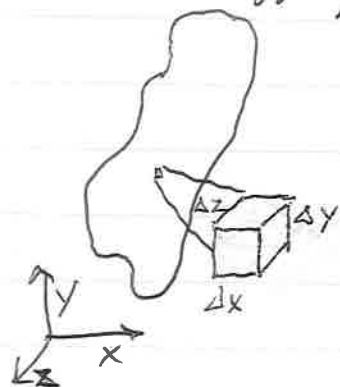


Electric Field of a 4.1 Continuous Charge Distribution

Continuous charge distributions

- effectively result when have
huge # of charges



Each small region with
 $\Delta x \Delta y \Delta z$ volume

- holds Δq charge

Total $\vec{E}$ -field by summing

$$\vec{E} \approx k \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

As $\Delta q \rightarrow 0$,

$$\boxed{\vec{E} = k \int \frac{dq}{r^2} \hat{r}}$$

Charge Distributions

4.2

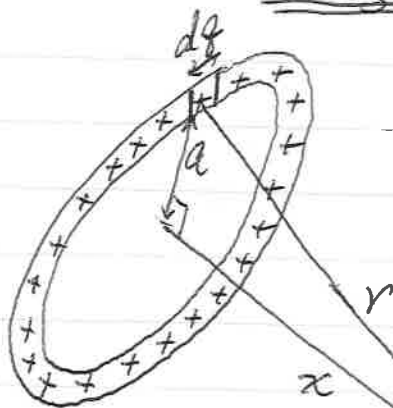
Charge is often distributed uniformly in a 1, 2 or 3 dimensional way

	<u>charge density</u> *	<u>dq</u>
line, l	$\lambda = Q/l$	λdl
surface, A	$\sigma = Q/A$	σdA
volume, V	$\rho = Q/V$	ρdV

* if Q is distributed uniformly, $Q = \text{total charge}$

Electric field from a Ring of Charge

4.3



$Q = \text{total charge}$

What is $\vec{E}$ -field
at P a distance
 x from ring's
center on its axis?



We need to sum $d\vec{E}$ vectorially
for all elements dq !

- components $\perp$ to x cancel

- components $\parallel$ to x !

$$dE \cos \theta = k \frac{dq}{r^2} \cos \theta$$

$$= k \frac{dq}{(a^2 + x^2)} \frac{x}{\sqrt{a^2 + x^2}}$$

$$= \underline{\underline{k x dq / [a^2 + x^2]^{3/2}}}$$

4.4

Normally, we would integrate over a linear charge density λ .

But we have a finite circumference with known total charge.

Since x and a are constant for all dq , we have

$$\begin{aligned}\underline{\underline{E}} &= \int dE \cos \theta \\ &= \int k \frac{x \, dq}{[a^2 + x^2]^{3/2}} = \frac{kx}{[a^2 + x^2]^{3/2}} \underbrace{\int dq}_{Q} \\ &= \boxed{k \frac{xQ}{[a^2 + x^2]^{3/2}}}\end{aligned}$$

Note: when $x \gg a$

$$\underline{\underline{E \approx k \frac{Q}{x^2}}} \quad \therefore \text{acts like a point charge.}$$

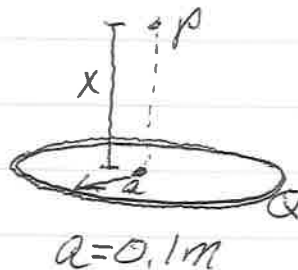
Example

A ring of charge lies flat on a table with total charge $Q = 0.1 \mu\text{C}$ and radius 0.1 m . What is the electric field 1 m from the ring?

Is $a \ll x$?

Not sure:

So use more general expression



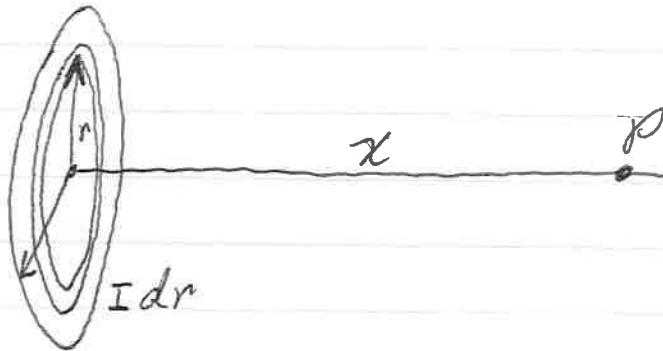
$$\begin{aligned}
 E &= k \frac{x Q}{[a^2 + x^2]^{3/2}} \\
 &= 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2} \frac{(1 \text{ m} \cdot 0.1 \mu\text{C})}{[1.01 \text{ m}^2]^{3/2}} \\
 &= 9 \times 10^3 \frac{\text{N}}{\text{C}} / 1.015 \\
 &= \boxed{8.91 \times 10^3 \frac{\text{N}}{\text{C}}}
 \end{aligned}$$

So using 'a' in denominator only makes 1.5% difference

Electric Field from a Charged Disk

4.5

We can consider a charged disk as a series of concentric charged rings.



We know the field for each ring

$$dE = k \frac{x dq}{[r^2 + x^2]^{3/2}}$$

To determine the E-field from the disk

- integrate from $r=0$ to R

Using a uniform charge
~~per~~ per area, σ , we can
 write

$$dq = \sigma dA = \sigma(2\pi r)(dr)$$

σ per ring

Setting up the integral

$$\begin{aligned} \underline{E} &= \int k \frac{\sigma(2\pi r)dr}{[r^2 + x^2]^{3/2}} \\ &= \frac{\sigma(2\pi)}{4\pi\epsilon_0} x \int_0^R \frac{2r dr}{[r^2 + x^2]^{3/2}} \\ &= \frac{\sigma x}{4\epsilon_0} \left[\frac{(r^2 + x^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R \\ &= -\frac{\sigma x}{4\epsilon_0} \left[\frac{+1}{\sqrt{R^2 + x^2}} - \frac{1}{x} \right] \\ &= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2 + x^2}} \right] \end{aligned}$$

If we go to very large distance
 $R \gg x$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

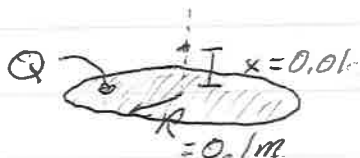
E-field from
 infinite
 sheet

Example

4.7

Calculate the electric field from a uniformly charged disk a distance along the disk axis of 1cm. The disk has $Q = 5\mu\text{C}$ and radius 0.1m.

$x \ll R$



Calculate charge density

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi R^2}$$
$$= \frac{5 \times 10^{-6} \text{ C}}{\pi (0.01 \text{ m})^2}$$

$$\underline{\underline{\sigma = 1.5 \times 10^{-4} \frac{\text{C}}{\text{m}^2}}}$$

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{R^2 + x^2}} \right]$$

$$= \frac{1.5 \times 10^{-4} \frac{\text{C}}{\text{m}^2}}{2(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2})} \left[1 - \frac{0.01 \text{ m}}{\sqrt{0.0101 \text{ m}^2}} \right]$$

need to check

$$= 8.5 \times 10^3 \frac{\text{N}}{\text{C}} [1 - 0.995]$$

$$\underline{\underline{E = 7.7 \times 10^3 \frac{\text{N}}{\text{C}}}}$$

so edge effects
10.9%!!